

PROPERTIES OF UNSATURATED  
POROUS MEDIA

by

G. E. Laliberte, A. T. Corey  
and R. H. Brooks

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## ABSTRACT

Some of the problems associated with satisfying the scaling criteria of the Brooks-Corey modeling theory for partially saturated porous media are examined critically.

The effect of the porosity of disturbed media on the hydraulic properties which are significant in the modeling theory is determined experimentally by varying porosity. The results indicate that the pore-size distribution index is changed only slightly over a wide range of porosities but permeability and bubbling pressure may be changed several fold over the same range. Evidently, bubbling pressure and permeability may be adjusted to suit the size of the model by changing the porosity without appreciably changing the pore-size distribution.

A functional relationship among the hydraulic properties which are significant in the modeling theory is developed, beginning with the fundamental equations used in the Burdine approach for relating permeability, saturation and capillary pressure. The relationship, involving saturated permeability, effective porosity, bubbling pressure and pore-size distribution index, is substantiated experimentally for three media. The relationship can be used to estimate permeability as a function of either capillary pressure or saturation, the only information required being capillary pressure-desaturation data.

A study was made to determine if disturbed materials are suitable for modeling undisturbed porous media. The results indicate that a medium obtained by pulverizing the material at the site of the prototype would usually not be suitable for a model because of changes in the pore-size distribution during pulverization. Unconsolidated media are available, however, which have the range of properties necessary for modeling any of the undisturbed media found in this study. The most difficult problem may be to simulate the transition from saturated conditions to partially saturated conditions which evidently is characteristic of some undisturbed media.

## PREFACE

Colorado State University's contribution to W-51 Regional Research Project entitled "Factors Influencing the Flow of Subsoil Water in the Immediate Proximity of and into Drainage Facilities" includes a study of the possibility of using physical models of field drainage systems. Work presented in Hydrology Paper No. 9 indicated that the theory of similitude proposed by Brooks and Corey in Hydrology Paper No. 3 was valid and could be used as a basis for constructing models of subsoil drains.

The study presented herein was conducted to delineate and help solve some of the obvious practical problems encountered in modeling actual field systems involving flow in partially saturated porous media. Additional details of this study have been presented in the senior author's dissertation with the same title, presented at Colorado State University in August 1966.

The authors are grateful to Dr. Arnold Klute, Professor of Soil Physics, Department of Agronomy, University of Illinois for his critical review of this paper and his many helpful suggestions.

## TABLE OF CONTENTS

	<u>Page</u>
LIST OF FIGURES . . . . .	vi
LIST OF TABLES (TEXT) . . . . .	vii
LIST OF TABLES (APPENDICES) . . . . .	viii
LIST OF SYMBOLS . . . . .	xi
INTRODUCTION . . . . .	1
BACKGROUND AND THEORY . . . . .	3
EXPERIMENTAL TECHNIQUES . . . . .	5
Fluids and Media . . . . .	5
Capillary Pressure-Permeability Measurement for the Porosity Experiment . . . . .	5
Capillary Pressure-Saturation Measurement for the Porosity Experiment . . . . .	6
Capillary Pressure-Permeability Measurement for the Undisturbed Media Experiment . . . . .	8
RESULTS AND DISCUSSION . . . . .	10
Porosity Experiment . . . . .	10
Undisturbed Media Experiment . . . . .	24
CONCLUSIONS . . . . .	31
BIBLIOGRAPHY . . . . .	32
APPENDIX A. Properties of Wetting Fluid and Media . . . . .	33
APPENDIX B. Capillary Pressure-Permeability Data . . . . .	34
APPENDIX C. Capillary Pressure-Desaturation Data . . . . .	39

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Schematic diagram of saturation-capillary pressure apparatus. . .	7
2	Sleeve-type sampler (ready for assembly) . . . . .	8
3	Undisturbed soil sample during test showing inflow and outflow pressure controllers and tensiometers . . . . .	8
4	Relative permeability as a function of capillary pressure for Touchet silt loam (GE 3) packed at five different porosities. . . .	11
5	Relative permeability as a function of capillary pressure for Columbia sandy loam packed at five different porosities . . . . .	11
6	Relative permeability as a function of capillary pressure for an unconsolidated sand packed at five different porosities . . . .	12
7	Saturation and effective saturation as functions of capillary pressure for Touchet silt loam (GE 3) . . . . .	13
8	Saturation and effective saturation as functions of capillary pressure for Touchet silt loam (GE 3) . . . . .	13
9	Saturation and effective saturation as functions of capillary pressure for Touchet silt loam (GE 3) . . . . .	14
10	Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam . . . . .	14
11	Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam . . . . .	15
12	Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam . . . . .	15
13	Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam . . . . .	16
14	Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand . . . . .	16
15	Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand . . . . .	17
16	Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand . . . . .	17
17	Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand . . . . .	18
18	Saturated permeability as a function of porosity for three disturbed media . . . . .	18
19	Bubbling pressure as a function of porosity for three disturbed media . . . . .	19
20	Pore-size distribution index as a function of porosity for three disturbed media . . . . .	19

LIST OF FIGURES - continued

<u>Figure</u>		<u>Page</u>
21	Permeability as a function of capillary pressure for Fort Collins clay loam . . . . .	25
22	Permeability as a function of capillary pressure for Weld loam . . . . .	25
23	Permeability as a function of capillary pressure for Cass sandy loam (5-inch depth) . . . . .	26
24	Permeability as a function of capillary pressure for Cass sandy loam (12-inch depth) . . . . .	26
25	Permeability as a function of capillary pressure for Cass sandy loam (20-inch depth) . . . . .	27
26	Permeability as a function of capillary pressure for Valentine loamy sand . . . . .	27
27	Permeability as a function of capillary pressure for a semi-consolidated sand . . . . .	28

LIST OF TABLES (TEXT)

<u>Table</u>		<u>Page</u>
1	POROSITY, SATURATED PERMEABILITY, BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX AS FUNCTIONS OF BULK DENSITY FOR THREE DISTURBED MEDIA . . . . .	20
2	POROSITY, BUBBLING PRESSURE, PORE-SIZE DISTRIBUTION INDEX, RESIDUAL SATURATION AND EFFECTIVE POROSITY AS FUNCTIONS OF BULK DENSITY FOR THREE DISTURBED MEDIA . . . . .	21
3	COMPARISON OF BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX DETERMINED FROM $K(p_c)$ AND $S(p_c)$ DATA FOR THREE DISTURBED MEDIA . . . . .	22
4	SUMMARY OF HYDRAULIC PROPERTIES USED IN CALCULATING THE PRODUCT $\frac{\phi_e \sigma^2}{K_o p_b^2} \left( \frac{\lambda}{\lambda + 2} \right)$ FOR THREE DISTURBED MEDIA ( $\sigma = 22.9$ DYNES/CM) . . . . .	23
5	POROSITY, SATURATED PERMEABILITY, BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX FOR SEVERAL MEDIA . . . . .	29
6	POROSITY, SATURATED PERMEABILITY, BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX FOR CASS SANDY LOAM . . . . .	29

LIST OF TABLES (APPENDICES)

<u>Table</u>	<u>Page</u>
A-1	DYNAMIC VISCOSITY AND DENSITY OF SOLTROL "C" OIL USED IN EXPERIMENTS . . . . . 33
A-2	GRAIN-SIZE ANALYSIS OF MATERIALS USED. . . . . 33
A-3	PARTICLE DENSITY OF MATERIALS USED. . . . . 33
B-1	CAPILLARY PRESSURE-PERMEABILITY DATA FOR TOUCHET SILT LOAM (GE 3) . . . . . 34
B-2	CAPILLARY PRESSURE-PERMEABILITY DATA FOR TOUCHET SILT LOAM (GE 3) . . . . . 34
B-3	CAPILLARY PRESSURE-PERMEABILITY DATA FOR TOUCHET SILT LOAM (GE 3) . . . . . 34
B-4	CAPILLARY PRESSURE-PERMEABILITY DATA FOR TOUCHET SILT LOAM (GE 3) . . . . . 34
B-5	CAPILLARY PRESSURE-PERMEABILITY DATA FOR TOUCHET SILT LOAM (GE 3) . . . . . 34
B-6	CAPILLARY PRESSURE-PERMEABILITY DATA FOR COLUMBIA SANDY LOAM. . . . . 34
B-7	CAPILLARY PRESSURE-PERMEABILITY DATA FOR COLUMBIA SANDY LOAM . . . . . 34
B-8	CAPILLARY PRESSURE-PERMEABILITY DATA FOR COLUMBIA SANDY LOAM . . . . . 34
B-9	CAPILLARY PRESSURE-PERMEABILITY DATA FOR COLUMBIA SANDY LOAM . . . . . 35
B-10	CAPILLARY PRESSURE-PERMEABILITY DATA FOR COLUMBIA SANDY LOAM. . . . . 35
B-11	CAPILLARY PRESSURE-PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND . . . . . 35
B-12	CAPILLARY PRESSURE-PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND . . . . . 35
B-13	CAPILLARY PRESSURE-PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND . . . . . 35
B-14	CAPILLARY PRESSURE-PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND . . . . . 35
B-15	CAPILLARY PRESSURE-PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND . . . . . 35
B-16	CAPILLARY PRESSURE-PERMEABILITY DATA FOR FORT COLLINS CLAY LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY) . . . . . 35
B-17	CAPILLARY PRESSURE-PERMEABILITY DATA FOR FORT COLLINS CLAY LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY) . . . . . 36



LIST OF TABLES (APPENDICES) - continued

<u>Table</u>		<u>Page</u>
B-18	CAPILLARY PRESSURE-PERMEABILITY DATA FOR FORT COLLINS CLAY LOAM (DISTURBED SAMPLE PASSED THROUGH A NO. 14 SIEVE) . . . . .	36
B-19	CAPILLARY PRESSURE-PERMEABILITY DATA FOR FORT COLLINS CLAY LOAM (DISTURBED SAMPLE PASSED THROUGH A NO. 48 SIEVE) . . . . .	36
B-20	CAPILLARY PRESSURE-PERMEABILITY DATA FOR WELD LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY) . . . . .	36
B-21	CAPILLARY PRESSURE-PERMEABILITY DATA FOR WELD LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY) . . . . .	36
B-22	CAPILLARY PRESSURE-PERMEABILITY DATA FOR WELD LOAM (DISTURBED SAMPLE PASSED THROUGH A NO. 14 SIEVE) . . . . .	36
B-23	CAPILLARY PRESSURE-PERMEABILITY DATA FOR WELD LOAM (DISTURBED SAMPLE PASSED THROUGH A NO. 48 SIEVE) . . . . .	36
B-24	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY AT 5-INCH DEPTH) . . . . .	36
B-25	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 5-INCH DEPTH) . . . . .	37
B-26	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY AT 12-INCH DEPTH) . . . . .	37
B-27	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 12-INCH DEPTH) . . . . .	37
B-28	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 14 SIEVE) . . . . .	37
B-29	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 48 SIEVE) . . . . .	37
B-30	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY AT 20-INCH DEPTH) . . . . .	37
B-31	CAPILLARY PRESSURE-PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 20-INCH DEPTH) . . . . .	37

LIST OF TABLES (APPENDICES) - continued

<u>Table</u>		<u>Page</u>
B-32	CAPILLARY PRESSURE-PERMEABILITY DATA FOR VALENTINE LOAMY SAND (UNDISTURBED SAMPLE TAKEN VERTICALLY AT 12-INCH DEPTH) . . . . .	38
B-33	CAPILLARY PRESSURE-PERMEABILITY DATA FOR VALENTINE LOAMY SAND (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 12-INCH DEPTH) . . . . .	38
B-34	CAPILLARY PRESSURE-PERMEABILITY DATA FOR VALENTINE LOAMY SAND (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 14 SIEVE) . . . . .	38
B-35	CAPILLARY PRESSURE-PERMEABILITY DATA FOR VALENTINE LOAMY SAND (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 48 SIEVE) . . . . .	38
B-36	CAPILLARY PRESSURE-PERMEABILITY DATA FOR A SEMI-CONSOLIDATED SAND (UNDISTURBED CORE) . . . . .	38
B-37	CAPILLARY PRESSURE-PERMEABILITY DATA FOR A SEMI-CONSOLIDATED SAND (DISTURBED SAMPLE PASSED THROUGH A NO. 14 SIEVE) . . . . .	38
B-38	CAPILLARY PRESSURE-PERMEABILITY DATA FOR A SEMI-CONSOLIDATED SAND (DISTURBED SAMPLE PASSED THROUGH A NO. 35 SIEVE) . . . . .	38
C-1	CAPILLARY PRESSURE-DESATURATION DATA FOR TOUCHET SILT LOAM (GE 3) . . . . .	39
C-2	CAPILLARY PRESSURE-DESATURATION DATA FOR TOUCHET SILT LOAM (GE 3) . . . . .	39
C-3	CAPILLARY PRESSURE-DESATURATION DATA FOR TOUCHET SILT LOAM (GE 3) . . . . .	39
C-4	CAPILLARY PRESSURE-DESATURATION DATA FOR COLUMBIA SANDY LOAM . . . . .	39
C-5	CAPILLARY PRESSURE-DESATURATION DATA FOR COLUMBIA SANDY LOAM . . . . .	39
C-6	CAPILLARY PRESSURE-DESATURATION DATA FOR COLUMBIA SANDY LOAM . . . . .	40
C-7	CAPILLARY PRESSURE-DESATURATION DATA FOR COLUMBIA SANDY LOAM . . . . .	40
C-8	CAPILLARY PRESSURE-DESATURATION DATA FOR AN UNCONSOLIDATED SAND . . . . .	40
C-9	CAPILLARY PRESSURE-DESATURATION DATA FOR AN UNCONSOLIDATED SAND . . . . .	40
C-10	CAPILLARY PRESSURE-DESATURATION DATA FOR AN UNCONSOLIDATED SAND . . . . .	40
C-11	CAPILLARY PRESSURE-DESATURATION DATA FOR AN UNCONSOLIDATED SAND . . . . .	40

LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
a	Constant in Gardner's equation for permeability . . . . .	varies
b	Constant in Gardner's equation for permeability . . . . .	varies
b	Subscript meaning "bubbling" . . . . .	none
b	Subscript meaning "bulk" . . . . .	none
c	Subscript meaning "capillary" . . . . .	none
°C	Degrees centigrade . . . . .	none
d	Differential operator . . . . .	none
e	Subscript meaning "effective" . . . . .	none
g	Acceleration due to gravity . . . . .	LT <sup>-2</sup>
k	Shape factor in the Kozeny-Carman equation . . . . .	none
K	Permeability - the permeability to the wetting fluid when the medium is occupied by more than one fluid phase . . . . .	L <sup>2</sup>
K <sub>o</sub>	Saturated permeability - the permeability to the wetting fluid at complete saturation . . . . .	L <sup>2</sup>
K <sub>r</sub>	Relative permeability - K/K <sub>o</sub> . . . . .	none
m	General subscript . . . . .	none
mb	Millibar, a unit of pressure . . . . .	FL <sup>-2</sup>
n	Positive constant for a medium in Gardner's equation for permeability . . . . .	none
N	Number of determinations of capillary pressure and relative permeability used in calculating $\eta$ . . . . .	none
N	Number of determinations of capillary pressure and effective saturation used in calculating $\lambda$ . . . . .	none
o	Subscript meaning "at complete saturation" . . . . .	none
p <sub>b</sub>	Bubbling pressure - approximately the minimum capillary pressure on the drainage cycle at which the non-wetting fluid is continuous . . . . .	FL <sup>-2</sup>
p <sub>c</sub>	Capillary pressure - the difference in pressure across the interface between the wetting fluid and the non-wetting fluid . . . . .	FL <sup>-2</sup>
q	Volume flux . . . . .	LT <sup>-1</sup>
S	Saturation - the ratio of the volume of the wetting fluid to the volume of the voids . . . . .	none
S <sub>e</sub>	Effective saturation - $(S - S_r)/(1 - S_r)$ . . . . .	none

LIST OF SYMBOLS - continued

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
$S_r$	Residual saturation - the saturation at which K approaches zero . . . . .	none
t	Time . . . . .	T
T	Tortuosity $(L_e/L)^2$ at complete saturation . . . . .	none
$\Delta$	Denotes a difference . . . . .	none
$\eta$	Pore-size distribution index, $-d(\log K) / d(\log p_c)$ . . . . .	none
$\theta$	Contact angle of interface between wetting and non-wetting fluid at the medium solids . . . . .	none
$\lambda$	Pore-size distribution index, $-d(\log S_e) / d(\log p_c)$ . . . . .	none
$\mu$	Dynamic viscosity of wetting fluid . . . . .	$FL^{-2}T$
$\mu^2$	Square microns, a unit of permeability . . . . .	$L^2$
$\rho$	Density of wetting fluid . . . . .	$FL^{-4}T^2$
$\rho_b$	Bulk density of the porous medium . . . . .	$FL^{-4}T^2$
$\rho_s$	Particle density - specific weight of the medium solids . . . . .	$FL^{-4}T^2$
$\sigma$	Surface tension (interfacial tension) of the wetting fluid . . . . .	$FL^{-1}$
$\Sigma$	Summation . . . . .	none
$\phi$	Porosity - the volume of the pore space expressed as a decimal fraction of the bulk volume of the medium . . . . .	none
$\phi_e$	Effective porosity $(1 - S_r)\phi$ . . . . .	none
$\nabla$	Gradient operator . . . . .	$L^{-1}$

## PROPERTIES OF UNSATURATED POROUS MEDIA

G. E. Laliberte, A. T. Corey and R. H. Brooks\*

### INTRODUCTION

In 1931, Richards (18) introduced an equation combining Darcy's law and the continuity condition. This second order non-linear differential equation describes steady and unsteady flow in both saturated and partially saturated media. Because of the complexity of the mathematical analysis involved, Richards' equation has remained unsolved except for simple cases.

When physical phenomena do not yield to mathematical analysis, the use of models often permits an insight into them. At present, however, the use of models for solving particular problems that deal with partially saturated media is practically nonexistent. Many investigators interested in the solution of groundwater problems have used sand tanks and Hele-Shaw models that do not adequately take into consideration partially saturated conditions. In 1956, Miller and Miller (14) presented a theory describing the criteria of similitude for flow in partially saturated porous media. Their theory and analagous theories developed by investigators in the petroleum industry have been reviewed by Corey et al. (8).

Recently, similitude requirements have been specified by Brooks and Corey (3) for modeling unsteady flow in partially saturated systems. They developed the theory by scaling the Richards equation with system parameters of length, pressure and time. The permeability of the fully saturated medium,  $K_0$ , was chosen as the parameter for scaling permeability,  $K$ . The bubbling pressure,  $p_b$ , of the particular medium and fluid was proposed for scaling capillary pressure,  $p_c$ . Bubbling pressure was found by Brooks and Corey to be closely related to the largest pores forming a continuous network within a porous medium. It is a constant for a given fluid and medium.

The exact method of determination of bubbling pressure is discussed in the section on experimental techniques. The length parameter selected was the bubbling pressure divided by the difference in the specific weights of the wetting and non-wetting fluid,  $\Delta\rho g$ . The time parameter necessary to complete the scaling theory is given by the expression  $p_b \mu \phi_e / K_0 (\Delta\rho g)^2$ . In this expression,  $\mu$  is the dynamic viscosity of the wetting fluid,  $\phi_e$  is the "effective" or "drainable" porosity of the medium and the other terms are as previously defined. For systems in which air is the non-wetting fluid, the specific weight of air may be neglected and  $\Delta\rho g$  may be replaced by  $\rho g$ , the specific weight of the wetting fluid.

In order for the model and prototype to behave similarly, the following scaling criteria must be met:

1. The functional relationships among scaled permeability, saturation and capillary pressure must be identical for both systems;
2. The macroscopic boundaries of the model must have a shape and orientation similar to those of the prototype;
3. The size of the model defined by a characteristic macroscopic dimension,  $L$ , must be such that  $\rho g L / p_b$  is the same as for the prototype;
4. The initial conditions in terms of scaled variables are identical in both systems, the scaling factor for time,  $t$ , being  $p_b \mu \phi_e / K_0 (\rho g)^2$ .

The last of these requirements can be eliminated for steady-state systems. Scott and

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Corey (20) used these criteria for modeling steady flow in sand columns. Their results established the validity of the theory for steady flow systems. Excellent verification of the theory for unsteady flow was obtained by Corey et al. (8) using drainage from two similar sand columns.

However, there are still many problems associated with satisfying these criteria that need to be overcome before the use of models for partially saturated media can be made practical. This study was initiated to resolve some of these difficulties.

The first criterion, that the functional relationships among scaled permeability, saturation and capillary pressure be identical in both systems, has been found by Brooks and Corey to be satisfied by media having similar pore-size distributions. They found that if two materials have similar scaled capillary pressure-permeability relationships they also will have similar scaled capillary pressure-saturation relationships. In fact, the absolute value of the negative slope of the straight line on a log-log plot of effective saturation as a function of capillary pressure, symbolized by  $\lambda$ , was called the pore-size distribution index. Evidently, the pore-size distribution index is a parameter of considerable significance in modeling partially saturated media.

According to the Brooks-Corey theory, significant porous medium properties (in addition to the bubbling pressure and pore-size distribution index) are the effective porosity and the saturated permeability. In fact, Corey (7) suggested that there must be some functional relationship among the parameters  $\phi_e$ ,  $K_o$ ,  $p_b$  and  $\lambda$ . Such a relationship must exist in a given medium at least, if bulk density is the only parameter which varies. Knowledge of such a relationship would permit calculation of unscaled permeability as a function of capillary pressure from only the capillary pressure-desaturation function for a medium. It also would provide an insight into probable effects of disturbing porous media insofar as disturbance changes the parameters indicated above.

In order to employ the modeling techniques of Brooks and Corey for studying prototype systems, it is necessary to measure the pertinent properties of the material as it exists undisturbed at the site. It is also necessary to locate a suitable medium for a model having a pore-size distribution similar to the medium at the site. In this connection, it would be desirable to determine whether or not the pore-size distribution of undisturbed earth materials can be characterized by the index proposed by Brooks and Corey.

This paper reports the results of a study designed to:

1. Develop theory for the purpose of defining a functional relationship among effective porosity, saturated permeability, bubbling pressure and pore-size distribution index;

2. Experimentally test the validity of this relationship over a wide range of soil textures by changing the bulk density;

3. Determine whether undisturbed earth materials have properties that can be modeled using disturbed earth materials and, in particular, to determine if the pore-size distribution of undisturbed materials can be characterized by a single dimensionless number as is the case for most disturbed materials;

4. Determine the range of effective porosity, saturated permeability, bubbling pressure and pore-size distribution index that can be encountered at field sites, providing undisturbed media can be characterized by these parameters.

Hopefully, this information will permit the construction of valid models of partially saturated systems or at least yield a better understanding of the hydraulic properties of porous media and their interrelationship.

## BACKGROUND AND THEORY

In a previous publication of this series, Hydrology Paper No. 3, Brooks and Corey have reviewed theory published in the petroleum engineering literature which relates permeability of homogeneous and isotropic porous media to pore-size distribution. Critical portions of the theory were contributed by Kozeny (12), Purcell (17) and Burdine (4). Brooks and Corey demonstrated that the theory was valid for the calculation of relative permeability  $K/K_0$  from a capillary pressure-saturation curve.

Recently, several other schemes have been proposed for the calculation of permeability values for flow in partially saturated media from pore-size distribution data. A method specifically pertinent to soils was proposed first by Childs and Collis-George (6) and was later modified by Marshall (13) and Millington and Quirk (15). In each case, information on pore-size distribution is deduced from capillary pressure-desaturation data. Jackson et al. (11) compared the latter methods of calculating relative permeability with direct measurements and found that the method proposed by Millington and Quirk gave the best results.

Each of the latter methods requires that the permeability of the fully saturated medium,  $K_0$ , be measured in order to calculate the permeability at other saturations. The method developed in the petroleum industry provides the possibility of calculating  $K_0$  from pore-size distribution data alone. In order to accomplish this, simplifying assumptions are made concerning the length of the actual flow path, the pore shape factor and the functional relationship between saturation and capillary pressure. The following development shows how this can be done and also provides a theoretical relationship among the parameters  $\phi_e$ ,  $K_0$ ,  $p_b$  and  $\lambda$ .

The equation for permeability of a fully saturated medium as presented by Wyllie and Spangler (21) is

$$K_0 = \frac{\phi \sigma^2 \cos^2 \theta}{kT} \int_0^1 \frac{dS}{p_c^2} \quad (1)$$

in which  $\phi$  is the porosity;  $\sigma$  is the interfacial tension of the wetting and non-wetting fluids;  $\theta$  is the angle of contact of the interfaces with the solid;  $k$  is a pore shape factor;  $T$  is tortuosity and  $S$  is saturation. All of these terms are defined in the List of Symbols on page x.

According to Brooks and Corey (3), the relationship between effective saturation and capillary pressure is given by

$$S_e = \left( \frac{p_b}{p_c} \right)^\lambda, \quad \text{for } p_c \geq p_b$$

and (2)

$$S_e = 1.0, \quad \text{for } p_c \leq p_b$$

where  $S_e$  is related to  $S$  by

$$S_e = \frac{S - S_r}{1 - S_r} \quad (3)$$

in which the residual saturation,  $S_r$ , is that value of  $S$  for which equations 2 hold. It is a value of  $S$  at which  $p_c$  is very large and  $K$  is very small. For some media,  $S_r$  has been found to be zero.

From equation 3, it follows that

$$dS = (1 - S_r) dS_e \quad (4)$$

and from equation

$$p_c = p_b S_e^{-\frac{1}{\lambda}}, \quad \text{for } p_c \geq p_b \quad (5)$$

A drainable porosity,  $\phi_e$  is defined such that

$$\phi_e = \phi (1 - S_r) \quad (6)$$

Equations 4, 5 and 6 are substituted into equation 1. Neglecting values of  $S < S_r$ , the indicated integration gives

$$K_0 = \frac{\phi_e \sigma^2 \cos^2 \theta}{p_b^2 kT} \left( \frac{\lambda}{\lambda + 2} \right) \quad (7)$$

This assumes that  $K = K_0$  when  $p_c \leq p_b$ .

Assuming further that the product  $kT$  is approximately 5, as was found by Carman (5), and that  $\cos^2\theta$  is nearly unity,

$$\frac{\phi_e \sigma^2}{K_o p_b^2} \left( \frac{\lambda}{\lambda + 2} \right) \approx 5 . \quad (8)$$

In the approximate form, the relationship contains only the properties  $\phi_e$ ,  $K_o$ ,  $p_b$ ,  $\lambda$  and the easily measurable fluid property  $\sigma$ . Apparently, equation 8 represents the relationship satisfying the first objective of this study.

For media having very uniform pores,  $\lambda$  is very large and the ratio  $\lambda/(\lambda + 2)$  approaches unity. In this case, equation 8 reduces to a form analogous to an equation which, according to Wyllie and Spangler (21), was first proposed in 1949 by Rose and Bruce (19). Evidently, the ratio  $\lambda/(\lambda + 2)$  can be regarded as a correction factor accounting for the non-uniformity of pores.

Brooks and Corey (3) found that their measured curves of  $K_r$  as a function of  $p_c$  could be approximated by the empirical equations

$$K = K_o, \text{ for } p_c \leq p_b$$

and

$$K = K_o \left( \frac{p_b}{p_c} \right)^\eta, \text{ for } p_c \geq p_b \quad (9)$$

where  $\eta$  is an exponent characteristic of particular media. By reasoning similar to that by which equation 8 was deduced, they showed that  $\eta$  is related to  $\lambda$  by

$$\eta = 2 + 3\lambda . \quad (10)$$

Data obtained by Brooks and Corey showed that equation 10 is valid within experimental error.

In practice, it has been found that with presently available experimental techniques,  $\eta$  can be measured with more precision than  $\lambda$ . Consequently, it is sometimes desirable to calculate  $\lambda$  from measured values of  $\eta$ .

The most serious defect of equations 7 and 8 results from the assumption that  $K$  equals  $K_o$  when  $p_c \leq p_b$ . In certain cases, a significant

transition exists from the range of invariant permeability to the range for which permeability is a power function of capillary pressure. This fact was pointed out by Gardner (9) who represented the functional relationship of permeability to capillary pressure by the equation

$$K = \frac{a}{b + p_c^n}, \quad (11)$$

where  $n$  is a positive dimensionless constant and  $a$  and  $b$  are also constants having units dependent on the units of permeability and capillary pressure and on the value of  $n$ . This equation represents permeability as a smooth function of capillary pressure and may often approximate the actual relationship very closely for many structured materials.

In a study of the Brooks-Corey modeling theory for unsaturated flow, Corey (7) observed changes in pore-size distribution with changes in the packing density of disturbed soils. The trend was for  $\eta$  to decrease as porosity increased, but no conclusive data were obtained.

Practically no information is available on the range of pore-size distribution index and bubbling pressure that can be expected under field conditions. However, in a study of volumetric moisture content as a function of suction, Perrier and Evans (16) noted displacement of the moisture curves toward moisture contents that were higher for disturbed soils than for undisturbed soils. They concluded that the effect was due to the greater compaction of the undisturbed samples. Variability of the data for the undisturbed samples was attributed to structural effects.

A device developed recently by Bower (2) for measuring the "air entry value" of agricultural soils in situ may be of some practical value in determining the range of bubbling pressures encountered under field conditions. Additional research is needed to determine whether or not the "air entry value" determined with Bower's device can be consistently related to the bubbling pressure as defined by Brooks and Corey.



## EXPERIMENTAL TECHNIQUES

The validity of the functional relationship expressed by equation 8 was tested by measuring experimentally the values of  $K_o$ ,  $p_b$  and  $\lambda$  for different values of  $\phi_e$ . The effective porosity was varied by altering the bulk density of three disturbed earth materials. For convenience, this portion of the study will be referred to as the porosity experiment.

The parameters  $K_o$ ,  $p_b$  and  $\eta$  were determined from capillary pressure-permeability curves. Utilizing equation 11 relating  $\eta$  and  $\lambda$ , values of  $\lambda$  were calculated from the experimentally determined values of  $\eta$ . Values of  $S_r$  were obtained from capillary pressure-desaturation curves using a method suggested by Brooks and Corey (3). In addition, these curves yielded a second set of values for  $p_b$  and  $\lambda$ . However, because of the greater reproducibility possible in the capillary pressure-permeability experiments, the values of  $p_b$  and of  $\lambda$  calculated from  $\eta$  were used to check equation 8. Effective porosity,  $\phi_e$ , was calculated from measured values of bulk density,  $\rho_b$ , particle density,  $\rho_s$ , and residual saturation,  $S_r$ .

Capillary pressure-permeability curves were also obtained for several undisturbed media to determine whether or not the pore-size distribution of undisturbed earth materials can be characterized by the index of Brooks and Corey and, if applicable, the range of the hydraulic properties that can be encountered at field sites. This portion of the study is referred to as the undisturbed media experiment.

### Fluids and Media

The wetting fluid used was a light hydrocarbon oil called Soltrol "C". Soltrol "C" is a core test fluid of a type commonly employed in the petroleum industry for laboratory model studies. This oil was obtained from the Special Products Division, Phillips Petroleum Company, Bartlesville, Oklahoma. It was selected instead of water primarily because soil structure is much more stable in the presence of a hydrocarbon than in water. The oil has more consistent wetting and interfacial properties in the presence of contaminants than water. Another very important advantage is that the model size can be reduced to one-half that required when oil is used as the wetting fluid instead of water. This effect is attributable primarily to the low (approximately 22.9 dynes/cm) surface tension of Soltrol "C". The dynamic viscosity, the density and the ratio of dynamic viscosity to specific weight are tabulated in Table A-1, Appendix A, for the range of temperatures encountered.

The non-wetting fluid in this study was air. For all measurements, the distribution of the non-wetting phase was static. Consequently, viscosity of the non-wetting fluid was not relevant. Moreover, because the density of air is small compared to that of Soltrol "C", the specific weight of the wetting fluid was neglected in this investigation.

Ten media were used in this study. For the porosity experiment, disturbed samples of three earth materials ranging in texture from silt loam to sand were used. For the undisturbed media experiment, seven undisturbed samples were taken at five locations. Six of the media consist of soil materials and another came from an outcrop of semi-consolidated sand. The six soils include a wide range of textures from clay loam to loamy sand. The semi-consolidated sand is fine in texture and has a relatively uniform pore-size distribution. The grain-size analyses obtained with a hydrometer and particle densities obtained with a pycnometer are presented in Tables A-2 and A-3 of Appendix A.

### Capillary Pressure-Permeability Measurement for the Porosity Experiment

All capillary pressure-permeability tests on disturbed media were made with columns of the material packed into acrylic tubes. The experimental equipment was similar to that used by Anat et al. (1).

For the Touchet silt loam (GE 3), the Columbia sandy loam and the unconsolidated sand, used in the porosity study, a soil column packer of the type designed by Jackson et al. (10) was used. To eliminate heterogeneity in horizontal planes caused by filling and vibrating simultaneously, the columns were first filled and later vibrated according to the method described by Anat et al. (1). Even this procedure resulted in large particles and aggregates being concentrated at the wall of the column near the upper end of the columns. This problem was resolved by making the columns extra long and then removing the extra length after the filling and vibrating operations as described by Corey et al. (8). In order to obtain the desired porosities, further packing was usually necessary. This was accomplished by gently tapping the side and top of the columns with a rubber mallet. Experiments later showed that this procedure resulted in a uniform porosity throughout the column.

The porosity,  $\phi$ , of each material was determined using measured values of the bulk density,  $\rho_b$ , and particle density,  $\rho_s$ , in the equation

$$\phi = 1 - \frac{\rho_b}{\rho_s} \quad (12)$$

The bulk density was measured by weighing the amount of air-dry soil in a known volume of the column. It was realized that the use of air-dry soil resulted in calculated values of bulk density which were slightly high. However, if the small amount of hygroscopic moisture (less than one percent by weight) which adsorbs on the soil particles is considered as part of the porous matrix, the use of air-dry soil is justifiable. The particle density was measured using a pycnometer bottle.

The procedure used in vacuum-saturating the column and in obtaining the capillary pressure-permeability data for the porosity study was the same as that employed by Corey et al. (8). The method of measuring permeability is identical in principle to that originally employed by Richards (18).

After the data had been collected and relative permeability,  $K_r$ , had been calculated, the values of  $\eta$  and  $p_b$  were calculated using a least squares analysis. The equation for  $\eta$  is

$$\eta = \frac{\frac{1}{N} \sum_{m=1}^N (\log p_c)_m \sum_{m=1}^N (\log K_r)_m - \sum_{m=1}^N (\log p_c \log K_r)_m}{\sum_{m=1}^N \left[ (\log p_c)_m^2 \right] - \frac{1}{N} \left[ \sum_{m=1}^N (\log p_c)_m \right]^2} \quad (13)$$

where  $N$  is the number of determinations of  $p_c$  and  $K_r$ , in the range  $p_c > p_b$ , for which the relationship approximates a straight line on a log-log plot. The bubbling pressure,  $p_b$ , was determined from the same values of  $p_c$  and  $K_r$  using the equation

$$p_b = \text{antilog} \left\{ \frac{1}{N} \left[ \sum_{m=1}^N (\log p_c)_m + \frac{1}{\eta} \sum_{m=1}^N (\log K_r)_m \right] \right\} \quad (14)$$

#### Capillary Pressure-Saturation Measurement for the Porosity Experiment

Initially, the capillary pressure-desaturation curve determinations for the bulk density experiment were made using columns of soil packed into the

pressure controllers used previously in the permeability determinations and converted for use in this study. These controllers, consisting of capillary barriers fitted inside 4-cm. sections of machined acrylic tubing, served the triple purpose of pressure control, pressure measurement and removal of the wetting fluid. A detailed diagram of the pressure controller was presented by Anat et al. (1) in Figure 10 of Hydrology Paper No. 7. Later, when it was discovered that the affect of column height in this static system was critical for the unconsolidated sand, it became necessary to use different equipment.

The difficulty arises with the linear pressure distribution associated with static conditions. For media having relatively uniform pore-size distributions and low bubbling pressures, the deviation from a linear distribution of saturation is appreciable. Consequently, if the measured saturation is a mean or weighted value for the entire sample, as in this experiment, the saturation value does not correspond to the capillary pressure at the midpoint of the sample. For materials having relatively wide distributions of pore size and high bubbling pressures, the error is negligible if the column height is small.

In this experiment, no difficulty was encountered for the Touchet silt loam (GE 3) or the Columbia sandy loam. However, for the unconsolidated sand, the error became apparent during the analysis, making it necessary to repeat the experiment using a smaller column height. For the repeat run, a column height of approximately 1 cm. was used instead of 4 cm. as in the original run.

The media in this study were packed manually. The filling operation was accomplished using a funnel with a long spout. The funnel was first filled with the lower end of the spout resting on the bottom of the container. As the funnel was slowly withdrawn, a random motion was imparted to its lower end such that a minimum of segregation of particles or aggregates would occur. Final adjustment of the bulk density was achieved by gentle tapping with a rubber mallet as previously described. It was found that using this manual procedure, homogeneous packing was attained more consistently than if the mechanical packer was used.

After vacuum saturation, each pressure controller was encased in clear plastic wrap in order to minimize the occurrence of hydraulic gradients in response to evaporation. The wrap remained around the controller for the remainder of the experiment.

In all measurements, capillary pressures were controlled and measured by systems of leveling tanks connected to the pressure controllers containing the media as shown in Figure 1. The saturations were determined by weighing the entire controller containing the medium after clamping off the pressure control lead and disconnecting it from the pressure regulation system.

The capillary pressures were changed in increments by adjusting the elevations of the leveling tanks. The advance of the meniscus in a horizontal indicator tube connected to the pressure controller provided a measure of the response of saturation to change in capillary pressure. When the advance of the meniscus ceased following each incremental change in capillary pressure, the system was considered to be in equilibrium and capillary pressure and saturation measurements were made. Before weighing the sample each time, any air in the pressure control lead was removed. Corrections to the capillary pressure were made for difference in pressure across the meniscus in the indicator tube and for difference in elevation between the meniscus and the midpoint of the column.

The analytical procedure used in determining the residual saturation involved a trial and error technique that is outlined by Brooks and Corey (3). After the effective saturation,  $S_e$ , had been calculated, the value of  $\lambda$  and  $p_b$  were calculated using a least squares analysis. The equation for  $\lambda$  is

$$\lambda = \frac{\frac{1}{N} \sum_{m=1}^N (\log p_c)_m \sum_{m=1}^N (\log S_e)_m - \sum_{m=1}^N (\log p_c \log S_e)_m}{\sum_{m=1}^N [(\log p_c)_m]^2 - \frac{1}{N} \left[ \sum_{m=1}^N (\log p_c)_m \right]^2} \quad (15)$$

where  $N$  is the number of determinations of  $p_c$  and  $S_e$ , in the range  $p_c > p_b$ , for which the relationship approximates a straight line on a log-log plot. The bubbling pressure was determined from the same values of  $p_c$  and  $S_e$  using the equation

$$p_b = \text{antilog} \left\{ \frac{1}{N} \left[ \sum_{m=1}^N (\log p_c)_m + \frac{1}{\lambda} \sum_{m=1}^N (\log S_e)_m \right] \right\} \quad (16)$$

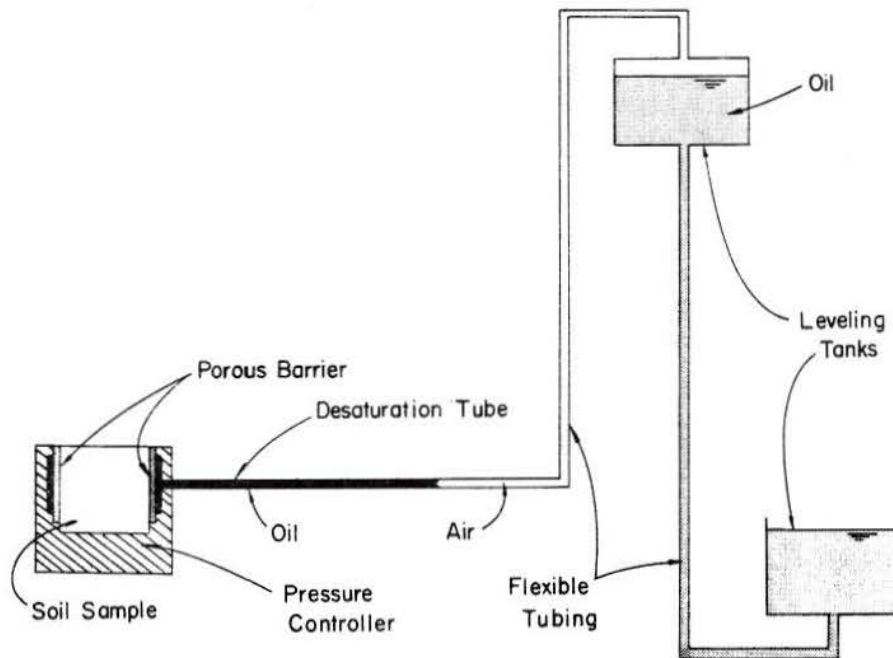


Figure 1. Schematic diagram of saturation-capillary pressure apparatus

### Capillary Pressure-Permeability Measurement for the Undisturbed Media Experiment

For this experiment, undisturbed samples were taken at five locations. At one location, samples were taken at three depths resulting in a total of seven materials being included in this experiment. Six of the materials consist of soil material and another came from an outcrop of semi-consolidated sand. The soils include a wide range of textures from clay loam to loamy sand. The semi-consolidated sand is fine in texture and has a relatively uniform pore-size distribution.

The undisturbed soil samples were obtained using a sleeve-type sampler, a photograph of which is shown in Figure 2. At each site, samples were obtained in both horizontal and vertical directions. The horizontal samples were taken such that the central axis was at the 12-inch depth and the vertical sample extended from 10 to 14 inches below the soil surface. At one site, horizontal and vertical samples were also taken at the 5-inch and 20-inch depths.

Samples were obtained by forcing the sampler (with its sleeve insert) into the soil during a period when the soil was relatively dry, at which time compaction of the sample was slight. The inner acrylic sleeve containing the sample was removed from the sampler at the site and protected from disturbance during transportation to the laboratory.

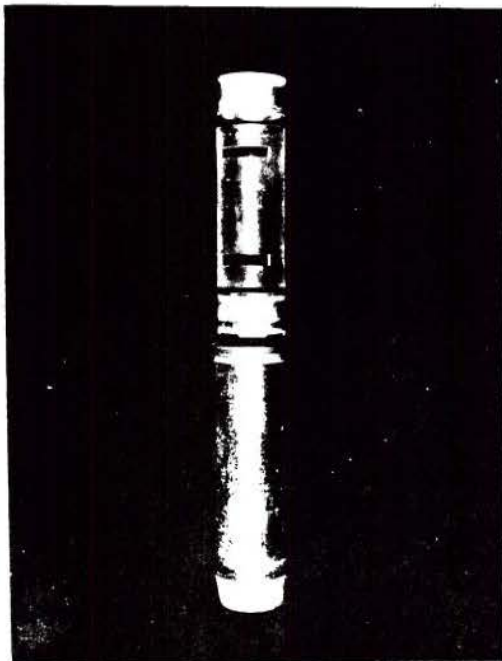


Figure 2. Sleeve-type sampler (ready for assembly).

All permeability measurements on the undisturbed material were made in the laboratory without removing the soil from the sleeve. A photograph of the apparatus used is shown in Figure 3. The methods of determining the capillary pressure-permeability relations and bulk density were identical in principle to that used for the disturbed materials. The procedure and equipment used have been described by Anat et al. (1) in Hydrology Paper No. 7. However, there were two significant innovations in equipment and technique that may be worthy of mention.

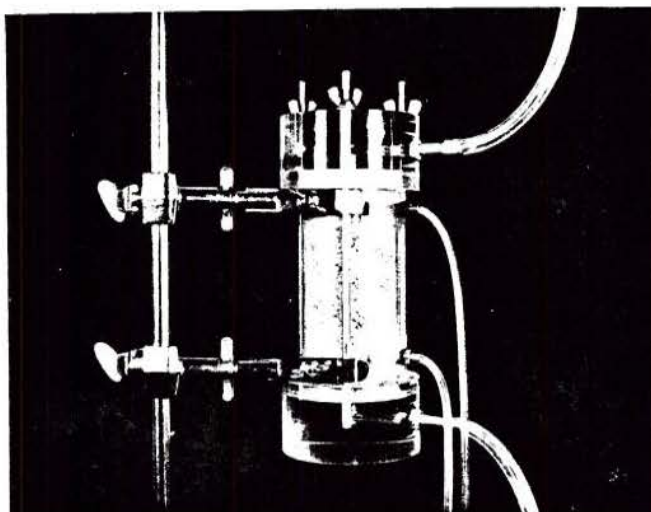


Figure 3. Undisturbed soil sample during test showing inflow and outflow pressure controllers and tensiometers.

The tensiometers, for measuring capillary pressures, and the hydraulic gradients during flow were not an integral part of the acrylic sleeve confining the sample. They were independently detachable and this feature proved to be a convenience of some importance. Openings in the sleeve of appropriate size permitted the tensiometers to be placed in contact with the soil flush with the inside wall of the acrylic sleeve. In the event of any malfunction of the tensiometer, such as air entrapment, the tensiometer could be quite conveniently removed and either restored or replaced.

The second innovation involved a change of technique. In the procedure described by Corey et al. (8), the capillary pressure is changed in increments by changing the elevations of the supply and the outflow siphon, keeping the elevation of the sample constant. In this experiment, however, the capillary pressure was changed by raising the elevation of the sample after pinching off the manometer leads, keeping the elevation of the supply and the outflow approximately constant except for minor

adjustments necessary to compensate for change in head loss in the system. The manometer leads were reopened after a sufficient time had lapsed to permit the fluid in the sample to equilibrate under its new regime. This procedure requires a much shorter time for the manometers to reach equilibrium since the level in the manometers is controlled by the elevation of the inflow and outflow siphons and these were changed only slightly.

The procedure used for the case of the semi-consolidated sand differed from that of the soil materials in detail but not in principle. A block of the outcrop was removed and transported to the laboratory where a specimen was obtained with a diamond core drill. Measurements of permeability of the semi-consolidated sand were made using an apparatus operating on the same principle as that used for unconsolidated soils.

After completing permeability measurements on the undisturbed earth materials, five were crushed sufficiently to pass through a number 14 sieve and the measurements were repeated. They were later crushed to pass through a number 48 sieve and the measurements were repeated again. The packing in this phase of the study was performed manually. In each case, an attempt was made to reproduce the original porosity of the undisturbed material.

Determination of bulk density and particle density in the determination of porosity for the undisturbed media was accomplished at the completion of the permeability measurements. Calculation of  $\phi$ ,  $\eta$ , and  $p_b$  was carried out using equations 12, 13 and 14 as in the case of the disturbed soils.

## RESULTS AND DISCUSSION

The experimental results are discussed in terms of how well the hydraulic properties of disturbed media satisfy equation 8, and whether the measured properties of undisturbed earth materials permit the hydraulic behavior of soils and rocks under field conditions to be modeled using a scaling theory developed from equations describing the hydraulic behavior of undisturbed media.

### Porosity Experiment

The capillary pressure-permeability data for this experiment are tabulated in Tables B-1 to B-15, Appendix B. The data are presented graphically in Figures 4, 5 and 6. The capillary pressure-desaturation data are tabulated in Tables C-1 to C-11, Appendix C. These data are presented graphically in Figures 7 to 17.

For the three media studied, it was found generally that saturated permeability increased, bubbling pressure decreased, and pore-size distribution index decreased slightly as porosity increased, or what amounts to the same, as bulk density decreased. Table 1 summarizes the data obtained in the capillary pressure-permeability determinations. Figures 18 to 20 demonstrate the dependence of these parameters on porosity for the three media studied.

Definite relationships were indicated for saturated permeability and bubbling pressure. For pore-size distribution index, however, the relationship was not so obvious. In fact, the results indicate that pore-size distribution index is changed only slightly over a range of porosities. For modeling purposes, one requirement of similitude is identical values of  $\eta$ . Evidently, bubbling pressure and permeability may be adjusted to suit the size of the model by changing the packing density without appreciably changing the pore-size distribution index.

The results from the capillary pressure-desaturation data, which are summarized in Table 2, were indicative of the same general relationships obtained from the capillary pressure-permeability data for bubbling pressure and pore-size distribution index. The results also disclosed that, in this experiment, residual saturation showed only a slight dependence on bulk density or porosity for a particular material.

Values of  $\lambda$  obtained from capillary pressure-desaturation data were compared with corresponding values obtained from capillary pressure-permeability data. The value of  $\lambda$  from the permeability data were calculated using values of  $\eta$  taken from Figure 20 for the appropriate porosity value substituted into equation 10. Similar comparisons were made for bubbling pressure using the curves of Figure 19. The results of the comparisons are tabulated in Table 3.

For the Touchet silt loam (GE 3) and the Columbia sandy loam, the bubbling pressure values differ in each case by less than 4 percent and the agreement for pore-size distribution index is within 3 percent in all cases but one. Evidently, equation 10 relating  $\lambda$  and  $\eta$  is valid. Such a statement substantiates the findings of Brooks and Corey (3), that if two materials have similar scaled capillary pressure-permeability relationships, they also will have similar scaled capillary pressure-effective saturation relationships. The limited variation in corresponding values of  $p_b$  and  $\lambda$  is also indicative of the magnitude of the analytical and experimental error involved in the determination of these parameters.

For the unconsolidated sand, however, the difference in bubbling pressure is almost 8 percent in one case and for pore-size distribution index the difference approached 6 percent. Although these differences are not great, they are slightly higher than the corresponding differences measured for the Touchet silt loam. The values of  $\lambda$  obtained from the capillary pressure-desaturation data are generally lower than the values calculated from the capillary pressure-permeability data. Since differences of much larger magnitude but in the same direction were observed for the columns of 4-cm height, it is quite probable that the differences observed for this material using a column of 1-cm height reflect the errors associated with column height discussed in the section dealing with experimental techniques. It was concluded that an improved method of determining capillary pressure-desaturation curves for materials having low bubbling pressures and high values of the pore-size distribution index is needed.

In order to check the validity of equation 8, values of the product  $\frac{\phi_e \sigma^2}{K_o p_b^2} \left( \frac{\lambda}{\lambda + 2} \right)$  were

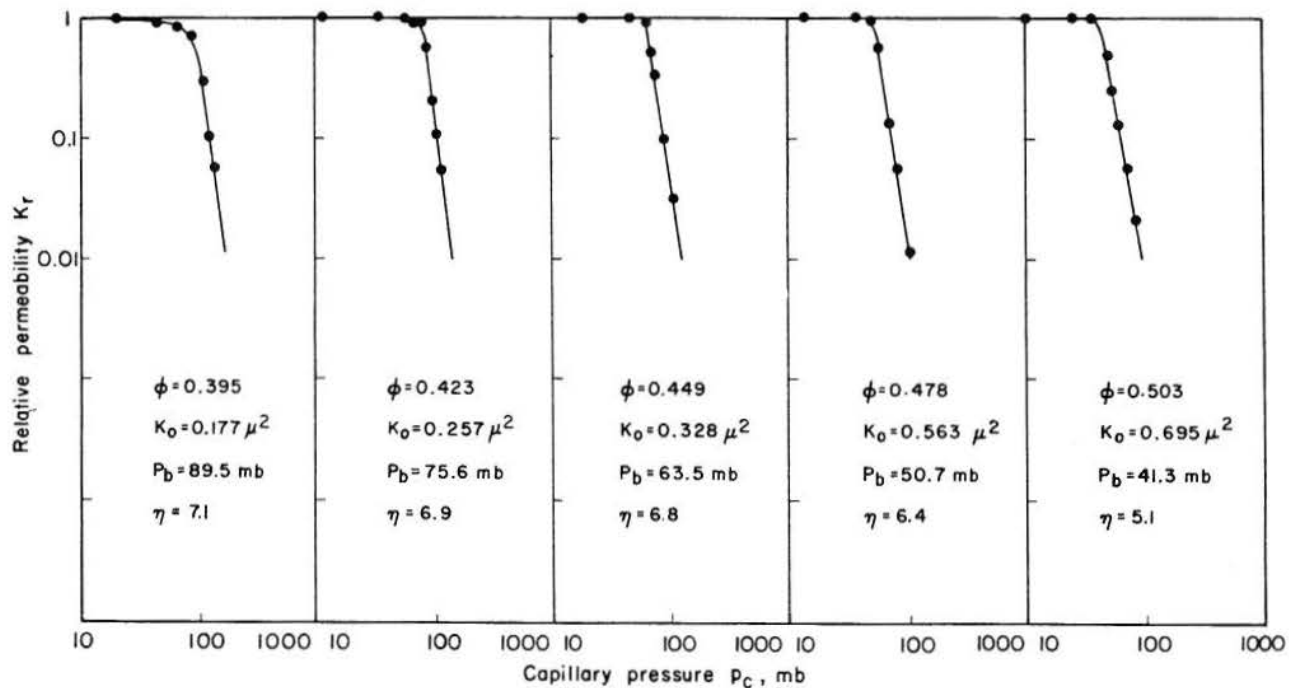


Figure 4. Relative permeability as a function of capillary pressure for Touchet silt loam (GE 3) packed at five different porosities.

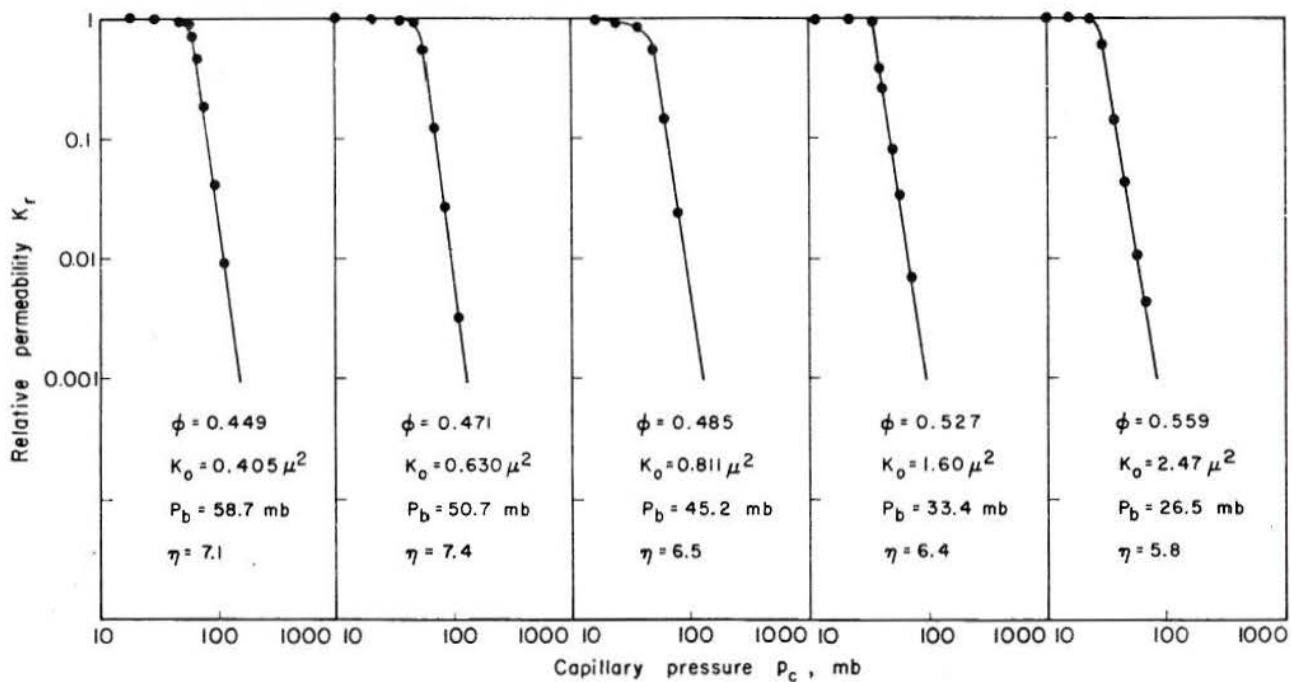


Figure 5. Relative permeability as a function of capillary pressure for Columbia sandy loam packed at five different porosities.

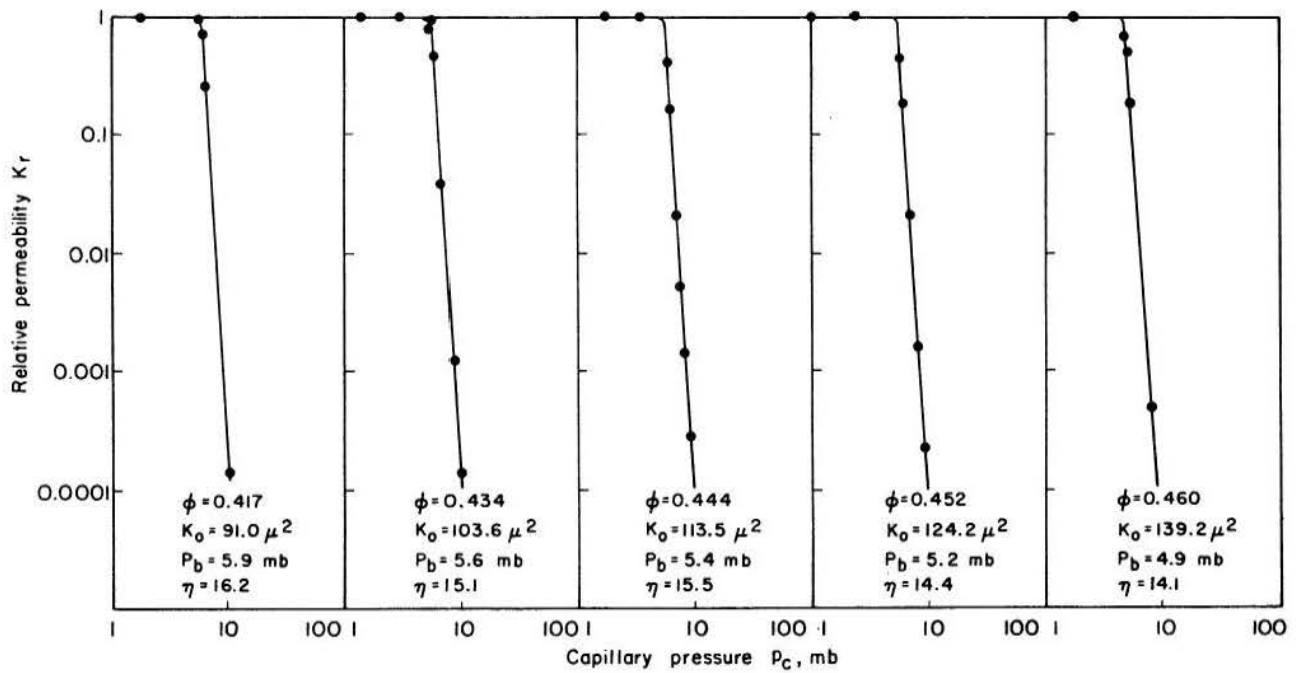


Figure 6. Relative permeability as a function of capillary pressure for an unconsolidated sand packed at five different porosities.



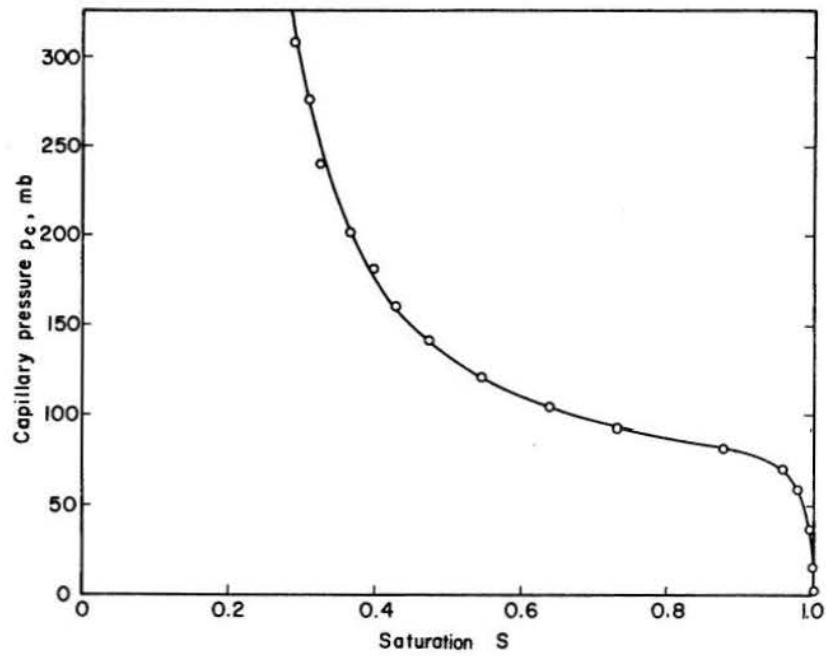


Figure 7. Saturation and effective saturation as functions of capillary pressure for Touchet silt loam (GE 3).

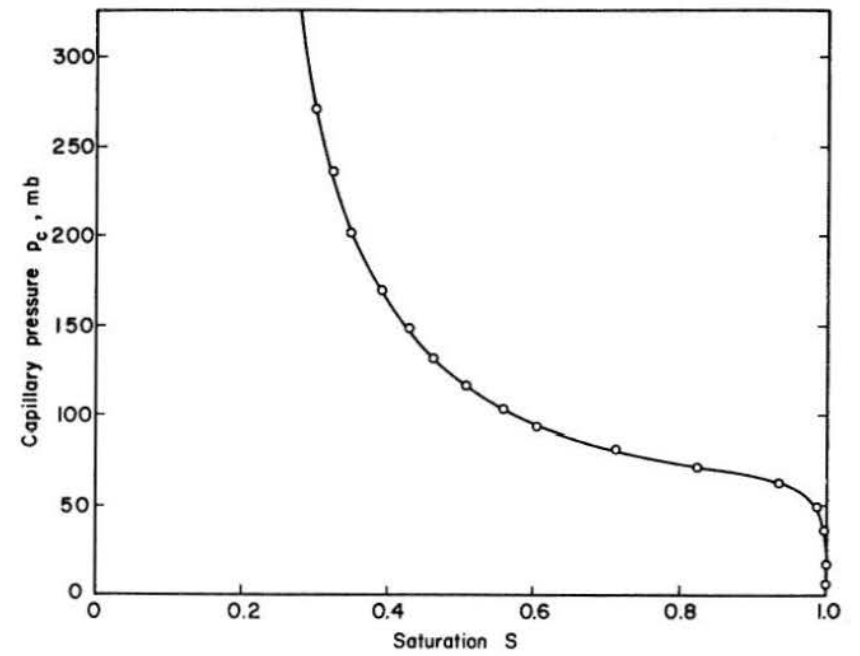


Figure 8. Saturation and effective saturation as functions of capillary pressure for Touchet silt loam (GE 3).

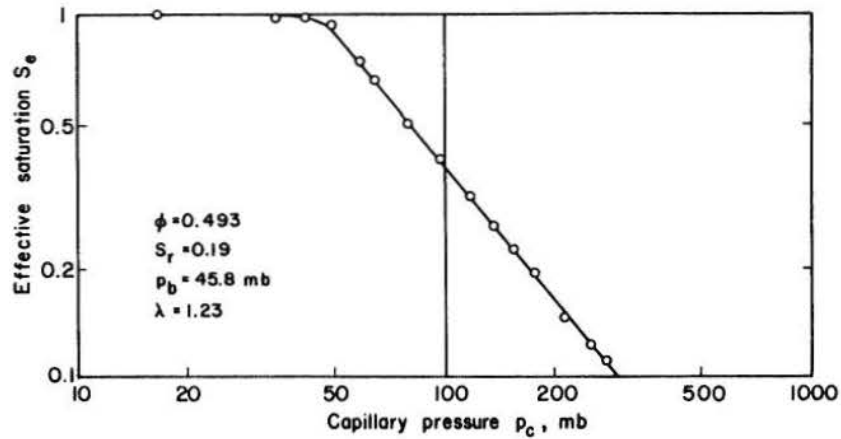
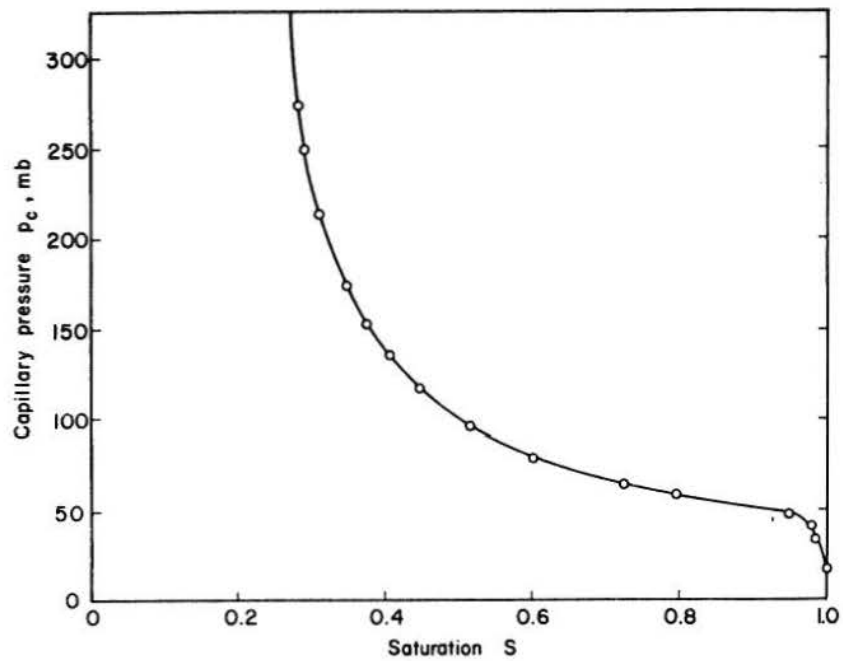


Figure 9. Saturation and effective saturation as functions of capillary pressure for Touchet silt loam (GE 3).

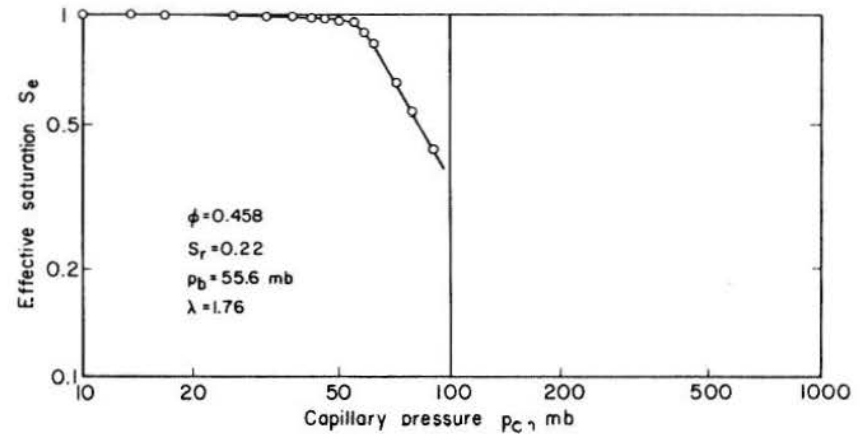
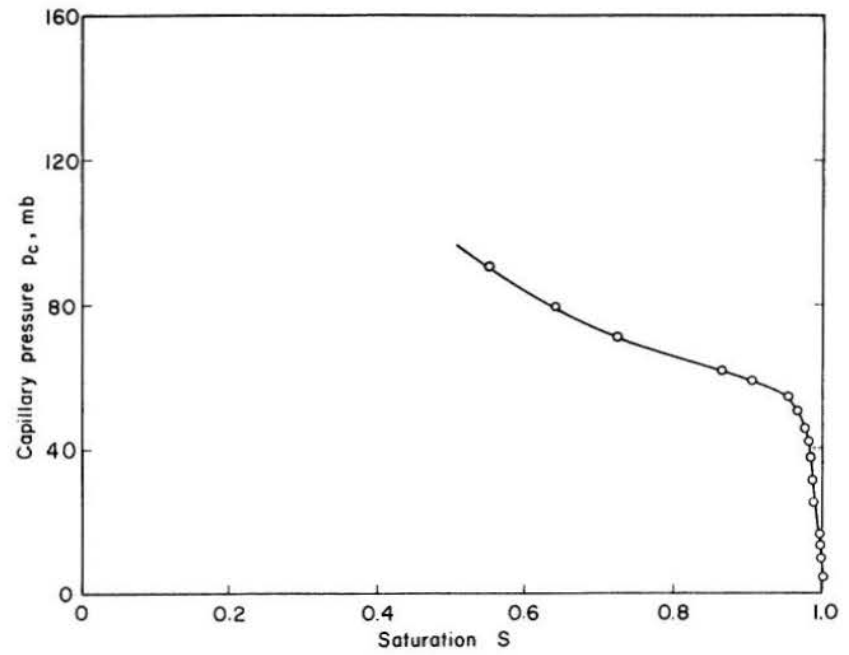


Figure 10. Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam.

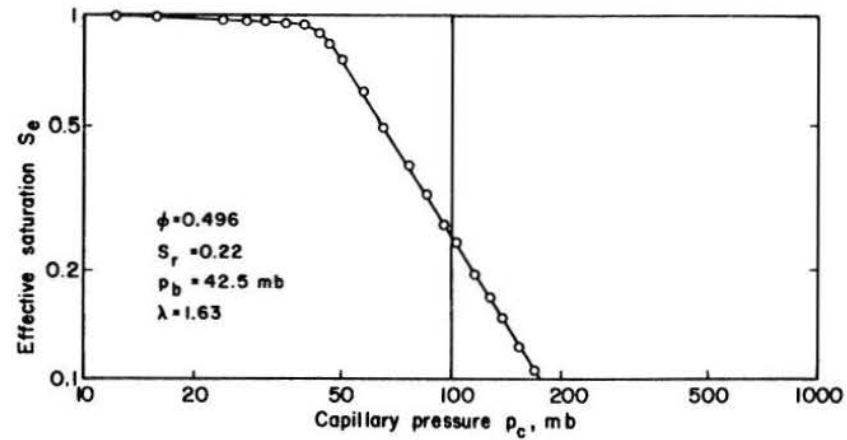
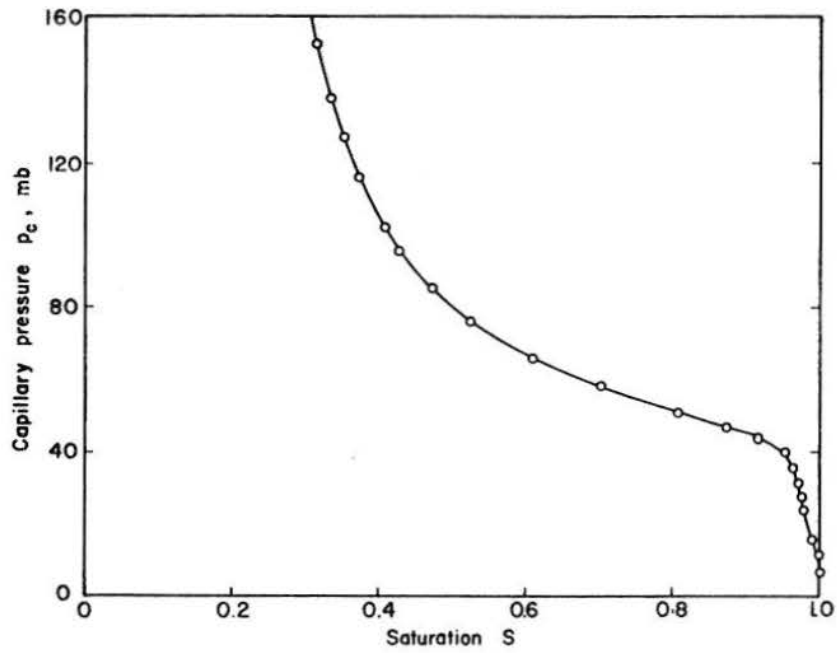


Figure 11. Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam.

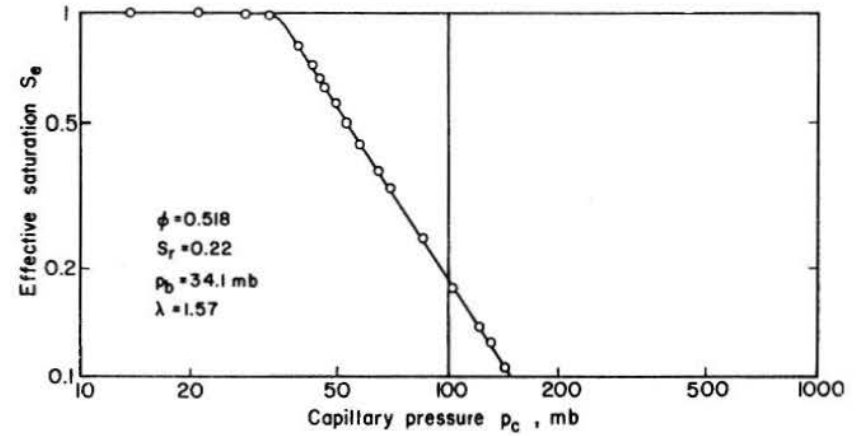
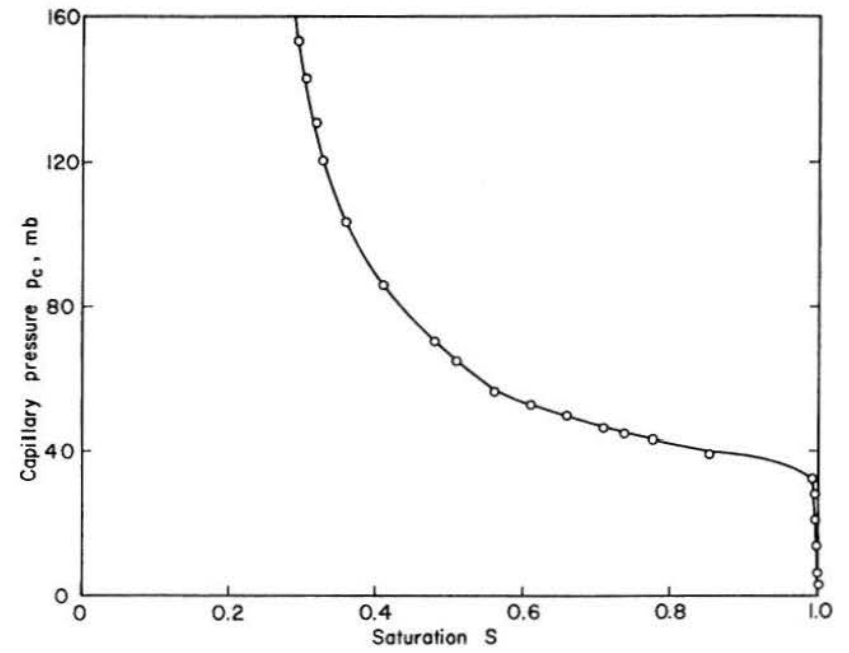


Figure 12. Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam.

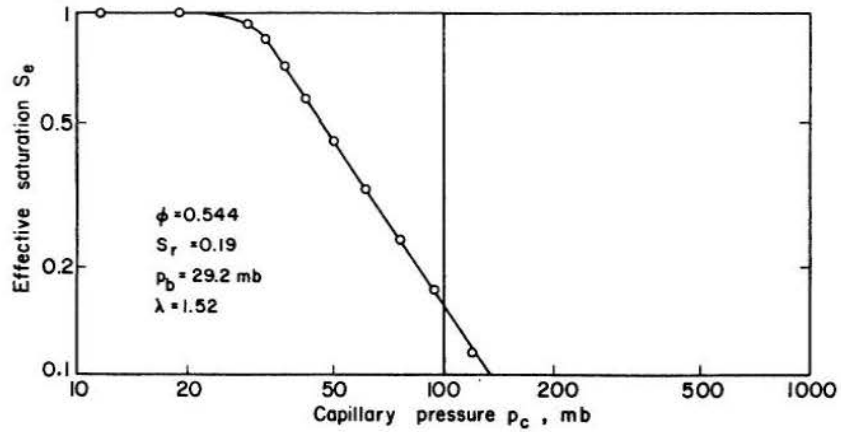
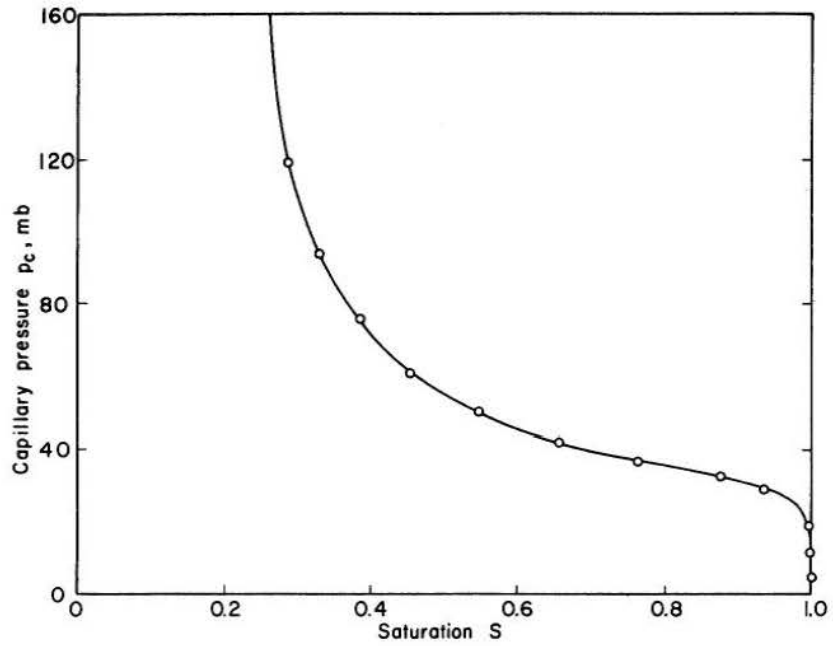


Figure 13. Saturation and effective saturation as functions of capillary pressure for Columbia sandy loam.

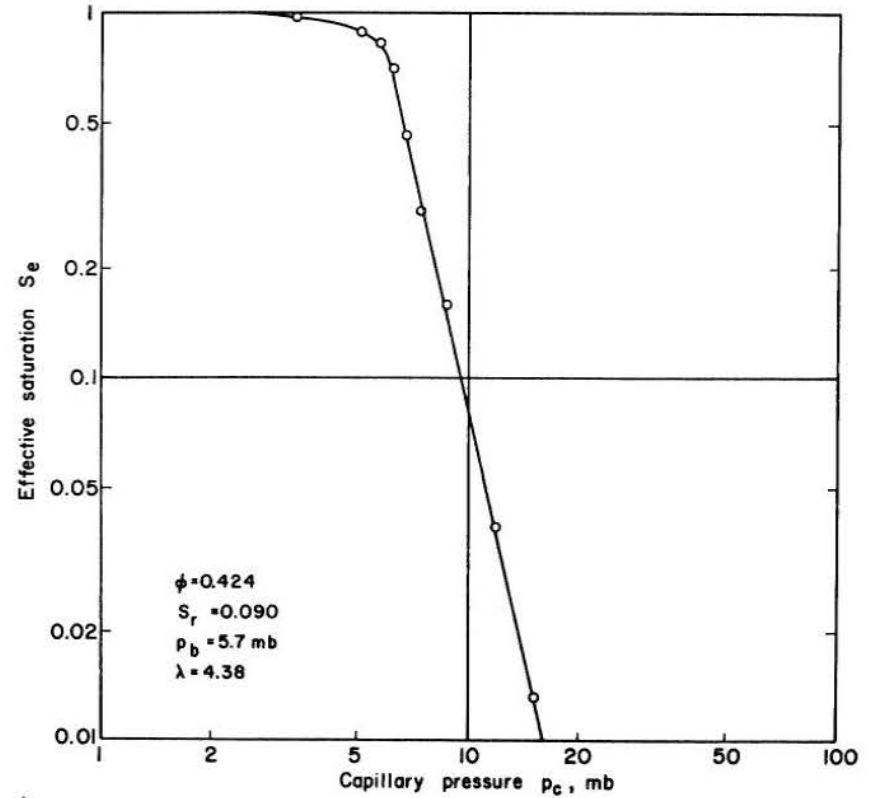
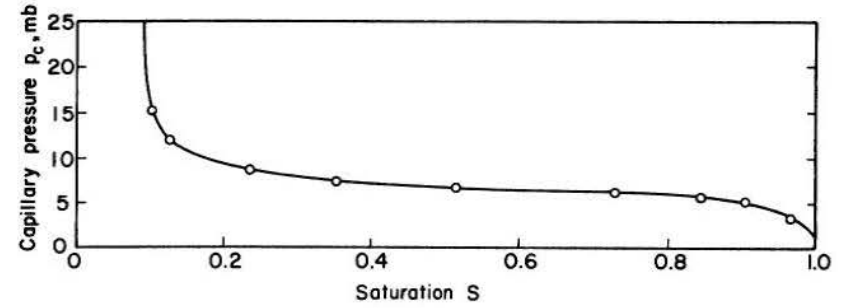


Figure 14. Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand.

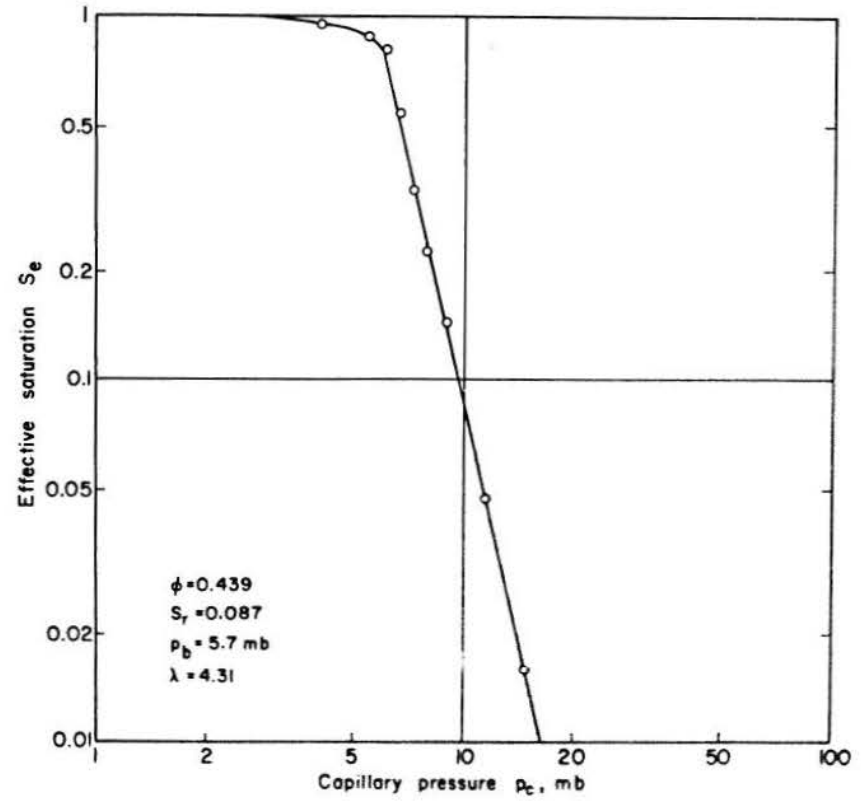
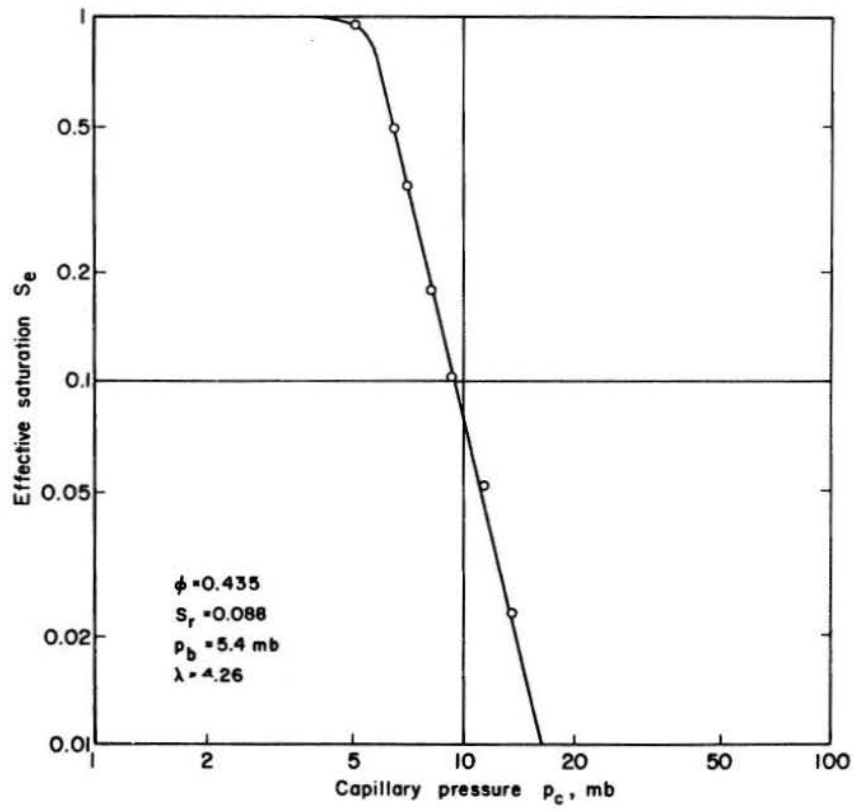
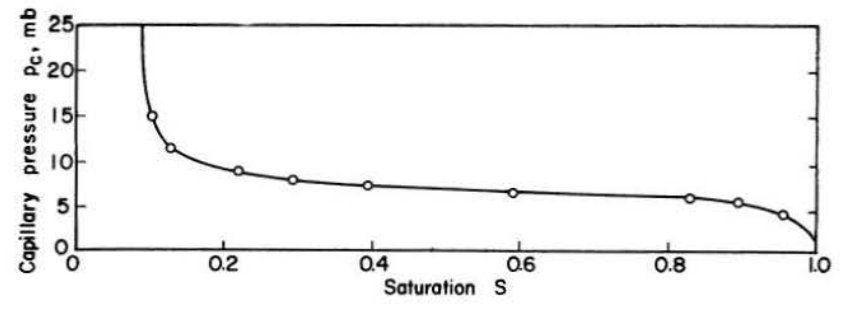
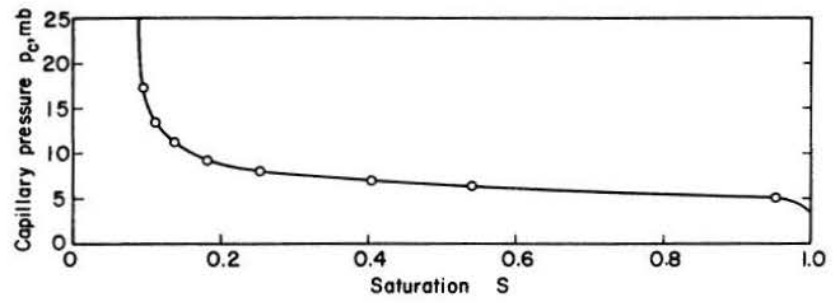


Figure 15. Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand.

Figure 16. Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand.

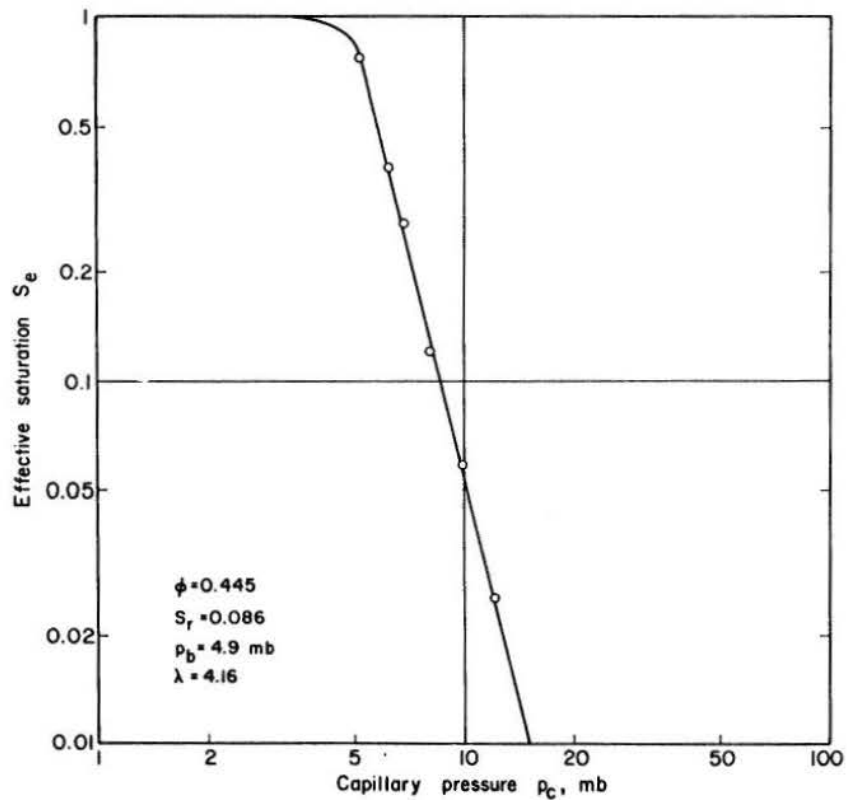
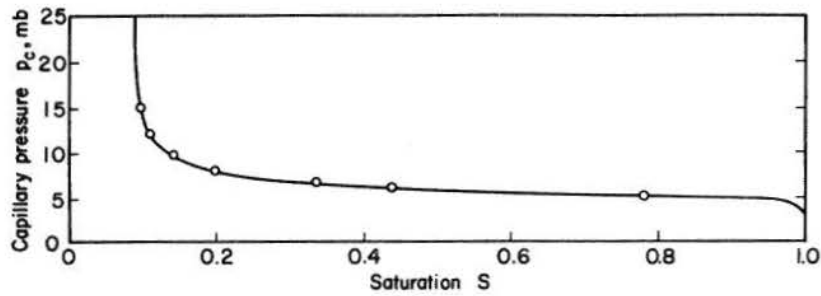


Figure 17. Saturation and effective saturation as functions of capillary pressure for an unconsolidated sand.

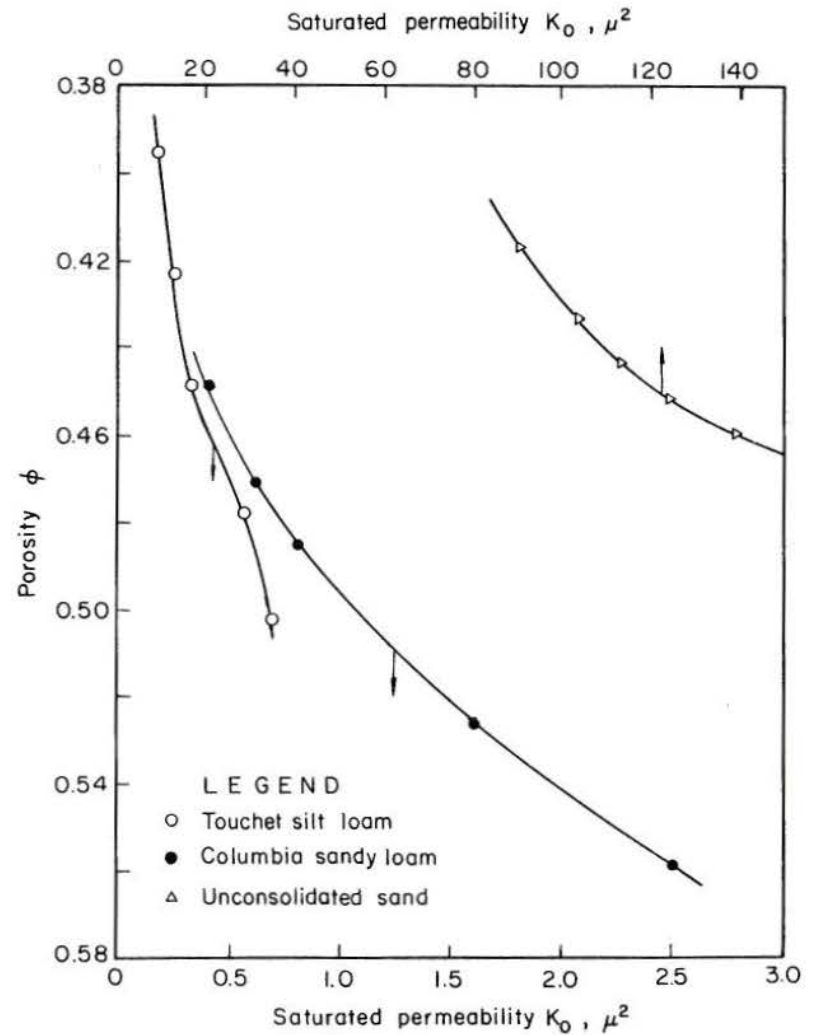


Figure 18. Saturated permeability as a function of porosity for three disturbed media.

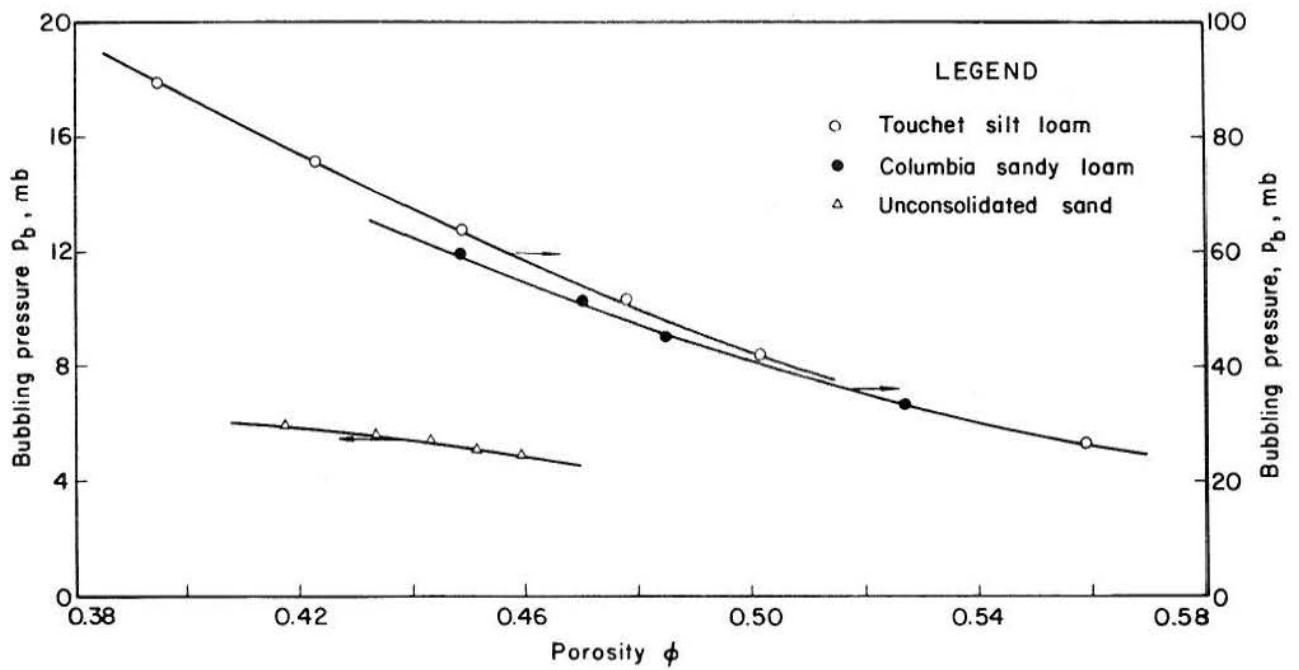


Figure 19. Bubbling pressure as a function of porosity for three disturbed media.

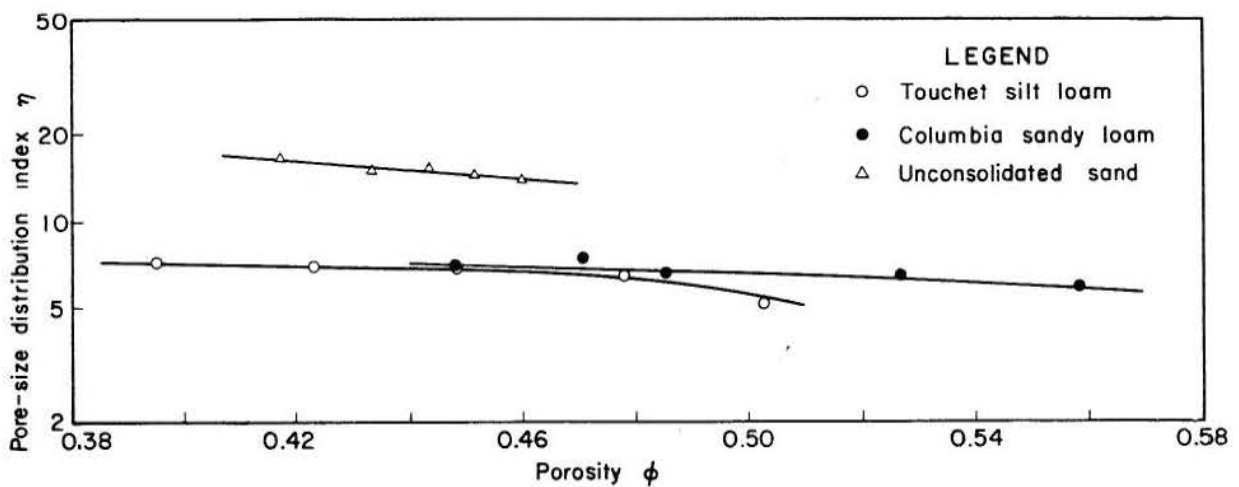


Figure 20. Pore-size distribution index as a function of porosity for three disturbed media.

TABLE 1. POROSITY, SATURATED PERMEABILITY, BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX AS FUNCTIONS OF BULK DENSITY FOR THREE DISTURBED MEDIA

$\rho_b$ gms/cm <sup>3</sup>	$\phi$	$K_o$ $\mu^2$	$p_b$ mb	$\eta$	$\lambda$ Eq. 12
Touchet silt loam (GE 3)					
1.57	0.395	0.177	89.5	7.1	1.70
1.50	0.423	0.257	75.6	6.9	1.64
1.43	0.449	0.328	63.5	6.8	1.59
1.36	0.478	0.563	50.7	6.4	1.47
1.29	0.503	0.695	41.3	5.1	1.02
Columbia sandy loam					
1.47	0.449	0.405	58.7	7.1	1.70
1.41	0.471	0.630	50.7	7.4	1.81
1.37	0.485	0.811	45.2	6.5	1.50
1.26	0.527	1.60	33.4	6.4	1.49
1.18	0.559	2.47	26.5	5.8	1.27
Unconsolidated sand					
1.58	0.417	91.0	5.9	16.2	4.75
1.53	0.434	103.6	5.6	15.1	4.37
1.51	0.444	113.5	5.4	15.5	4.49
1.48	0.452	124.2	5.2	14.4	4.13
1.46	0.460	139.2	4.9	14.1	4.02



TABLE 2. POROSITY, BUBBLING PRESSURE, PORE-SIZE DISTRIBUTION INDEX, RESIDUAL SATURATION AND EFFECTIVE POROSITY AS FUNCTIONS OF BULK DENSITY FOR THREE DISTURBED MEDIA

$\rho_b$ gms/cm <sup>3</sup>	$\phi$	$P_b$ mb	$\lambda$	$S_r$	$\phi_e$
Touchet silt loam (GE 3)					
1.48	0.430	72.8	1.67	0.22	0.335
1.40	0.463	59.2	1.47	0.22	0.361
1.32	0.493	45.8	1.23	0.19	0.399
Columbia sandy loam					
1.44	0.458	55.6	1.76	0.22	0.357
1.34	0.496	42.5	1.63	0.22	0.387
1.28	0.518	34.1	1.57	0.22	0.404
1.22	0.544	29.2	1.52	0.19	0.441
Unconsolidated sand					
1.56	0.424	5.7	4.38	0.090	0.386
1.53	0.435	5.4	4.26	0.088	0.397
1.52	0.439	5.7	4.31	0.087	0.401
1.50	0.445	4.9	4.16	0.086	0.407

TABLE 3. COMPARISON OF BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX DETERMINED FROM  $K(p_c)$  AND  $S(p_c)$  DATA FOR THREE DISTURBED MEDIA

$\phi$	$p_b$ (mb)		$\lambda$	
	$K(p_c)$	$S(p_c)$	$K(p_c) + \text{Eq. 12}$	$S(p_c)$
Touchet silt loam (GE 3)				
0.430	72.1	72.8	1.63	1.67
0.463	57.0	59.2	1.52	1.47
0.493	44.9	45.8	1.26	1.23
Columbia sandy loam				
0.458	55.2	55.6	1.71	1.76
0.496	42.2	42.5	1.64	1.63
0.518	35.5	34.1	1.55	1.57
0.544	29.4	29.2	1.41	1.52
Unconsolidated sand				
0.424	5.8	5.7	4.67	4.38
0.435	5.6	5.4	4.50	4.26
0.439	5.5	5.7	4.43	4.31
0.445	5.3	4.9	4.33	4.16

calculated. Inspection of the calculated values indicates that the error introduced by substituting a value of 5 for  $kT/\cos^2\theta$  is less than 25 percent for the three soils studied. Accepting this level of accuracy, it is possible to write an equation for saturated permeability, that is,

$$K_o = \frac{\phi_e \sigma^2}{5p_b^2} \left( \frac{\lambda}{\lambda + 2} \right) \quad (17)$$

A summary of the hydraulic properties used in the calculations along with computed values of this product is presented in Table 4. The values of residual saturation,  $S_r$ , used in computing effective porosity,  $\phi_e$ , were obtained by interpolation of values of  $S_r$  in Table 2.

In equation 17, values for all of the unknown quantities in the right-hand member can be obtained from capillary pressure-desaturation data. If a direct measurement of  $K_o$  is not available, equation

17 provides a means of calculating saturated permeability from capillary pressure-desaturation data alone. The capillary pressure-permeability relationship of equations 9.

$$K = K_o, \text{ for } p_c \leq p_b,$$

and

$$K = K_o \left( \frac{p_b}{p_c} \right)^\eta, \text{ for } p_c \geq p_b,$$

is defined completely by capillary pressure-desaturation data if  $K_o$  is calculated using equation 17,  $\eta$  is calculated using equation 10, that is,

$$\eta = 2 + 3\lambda,$$

and  $\lambda$  in this equation and  $p_b$  are obtained directly from the capillary pressure-desaturation data. Of

TABLE 4. SUMMARY OF HYDRAULIC PROPERTIES USED IN CALCULATING THE PRODUCT  $\frac{\phi_e \sigma^2}{K_o p_b^2} \cdot \left( \frac{\lambda}{\lambda + 2} \right)$  FOR THREE DISTURBED MEDIA ( $\sigma = 22.9$  DYNES/CM)

$p_b$ gms/cm <sup>3</sup>	$S_r$	$\phi_e$	$K_o$ $\mu^2$	$p_b$ mb	$\lambda$ Eq. 12	$\frac{\phi_e \sigma^2}{K_o p_b^2} \cdot \left( \frac{\lambda}{\lambda + 2} \right)$
Touchet silt loam (GE 3)						
1.57	0.22	0.308	0.177	89.5	1.70	5.2
1.50	0.22	0.330	0.257	75.6	1.64	5.3
1.43	0.22	0.349	0.328	63.5	1.59	6.1
1.36	0.20	0.382	0.563	50.7	1.47	5.8
1.29	0.18	0.412	0.695	41.3	1.02	6.2
Columbia sandy loam						
1.47	0.22	0.350	0.405	58.7	1.70	6.0
1.41	0.22	0.367	0.630	50.7	1.81	5.6
1.37	0.22	0.379	0.811	45.2	1.50	5.1
1.26	0.21	0.416	1.60	33.4	1.49	5.2
1.18	0.18	0.458	2.47	26.5	1.27	5.4
Unconsolidated sand						
1.58	0.091	0.379	91.0	5.9	4.75	4.4
1.53	0.088	0.395	103.6	5.6	4.37	4.3
1.51	0.086	0.405	113.5	5.4	4.49	4.4
1.48	0.085	0.413	124.2	5.2	4.13	4.4
1.46	0.084	0.422	139.2	4.9	4.02	4.4

course, it is necessary to know the values of  $\sigma$  and to determine  $\phi_e$  for calculation of  $K_0$  in equation 17.

The use of equation 8, along with the equations of Brooks and Corey, could be used as a method of predicting permeability as a function of capillary pressure (or of such related variables as saturation and volumetric moisture content) from pore-size distribution data. The calculations required are much simpler than in the existing methods of Childs and Collis-George (6), Marshall (13) and Millington and Quirk (15). This technique needs to be tested to determine its general applicability to other media.

A consideration of equation 7 and closer examination of the measured values of  $\frac{\phi_e \sigma^2}{K_0 p_b^2} \left( \frac{\lambda}{\lambda + 2} \right)$ , suggests that, for a particular soil, there is a characteristic relationship between the quantity  $kT/\cos^2\theta$  and porosity. For the unconsolidated sand,  $kT/\cos^2\theta$ , deduced from equation 7, remained almost invariant about a mean value of 4.4 for the range of porosities studied. For the Columbia sandy loam, the quantity first decreased with increasing porosity and then increased slightly about a mean of 5.5. For the Touchet silt loam, the values of  $kT/\cos^2\theta$  increased about a mean of 5.7 as porosity increased. This trend was consistent for this soil, with the exception of one value. Upon inspection of Figure 18, it is apparent that the measured value of saturated permeability for this porosity was unusually low. Accepting the possibility that the value of saturated permeability obtained was lower than the true value, unique relationships between  $kT/\cos^2\theta$  and porosity were observed for each of three soils.

Isolation of the variation within the quantity  $kT/\cos^2\theta$  can not be made with certainty. However, it is possible to conjecture with some degree of confidence that neither  $k$  nor  $\theta$  should vary appreciably with porosity for a particular soil. The value of shape factor,  $k$ , probably varies within narrow limits in most cases. In any case, it is doubtful that much variation in  $k$  could be attributed to variation in porosity. The value of contact angle is a function of the wetting and non-wetting fluid properties and of the chemistry of the surface of the solid particles. Since these factors are also independent of porosity, the value of contact angle should also be independent of porosity. On this basis, most if not all of the variation can be attributed either to tortuosity or to the transition from the range of invariant permeability to the range for which permeability is a power function of capillary pressure.

### Undisturbed Media Experiment

The capillary pressure-permeability relationship has been represented by Brooks and Corey (3) by the equations 9,

$$K = K_0, \text{ for } p_c \leq p_b,$$

and

$$K = K_0 \left( \frac{p_b}{p_c} \right)^\eta, \text{ for } p_c \geq p_b,$$

as previously stated. Equations 9 describe two straight lines on a log-log plot intersecting at the coordinates  $K = K_0$  and  $p_c = p_b$ . In the range  $p_c \leq p_b$ , permeability is invariant with capillary pressure while in the range  $p_c \geq p_b$ , permeability is a power function of capillary pressure, plotting as a straight line having a slope of minus  $\eta$  on a log-log plot.

Capillary pressure-permeability curves for this experiment are included in Figures 21 to 27. For undisturbed materials having large pores between large aggregates or in cracks or holes, the relationship between permeability and capillary pressure is not well described by equations 9. In these cases, there is a gradual transition from the invariant permeability to that represented by a straight line of slope minus  $\eta$ . In all cases, however, the curves approach a straight line at the higher capillary pressures.

Samples having a much larger ratio of volume to boundary area might have a less significant transition. The probability that large pores associated with structure, worm holes, cracks and root cavities will drain at capillary pressures less than  $p_b$ , might be decreased with samples having a much larger volume.

The properties of the materials studied are tabulated in Tables 5 and 6. A somewhat surprising result for the soil materials is that, with the exception of the saturated permeabilities, the properties of the undisturbed horizontal and vertical samples differed only slightly. The vertical samples for some soils had larger permeabilities when fully saturated than did the corresponding horizontal samples. This could result from a preponderance of vertical over horizontal cracks in these soils.

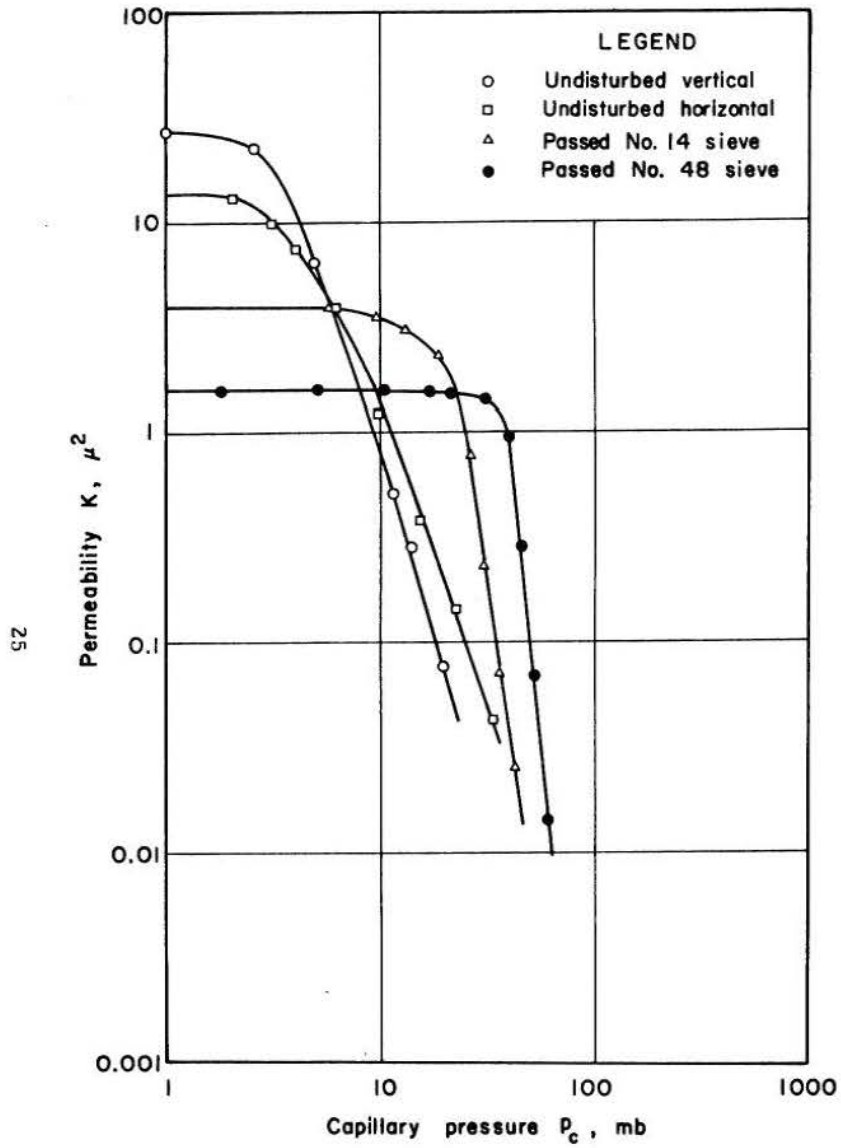


Figure 21. Permeability as a function of capillary pressure for Fort Collins clay loam.

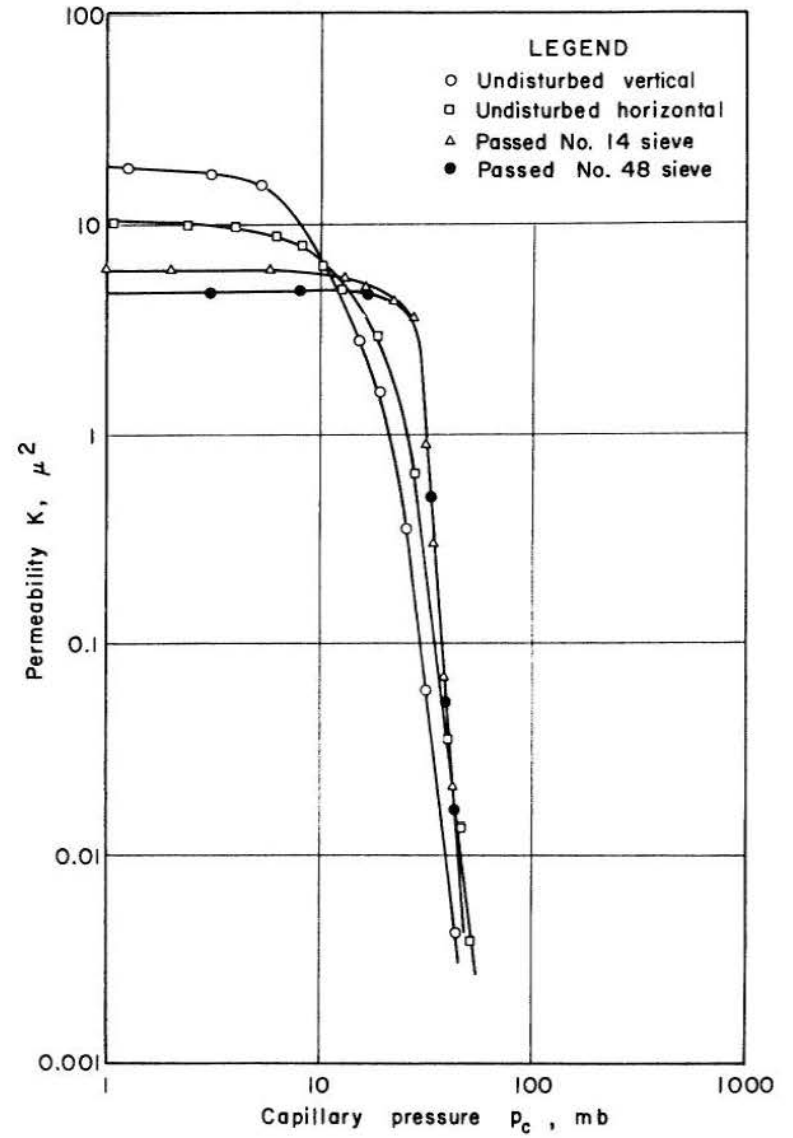


Figure 22. Permeability as a function of capillary pressure for Weld loam.

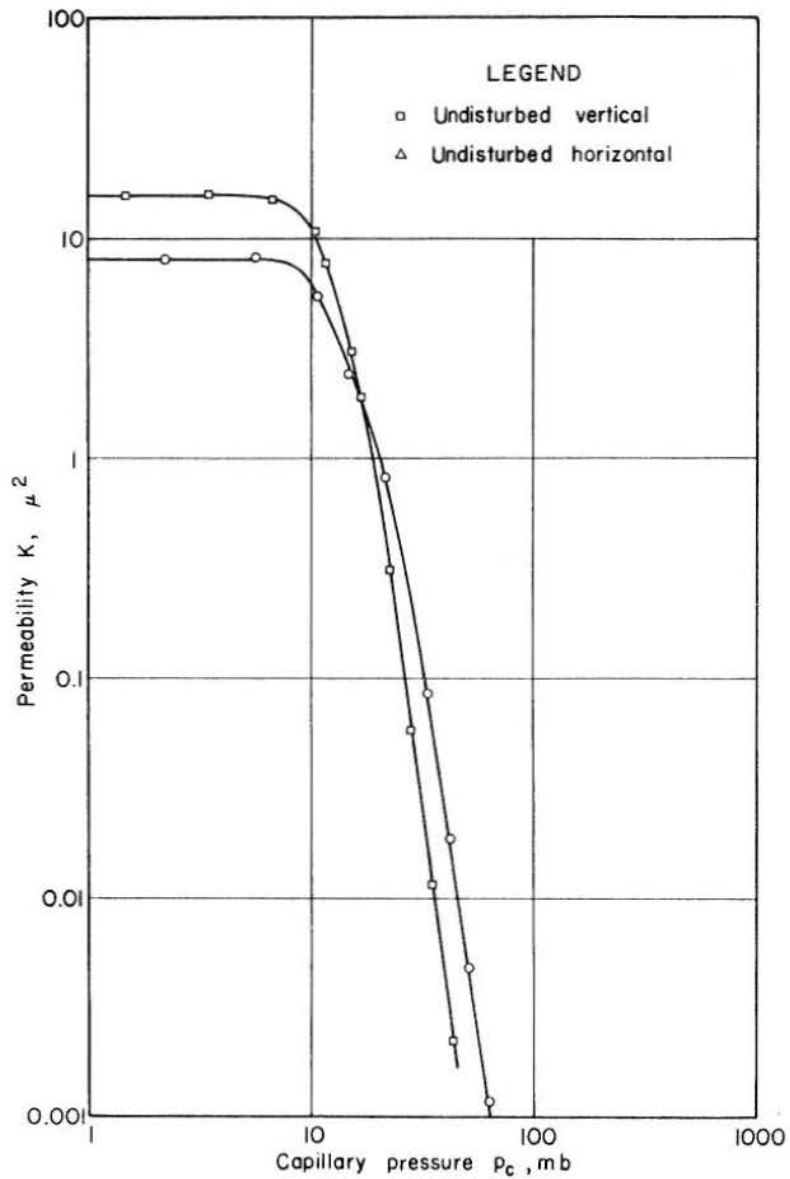


Figure 23. Permeability as a function of capillary pressure for Cass sandy loam (5-inch depth).

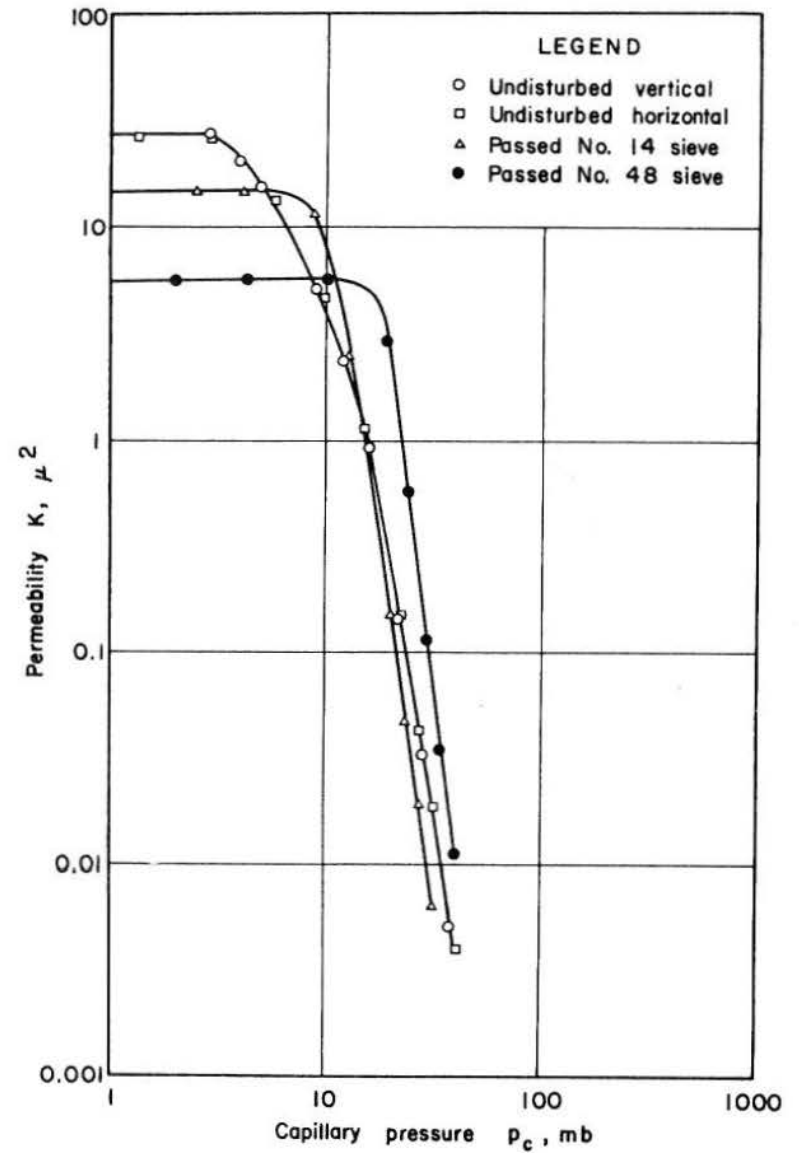


Figure 24. Permeability as a function of capillary pressure for Cass sandy loam (12-inch depth).

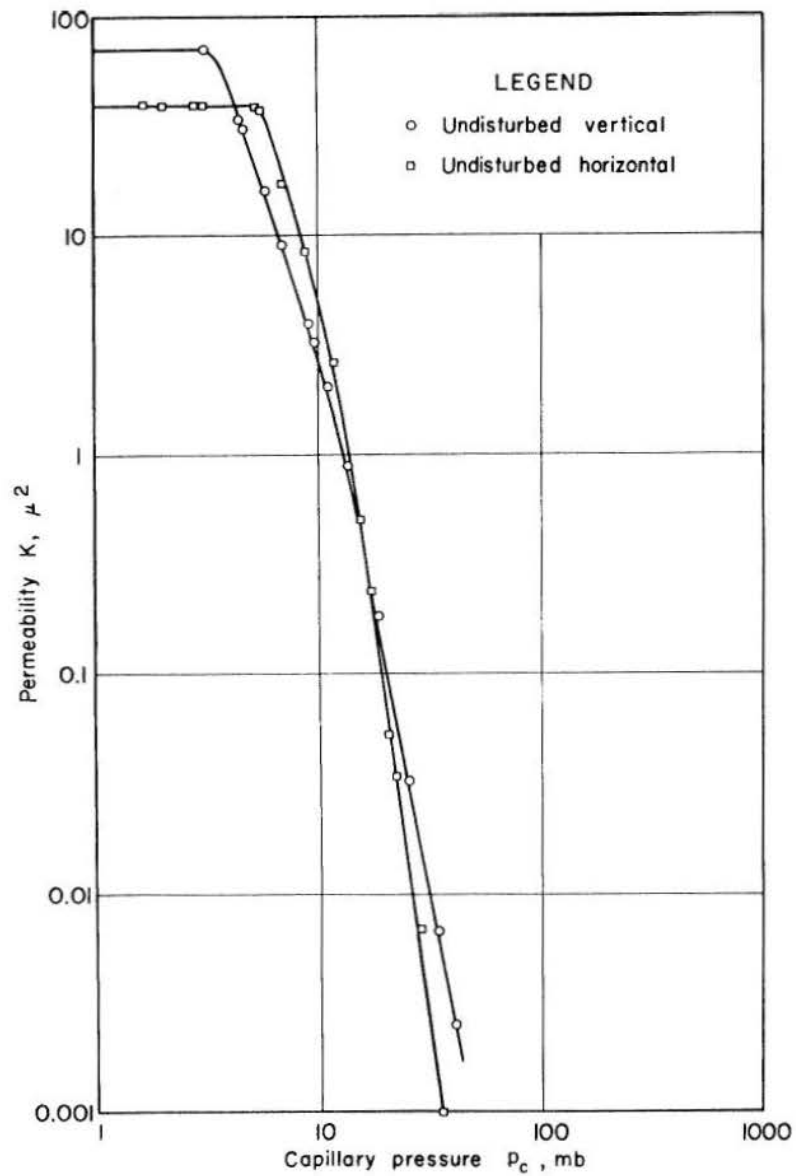


Figure 25. Permeability as a function of capillary pressure for Cass sandy loam (20-inch depth).

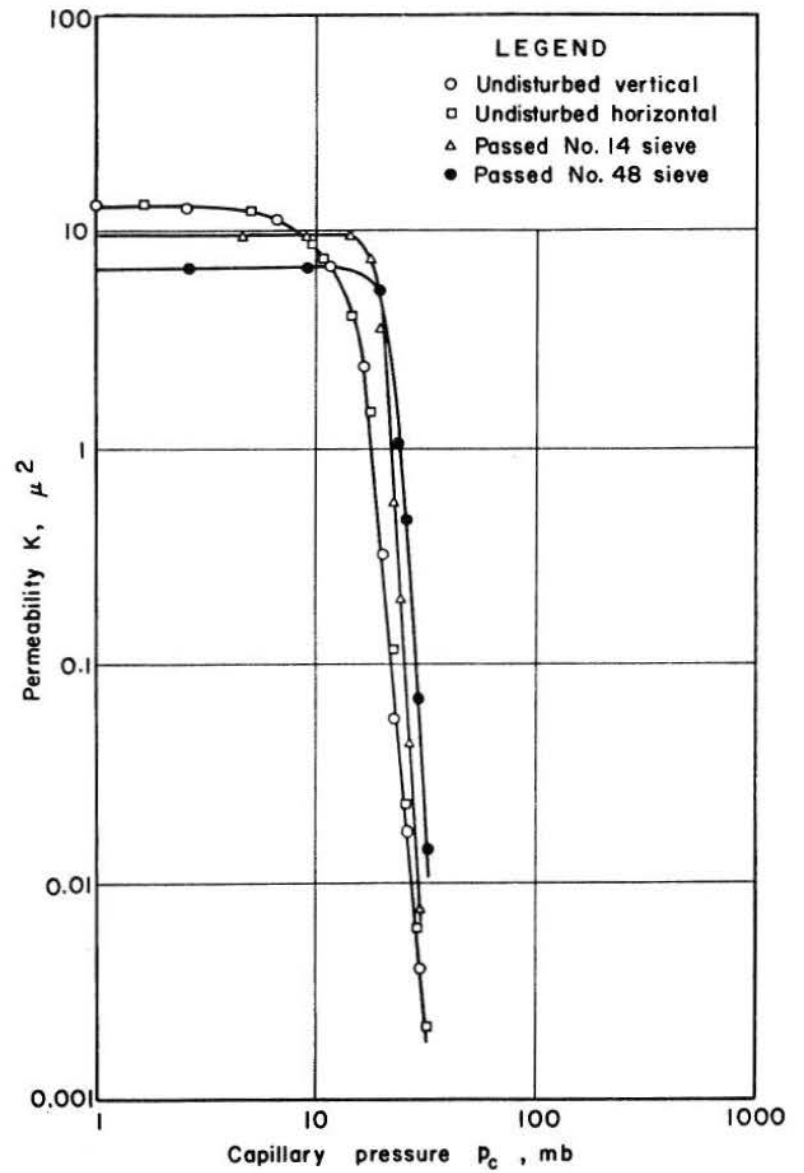


Figure 26. Permeability as a function of capillary pressure for Valentine loamy sand.

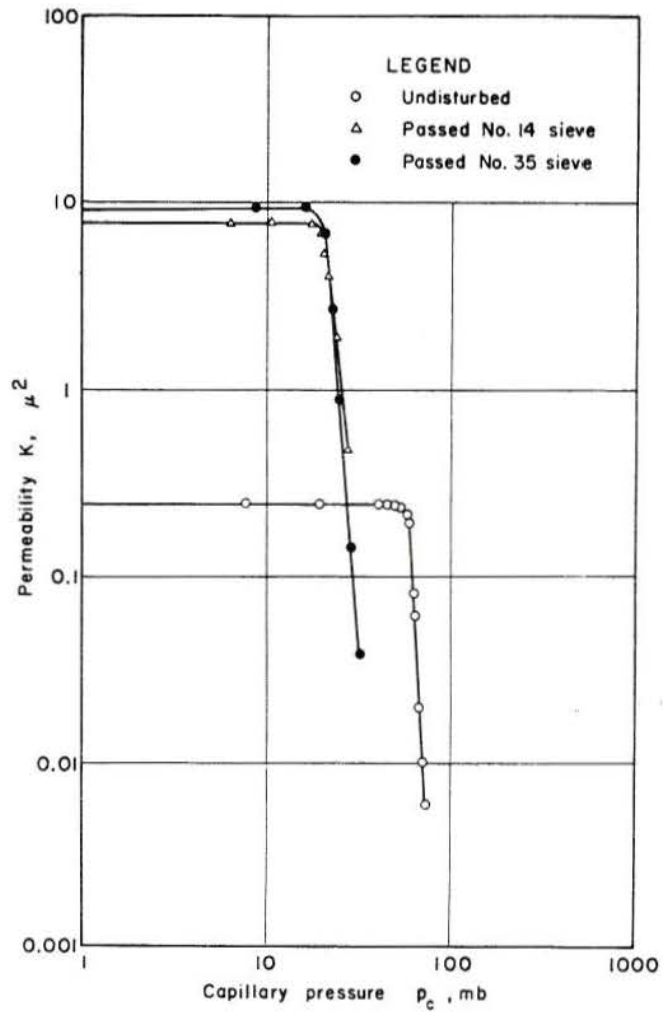


Figure 27. Permeability as a function of capillary pressure for a semi-consolidated sand.



TABLE 5. POROSITY, SATURATED PERMEABILITY, BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX FOR SEVERAL MEDIA

Treatment (orientation)		$\phi$	$K_o$ $\mu^2$	$p_b$ mb	$\eta$
Fort Collins clay loam	Undisturbed (vertical)	0.454	27.6	3.4	3.5
	Undisturbed (horizontal)	0.463	13.1	4.2	2.7
	Passed No. 14 sieve	0.472	3.99	20.0	6.8
	Passed No. 48 sieve	0.469	1.58	38.1	9.4
Weld loam	Undisturbed (vertical)	0.504	18.9	15.4	7.9
	Undisturbed (horizontal)	0.486	10.3	19.8	8.2
	Passed No. 14 sieve	0.483	6.08	27.1	12.8
	Passed No. 48 sieve	0.502	4.81	27.2	12.0
Valentine loamy sand	Undisturbed (vertical)	0.398	13.2	13.8	10.3
	Undisturbed (horizontal)	0.383	13.3	13.8	11.0
	Passed No. 14 sieve	0.396	9.59	19.0	15.6
	Passed No. 48 sieve	0.382	6.75	21.3	15.3
Semi- consolidated sand	Undisturbed	0.304	0.24	59.3	16.4
	Passed No. 14 sieve	0.505	7.87	19.9	8.2
	Passed No. 35 sieve	0.543	9.30	20.3	11.4

TABLE 6. POROSITY, SATURATED PERMEABILITY, BUBBLING PRESSURE AND PORE-SIZE DISTRIBUTION INDEX FOR CASS SANDY LOAM

Depth inches	Treatment (orientation)	$\phi$	$K_o$ $\mu^2$	$p_b$ mb	$\eta$
5	Undisturbed (vertical)	0.446	8.08	17.2	6.8
	Undisturbed (horizontal)	0.477	16.0	13.5	7.5
12	Undisturbed (vertical)	0.463	27.1	9.1	5.8
	Undisturbed (horizontal)	0.471	26.2	9.1	5.8
	Passed No. 14 sieve	0.473	14.9	10.0	6.5
	Passed No. 48 sieve	0.469	5.66	18.1	7.6
20	Undisturbed (vertical)	0.520	70.2	6.0	5.3
	Undisturbed (horizontal)	0.496	40.3	8.7	7.5

The samples of the material reported in Table 5 were taken just below the plowed layer. In order to determine if greater isotropy might exist at other depths, samples of the Cass sandy loam taken from three depths were studied. In this phase of the study, heterogeneity between horizontal and vertical samples camouflaged whatever isotropy may have been observed. Unusually large differences in porosity between horizontal and vertical samples were obtained and, although the measured values of  $K_o$ ,  $p_b$  and  $\eta$  were consistent with the porosity values, the effect of porosity differences undoubtedly was largely responsible for the variation in the properties.

In each case, the saturated permeability was lower for the disturbed soil samples than for the corresponding undisturbed material. The bubbling pressure was higher and the value of the  $\eta$  was higher, indicating a more limited range of pore sizes. For the Fort Collins clay loam and the Cass sandy loam,  $p_b$  and  $\eta$  increased the finer the soil was

pulverized. For the Weld loam and the Valentine loamy sand, little change was observed. For all four soils, permeability decreased the finer the soil was pulverized, as was expected. Again, failure to obtain exactly the same porosity in some cases probably masked whatever effect the finer sieve might have produced.

In the case of the semi-consolidated sand, pulverization produced a large increase in  $K_o$  and a large decrease in  $p_b$  and  $\eta$ . This is no doubt, a result of the fact that the pulverization did not break down the weak cementation of the sandstone completely, so that the pulverized material had more secondary porosity than the original material in the outcrop. The resulting porosity and average pore-size, therefore, was greater in the disturbed material. Since the sand studied had an unusually uniform pore size in the undisturbed state, conclusions can not be made from this study concerning the effect of pulverizing sandstones in general.

## CONCLUSIONS

Some of the problems associated with satisfying the scaling criteria for modeling partially saturated porous media were examined critically. The findings should resolve some of the problems encountered in making practical use of models. The theory developed describes how the hydraulic properties of porous media are related to each other.

Significant properties in a theory developed by Brooks and Corey (3) for modeling flow in partially saturated porous media are saturated permeability, effective porosity, bubbling pressure and pore-size distribution index. In the present study, a relationship was developed relating these hydraulic properties to each other. The relationship is given by equation 8, that is,

$$\frac{\phi_e \sigma^2}{K_o p_b^2} \left( \frac{\lambda}{\lambda + 2} \right) = 5 .$$

The validity of this relationship was tested experimentally by determining the dependence of these hydraulic properties on porosity. By varying the bulk density, it was found that as porosity increased for a particular soil, saturated permeability increased, bubbling pressure decreased and pore-size distribution index and residual saturation decreased only slightly over a wide range of porosities. Moreover, the relationship was valid within acceptable limits for estimating saturated permeability from values of the other hydraulic properties obtained from capillary pressure-desaturation data. The use of this relationship, combined with equations described previously by Brooks and Corey (3), was proposed as a simplified procedure for determining permeability as a function of capillary pressure for partially saturated media. The only data required can be obtained from capillary pressure-desaturation data.

In addition, some definite trends were observed in the dependence of the product  $kT/\cos^2\theta$  on porosity. On the basis of these trends, the general equation was proposed as a tool of determining tortuosity.

In testing the validity of this relationship, it was necessary to obtain both capillary pressure-permeability data and capillary pressure-desaturation data. Comparisons were made of bubbling pressure and pore-size distribution index values obtained separately from each set of data.

The pore-size distribution index  $\lambda$  was calculated from the capillary pressure-desaturation data using equation 10,

$$\eta = 2 + 3\lambda$$

For two soils having high bubbling pressures and intermediate values of  $\lambda$ , the bubbling pressures differed by less than 4 percent and, except for one case, the pore-size distribution indexes differed by less than 3 percent over a wide range of porosities. For an unconsolidated sand, the bubbling pressures differed by about 8 percent and the pore-size distribution indexes by 6 percent. The larger differences in the case of the sand were attributed to errors introduced by the non-uniformity of capillary pressure (and therefore saturation) which exists under static conditions.

In order to apply the modeling theory of Brooks and Corey (3) for studying prototype systems, it is necessary to measure the pertinent properties of the material as it exists undisturbed at the site. In this phase of the study, it was found that undisturbed earth materials have properties producing a wide range of hydraulic behavior. Although disturbance by pulverization may produce very large changes in the properties of materials from a particular site, the range of properties observed is not wider than that previously observed by Brooks and Corey, for disturbed materials.

It appears, therefore, that it usually will be possible to find a suitable medium for use in a laboratory model for most naturally occurring earth materials. If the prototype is an aggregated soil, however, unconsolidated sands should not be used in the model since the latter have a much narrower range of pore sizes than structured soils.

The most difficult problem might be to simulate the portion of the relationship between permeability and capillary pressure occurring at low capillary pressures. Brooks and Corey, however, previously have observed somewhat similar behavior for crushed (but not pulverized) clays. It is possible that the transition in the capillary pressure-permeability function is affected by the ratio of sample volume to boundary area. Until the significance of this effect has been determined, it is not possible to conclude whether or not the capillary pressure-permeability function for undisturbed soils with structure can be characterized adequately by the pore-size distribution index alone.

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APPENDIX A  
Properties of Wetting Fluid and Media

TABLE A-1. DYNAMIC VISCOSITY AND DENSITY OF SOLTROL "C"  
OIL USED IN EXPERIMENTS

Temp. °C	Viscosity, $\mu$ centipoises	Density, $\rho$ grams/ml	$\mu/\rho g$ cm-seconds
20.0	1.589	0.7582	2.127
20.5	1.571	0.7579	2.108
21.0	1.555	0.7576	2.089
21.5	1.539	0.7573	2.070
22.0	1.524	0.7569	2.051
22.5	1.509	0.7566	2.032
23.0	1.494	0.7562	2.014
23.5	1.481	0.7559	1.996
24.0	1.468	0.7556	1.979
24.5	1.454	0.7553	1.962
25.0	1.440	0.7549	1.945
25.5	1.427	0.7546	1.927
26.0	1.414	0.7542	1.910
26.5	1.401	0.7539	1.893
27.0	1.388	0.7536	1.877
27.5	1.375	0.7533	1.861
28.0	1.362	0.7529	1.845
28.5	1.349	0.7526	1.829
29.0	1.337	0.7522	1.814
29.5	1.326	0.7519	1.799
30.0	1.315	0.7515	1.783

TABLE A-2. GRAIN-SIZE ANALYSIS OF MATERIALS USED

Sample Description	Sand ( $d > 50\mu$ ) percent	Silt ( $5\mu < d < 50\mu$ ) percent	Clay ( $d < 5\mu$ ) percent
Touchet silt loam (GE 3)	32	53	15
Columbia sandy loam	54	35	11
Unconsolidated sand	90	6	4
Fort Collins clay loam	44	26	30
Weld loam	55	24	21
Cass sandy loam (5 in.)	64	20	16
Cass sandy loam (12 in.)	66	18	16
Cass sandy loam (20 in.)	64	18	18
Valentine loamy sand	80	4	16
Semi-consolidated sand	73	14	13

TABLE A-3. PARTICLE DENSITY OF MATERIALS USED

Sample Description	Run No. 1 gms/cm <sup>3</sup>	Run No. 2 gms/cm <sup>3</sup>	Mean gms/cm <sup>3</sup>
Touchet silt loam (GE 3)	2.601	2.597	2.599
Columbia sandy loam	2.667	2.660	2.664
Unconsolidated sand	2.707	2.705	2.706
Fort Collins clay loam	2.568	2.568	2.568
Weld loam	2.601	2.590	2.596
Cass sandy loam (5 in.)	2.622	2.628	2.625
Cass sandy loam (12 in.)	2.662	2.664	2.663
Cass sandy loam (20 in.)	2.673	2.669	2.671
Valentine loamy sand	2.632	2.632	2.632
Semi-consolidated sand	2.589	2.613	2.601



TABLE B-9. CAPILLARY PRESSURE - PERMEABILITY DATA FOR COLUMBIA SANDY LOAM\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
27.8	1.851	0.998	0.0850	1.58	0.993	1.7
27.5	1.861	0.990	0.0851	1.60	0.998	3.6
27.2	1.871	0.954	0.0817	1.60	1.000	11.5
27.2	1.871	0.954	0.0796	1.56	0.975	21.0
24.0	1.979	1.017	0.0779	1.52	0.945	33.0
25.8	1.917	1.012	0.0321	0.607	0.378	35.1
25.5	1.927	0.925	0.0199	0.415	0.258	41.2
23.5	1.996	1.058	0.00689	0.130	0.0210	48.5
26.4	1.897	1.027	0.00251	0.0464	0.0289	50.6
23.3	2.003	0.898	0.000472	0.0105	0.00656	72.7

\*  $\rho_b = 1.26$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.527$ ,  $K_o = 1.60 \mu^2$ ,  
 $p_b = 33.4$  mb,  $\eta = 6.5$

TABLE B-10. CAPILLARY PRESSURE - PERMEABILITY DATA FOR COLUMBIA SANDY LOAM\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
26.2	1.904	0.983	0.124	2.40	0.971	1.6
26.6	1.890	0.993	0.130	2.47	1.000	4.4
26.8	1.884	1.012	0.132	2.47	0.999	10.2
26.5	1.893	0.840	0.108	2.45	0.990	15.9
26.4	1.897	0.957	0.123	2.43	0.985	22.6
24.3	1.969	1.112	0.0847	1.50	0.607	30.2
25.5	1.927	1.043	0.0189	0.349	0.141	37.0
23.9	1.982	1.057	0.00566	0.106	0.0430	45.8
26.7	1.887	1.048	0.00145	0.0262	0.0106	57.2
24.5	1.962	1.220	0.000708	0.0114	0.00461	76.2
23.3	2.003	1.433	0.000119	0.00177	0.000717	92.1

\*  $\rho_b = 1.18$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.559$ ,  $K_o = 2.47 \mu^2$ ,  
 $p_b = 26.5$  mb,  $\eta = 5.8$

TABLE B-11. CAPILLARY PRESSURE - PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
23.3	2.003	0.865	3.93	91.0	1.000	1.8
23.3	2.003	0.810	3.60	89.0	0.979	5.6
23.4	2.000	1.017	3.36	66.0	0.726	6.1
23.4	2.000	0.973	1.17	24.0	0.264	6.4
23.4	2.000	0.987	0.000722	0.0131	0.000144	10.2

\*  $\rho_b = 1.58$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.417$ ,  $K_o = 16.2 \mu^2$ ,  
 $p_b = 5.9$  mb,  $\eta = 16.2$

TABLE B-12. CAPILLARY PRESSURE - PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
26.9	1.880	0.930	5.11	103.3	0.997	1.4
26.9	1.880	0.963	5.31	105.6	1.000	3.1
27.0	1.877	0.990	5.18	98.2	0.948	5.6
27.1	1.874	0.973	4.16	80.2	0.774	5.5
27.0	1.877	0.950	2.45	48.4	0.468	6.0
27.0	1.877	0.953	0.201	3.96	0.0386	6.8
26.9	1.880	0.858	0.00603	0.152	0.00128	8.9
26.1	1.907	0.787	0.000592	0.0143	0.000138	10.1

\*  $\rho_b = 1.53$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.434$ ,  $K_o = 103.6 \mu^2$ ,  
 $p_b = 5.6$  mb,  $\eta = 13.1$

TABLE B-13. CAPILLARY PRESSURE - PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
25.3	1.934	0.505	2.96	113.2	0.997	1.8
25.3	1.927	0.457	2.69	113.5	1.000	3.6
25.9	1.914	0.943	2.29	46.5	0.410	5.8
26.0	1.910	0.933	0.895	18.3	0.161	6.1
27.1	1.874	0.872	0.111	2.39	0.0210	6.8
27.0	1.877	0.900	0.0281	0.586	0.00516	7.6
27.1	1.874	0.890	0.00766	0.161	0.00142	8.3
26.1	1.907	0.855	0.00145	0.0324	0.000285	9.3

\*  $\rho_b = 1.51$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.444$ ,  $K_o = 113.5 \mu^2$ ,  
 $p_b = 9.4$  mb,  $\eta = 15.5$

TABLE B-14. CAPILLARY PRESSURE - PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
25.6	1.924	1.008	6.51	124.2	1.000	0.9
25.6	1.924	1.000	6.49	123.0	0.990	2.4
25.8	1.917	1.017	2.87	54.0	0.435	5.5
25.8	1.917	1.000	1.18	22.5	0.181	6.0
26.0	1.910	1.010	0.141	2.66	0.0214	6.8
27.0	1.877	1.026	0.0108	0.198	0.00160	8.2
25.8	1.917	1.015	0.00146	0.0276	0.000222	9.1
27.7	1.894	1.000	0.000469	0.00669	0.0000700	10.2
26.8	1.884	0.982	0.000191	0.00365	0.0000294	11.0

\*  $\rho_b = 1.48$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.452$ ,  $K_o = 124.2 \mu^2$ ,  
 $p_b = 3.2$  mb,  $\eta = 14.4$

TABLE B-15. CAPILLARY PRESSURE - PERMEABILITY DATA FOR AN UNCONSOLIDATED SAND\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
23.4	2.000	0.953	6.63	139.2	1.000	1.8
23.4	2.000	1.046	4.82	91.9	0.661	4.9
23.4	2.000	1.015	3.53	69.6	0.500	5.2
23.4	2.000	0.972	1.28	26.4	0.190	5.5
23.4	2.000	1.043	0.00360	0.0690	0.000496	8.4

\*  $\rho_b = 1.46$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.460$ ,  $K_o = 139.2 \mu^2$ ,  
 $p_b = 4.9$  mb,  $\eta = 14.1$

TABLE B-16. CAPILLARY PRESSURE - PERMEABILITY DATA FOR FORT COLLINS CLAY LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY)\*

Temp. °C	$\mu/\rho_g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K$ $\mu^2$	$K_r$	$p_c$ mb
23.7	1.989	0.994	1.38	27.6	1.000	0.5
23.7	1.989	1.000	1.12	22.3	0.808	2.6
23.7	1.989	1.000	0.322	6.41	0.232	4.9
23.7	1.989	0.975	0.0250	0.511	0.0185	11.5
23.6	1.993	1.038	0.0146	0.280	0.0101	13.9
23.6	1.993	0.975	0.00360	0.0776	0.00280	19.7

\*  $\rho_b = 1.40$  gms/cc,  $\rho_s = 2.57$  gms/cc,  $\phi = 0.454$ ,  $K_o = 27.6 \mu^2$ ,  
 $p_b = 3.7$  mb,  $\eta = 3.5$





TABLE B-25. CAPILLARY PRESSURE - PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 5-INCH DEPTH)\*

Temp. °C	$\mu/\rho g \times 10^6$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.2	1.938	0.991	0.809	15.8	0.891	1.5
25.1	1.941	0.968	0.786	16.0	1.000	3.5
25.1	1.941	1.005	0.787	15.2	0.952	6.6
24.8	1.952	1.081	0.606	10.9	0.685	10.1
24.7	1.955	1.005	0.398	7.74	0.484	11.6
24.6	1.959	1.025	0.164	3.13	0.196	15.0
24.3	1.969	1.008	0.0974	1.90	0.119	16.8
24.0	1.979	1.032	0.0165	0.316	0.0198	22.8
25.2	1.938	1.001	0.00304	0.0589	0.00369	28.4
24.7	1.955	0.985	0.000584	0.0116	0.000726	35.1
24.9	1.948	1.024	0.000120	0.00229	0.000144	43.9

\*  $\rho_b = 1.37$  gms/cc,  $\rho_s = 2.62$  gms/cc,  $\phi = 0.477$ ,  $K_o = 16.0 \mu^2$ ,  $P_b = 13.5$  mb,  $\eta = 7.5$

TABLE B-26. CAPILLARY PRESSURE - PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY AT 12-INCH DEPTH)\*

Temp. °C	$\mu/\rho g \times 10^6$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.1	1.941	0.861	1.20	27.1	1.000	2.9
25.2	1.938	1.005	1.05	20.2	0.746	4.0
25.1	1.941	1.000	0.796	15.4	0.569	5.0
25.1	1.941	1.015	0.262	5.00	0.184	9.2
24.9	1.948	1.018	0.120	2.30	0.0847	12.3
25.1	1.941	0.961	0.0450	0.909	0.0335	16.2
25.0	1.945	0.872	0.00651	0.145	0.00535	22.2
25.1	1.941	0.912	0.00155	0.0330	0.00121	29.1
25.0	1.945	0.699	0.000184	0.00513	0.000189	38.8

\*  $\rho_b = 1.43$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.464$ ,  $K_o = 27.1 \mu^2$ ,  $P_b = 9.1$  mb,  $\eta = 5.8$

TABLE B-27. CAPILLARY PRESSURE - PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 12-INCH DEPTH)\*

Temp. °C	$\mu/\rho g \times 10^6$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
24.0	1.979	1.000	1.32	26.2	1.000	1.4
24.0	1.979	0.985	1.29	25.9	0.991	3.0
23.8	1.986	0.994	0.658	13.2	0.503	5.8
24.0	1.979	1.102	0.235	4.59	0.176	9.8
24.3	1.969	1.029	0.0583	1.12	0.0426	15.3
23.2	2.007	1.104	0.00824	0.150	0.00572	22.5
23.7	1.989	1.045	0.00224	0.0427	0.00163	27.9
24.4	1.965	1.034	0.000984	0.0187	0.000713	32.2
25.9	1.914	0.991	0.000209	0.00403	0.000154	42.2

\*  $\rho_b = 1.41$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.471$ ,  $K_o = 26.2 \mu^2$ ,  $P_b = 9.1$  mb,  $\eta = 5.7$

TABLE B-28. CAPILLARY PRESSURE - PERMEABILITY DATA FOR CASS SANDY LOAM (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 14 SIEVE)\*

Temp. °C	$\mu/\rho g \times 10^6$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.0	1.945	0.983	0.748	14.8	0.893	2.5
25.0	1.945	0.950	0.728	14.9	1.000	4.2
25.0	1.945	0.985	0.587	11.6	0.777	8.9
25.0	1.945	1.022	0.131	2.50	0.168	13.2
25.0	1.945	0.988	0.00771	0.152	0.102	20.6
25.0	1.945	1.017	0.00247	0.0473	0.00317	24.3
25.8	1.917	0.997	0.000989	0.0190	0.00128	28.3
25.0	1.945	0.977	0.000318	0.00634	0.000425	33.2

\*  $\rho_b = 1.40$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.473$ ,  $K_o = 14.9 \mu^2$ ,  $P_b = 13.6$  mb,  $\eta = 6.5$

TABLE B-29. CAPILLARY PRESSURE - PERMEABILITY DATA FOR CASS SANDY LOAM (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 48 SIEVE)\*

Temp. °C	$\mu/\rho g \times 10^6$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
23.9	1.982	0.995	0.284	5.66	1.000	2.1
23.8	1.986	0.983	0.277	5.60	0.991	4.3
23.8	1.986	0.950	0.266	5.57	0.985	10.3
23.5	1.996	0.897	0.224	5.00	0.884	15.9
23.4	2.000	0.970	0.137	2.83	0.500	19.3
23.4	2.000	1.062	0.0308	0.570	0.101	24.6
23.4	2.000	1.033	0.00591	0.114	0.0202	30.0
23.5	1.996	0.975	0.00169	0.0346	0.00611	34.9
23.7	1.989	0.988	0.000557	0.0111	0.00196	41.4

\*  $\rho_b = 1.41$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.469$ ,  $K_o = 5.66 \mu^2$ ,  $P_b = 18.1$  mb,  $\eta = 7.6$

TABLE B-30. CAPILLARY PRESSURE - PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN VERTICALLY AT 20-INCH DEPTH)\*

Temp. °C	$\mu/\rho g \times 10^6$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
26.1	1.907	0.552	2.04	70.2	1.000	3.1
25.2	1.938	1.038	1.84	34.3	0.489	4.5
25.6	1.924	1.010	1.60	30.5	0.434	4.7
25.2	1.938	1.055	0.874	16.1	0.229	5.9
25.3	1.934	0.964	0.452	9.08	0.129	7.1
25.7	1.920	1.015	0.207	3.91	0.0557	9.2
25.0	1.945	0.961	0.162	3.28	0.0467	9.8
25.2	1.938	1.008	0.105	2.02	0.0288	11.2
25.1	1.941	0.926	0.0424	0.888	0.0126	13.6
26.1	1.907	0.990	0.00952	0.183	0.00261	18.6
25.3	1.934	1.046	0.00174	0.0322	0.000458	25.5
25.9	1.914	1.051	0.000375	0.00683	0.0000972	34.4
26.0	1.910	0.979	0.000130	0.00253	0.0000361	41.5

\*  $\rho_b = 1.28$  gms/cc,  $\rho_s = 2.67$  gms/cc,  $\phi = 0.520$ ,  $K_o = 70.2 \mu^2$ ,  $P_b = 6.0$  mb,  $\eta = 5.3$

TABLE B-31. CAPILLARY PRESSURE - PERMEABILITY DATA FOR CASS SANDY LOAM (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 20-INCH DEPTH)\*

Temp. °C	$\mu/\rho g \times 10^6$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
26.2	1.904	1.011	2.14	40.3	1.000	1.7
25.4	1.931	0.998	2.05	39.6	0.984	2.0
25.7	1.920	0.981	2.02	39.4	0.978	2.8
25.3	1.934	0.938	1.92	39.6	0.982	3.0
25.0	1.945	0.929	1.87	39.2	0.974	5.2
25.7	1.920	0.996	1.96	37.8	0.937	5.5
25.0	1.945	1.035	0.931	17.5	0.434	6.9
26.0	1.910	1.088	0.481	8.44	0.209	8.7
25.3	1.934	1.026	0.145	2.74	0.0679	11.9
25.6	1.924	1.068	0.0279	0.502	0.0125	15.5
24.9	1.948	1.025	0.0129	0.245	0.00608	17.3
25.3	1.934	1.021	0.00329	0.0623	0.00155	20.6
25.6	1.924	1.144	0.00205	0.0344	0.000855	22.4
25.9	1.914	1.039	0.000385	0.00709	0.000176	28.2
26.0	1.910	0.988	0.0000485	0.000938	0.0000233	35.6

\*  $\rho_b = 1.35$  gms/cc,  $\rho_s = 2.67$  gms/cc,  $\phi = 0.496$ ,  $K_o = 40.3 \mu^2$ ,  $P_b = 8.7$  mb,  $\eta = 7.5$

TABLE B-32. CAPILLARY PRESSURE - PERMEABILITY DATA FOR VALENTINE LOAMY SAND (UNDISTURBED SAMPLE TAKEN VERTICALLY AT 12-INCH DEPTH)\*

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.7	1.920	0.985	0.679	13.2	1.000	0.9
25.6	1.924	1.000	0.663	12.8	0.963	2.6
25.6	1.924	0.975	0.573	11.3	0.854	6.7
23.9	1.982	0.957	0.330	6.83	0.516	11.8
24.0	1.979	0.963	0.113	2.32	0.175	16.2
24.5	1.962	1.194	0.0198	0.321	0.0243	20.4
24.9	1.948	0.835	0.00241	0.0562	0.00425	23.0
24.9	1.948	0.862	0.000750	0.0170	0.00128	26.1
26.1	1.907	0.953	0.000203	0.00406	0.000307	30.4

\*  $\rho_b = 1.58$  gms/cc,  $\rho_s = 2.63$  gms/cc,  $\phi = 0.398$ ,  $K_o = 13.2 \mu^2$ ,  $P_b = 13.8$  mb,  $\eta = 10.3$

TABLE B-33. CAPILLARY PRESSURE - PERMEABILITY DATA FOR VALENTINE LOAMY SAND (UNDISTURBED SAMPLE TAKEN HORIZONTALLY AT 12-INCH DEPTH)\*

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.0	1.945	1.026	0.683	13.3	1.000	1.7
25.0	1.945	1.038	0.642	12.3	0.928	5.1
25.2	1.938	1.019	0.445	8.67	0.653	9.7
25.3	1.934	1.015	0.379	7.41	0.558	10.9
25.5	1.927	1.067	0.217	4.02	0.302	14.8
25.7	1.920	1.024	0.0781	1.46	0.110	17.8
23.9	1.982	1.129	0.00676	0.119	0.00893	22.8
25.0	1.945	1.078	0.00122	0.0227	0.00171	25.9
25.0	1.945	0.872	0.000281	0.00624	0.000472	29.4
25.3	1.934	0.915	0.000103	0.00217	0.000163	32.4

\*  $\rho_b = 1.62$  gms/cc,  $\rho_s = 2.63$  gms/cc,  $\phi = 0.383$ ,  $K_o = 13.3 \mu^2$ ,  $P_b = 13.8$  mb,  $\eta = 11.0$

TABLE B-34. CAPILLARY PRESSURE - PERMEABILITY DATA FOR VALENTINE LOAMY SAND (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 14 SIEVE)\*

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.0	1.945	0.998	0.454	8.85	0.923	1.5
24.8	1.952	0.935	0.459	9.59	1.000	4.7
25.0	1.945	0.940	0.462	9.57	0.998	9.1
25.8	1.917	0.958	0.473	9.46	0.986	14.3
25.8	1.917	0.987	0.386	7.51	0.783	17.7
26.0	1.910	1.032	0.194	3.58	0.374	19.8
25.0	1.945	0.983	0.0284	0.561	0.0585	22.9
25.0	1.945	1.040	0.0106	0.199	0.0207	24.4
25.5	1.927	1.082	0.00241	0.0429	0.00447	26.9
25.1	1.941	1.085	0.000421	0.00754	0.000786	30.1

\*  $\rho_b = 1.59$  gms/cc,  $\rho_s = 2.63$  gms/cc,  $\phi = 0.396$ ,  $K_o = 9.59 \mu^2$ ,  $P_b = 19.0$  mb,  $\eta = 15.6$

TABLE B-38. CAPILLARY PRESSURE - PERMEABILITY DATA FOR SEMI-CONSOLIDATED SAND (DISTURBED SAMPLE PASSED THROUGH A NO. 35 SIEVE)\*

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.0	1.945	0.99	0.450	8.82	0.96	9.0
25.0	1.945	0.99	0.470	9.21	1.00	16.5
25.0	1.945	1.00	0.337	6.55	0.71	20.9
25.0	1.945	1.07	0.138	2.51	0.27	22.8
25.0	1.945	0.99	0.0428	0.840	0.091	24.7
25.0	1.945	1.00	0.00716	0.139	0.015	29.1
25.0	1.945	1.10	0.00211	0.0372	0.0040	33.0

\*  $\rho_b = 1.19$  gms/cc,  $\rho_s = 2.60$  gms/cc,  $\phi = 0.543$ ,  $K_o = 9.21 \mu^2$ ,  $P_b = 20.3$  mb,  $\eta = 11.4$

TABLE B-35. CAPILLARY PRESSURE - PERMEABILITY DATA FOR VALENTINE LOAMY SAND (DISTURBED SAMPLE TAKEN AT 12-INCH DEPTH AND PASSED THROUGH A NO. 48 SIEVE)\*

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
23.5	1.996	1.005	0.335	6.66	0.987	2.7
23.4	2.000	0.968	0.327	6.75	1.000	9.2
23.6	1.993	0.832	0.219	5.25	0.778	19.2
23.6	1.993	0.715	0.0368	1.03	0.152	23.6
23.3	2.003	0.647	0.0100	0.310	0.0460	26.0
23.5	1.996	0.693	0.00163	0.0469	0.00695	29.5
23.3	2.003	0.827	0.000396	0.00960	0.00142	32.6

\*  $\rho_b = 1.63$  gms/cc,  $\rho_s = 2.63$  gms/cc,  $\phi = 0.381$ ,  $K_o = 6.75 \mu^2$ ,  $P_b = 21.3$  mb,  $\eta = 15.3$

TABLE B-36. CAPILLARY PRESSURE - PERMEABILITY DATA FOR A SEMI-CONSOLIDATED SAND (UNDISTURBED CORE)\*

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
26.2	1.904	1.07	0.0138	0.245	1.00	7.7
24.8	1.952	1.02	0.0128	0.245	1.00	19.6
24.8	1.952	1.10	0.0138	0.245	1.00	41.0
24.6	1.959	1.01	0.0124	0.240	0.98	45.6
26.1	1.907	1.01	0.0127	0.241	0.98	51.5
24.6	1.959	1.01	0.0121	0.236	0.97	54.6
24.5	1.962	1.06	0.0115	0.214	0.87	59.4
25.0	1.945	1.09	0.0108	0.193	0.79	60.0
25.0	1.945	1.02	0.00425	0.0808	0.33	64.2
24.5	1.962	1.14	0.00363	0.0627	0.26	64.7
24.5	1.962	1.17	0.00121	0.0201	0.082	67.8
25.0	1.945	1.10	0.000570	0.0100	0.041	72.0
25.0	1.945	1.10	0.000337	0.00594	0.024	74.6

\*  $\rho_b = 1.81$  gms/cc,  $\rho_s = 2.60$  gms/cc,  $\phi = 0.304$ ,  $K_o = 0.245 \mu^2$ ,  $P_b = 59.3$  mb,  $\eta = 16.4$

TABLE B-37. CAPILLARY PRESSURE - PERMEABILITY DATA FOR A SEMI-CONSOLIDATED SAND (DISTURBED SAMPLE, PASSED THROUGH A NO. 14 SIEVE)\*

Temp. °C	$\mu/\rho g \times 10^5$ cm-sec	$\frac{\Delta H}{\Delta L}$	$q \times 10^2$ cm/sec	$K \mu^2$	$K_r$	$P_c$ mb
25.0	1.945	0.97	0.370	7.44	1.00	6.5
25.0	1.945	0.97	0.369	7.42	1.00	10.9
25.0	1.945	0.93	0.349	7.27	0.98	17.6
25.0	1.945	0.98	0.329	6.51	0.87	20.1
25.0	1.945	0.87	0.226	5.07	0.68	20.6
25.0	1.945	0.85	0.168	3.84	0.52	21.8
25.0	1.945	0.75	0.0759	1.97	0.26	24.4
25.0	1.945	0.95	0.0220	0.450	0.060	27.6

\*  $\rho_b = 1.29$  gms/cc,  $\rho_s = 2.60$  gms/cc,  $\phi = 0.505$ ,  $K_o = 7.44 \mu^2$ ,  $P_b = 19.9$  mb,  $\eta = 8.2$

APPENDIX C  
Capillary Pressure-Desaturation Data

TABLE C-1. CAPILLARY PRESSURE - DESATURATION DATA FOR TOUCHET SILT LOAM (GE3)\*

$P_c$ mb	S	$S_e$
3.4	1.000	1.000
15.0	0.990	0.999
36.9	0.993	0.991
59.6	0.978	0.971
71.0	0.964	0.954
82.1	0.877	0.842
93.0	0.731	0.655
104.1	0.639	0.537
120.8	0.545	0.417
142.0	0.472	0.323
160.3	0.429	0.268
181.6	0.397	0.227
202.2	0.366	0.187
240.3	0.322	0.131
276.6	0.308	0.113
307.9	0.287	0.0856

\*  $\rho_b = 1.48$  gms/cc,  $\rho_s = 2.60$  gms/cc,  $\phi = 0.430$ ,  $S_r = 0.22$ ,  
 $p_b = 72.8$  mb,  $\lambda = 1.67$

TABLE C-2. CAPILLARY PRESSURE - DESATURATION DATA FOR TOUCHET SILT LOAM (GE3)\*

$P_c$ mb	S	$S_e$
6.4	1.000	1.000
17.1	1.000	1.000
36.4	0.994	0.993
49.3	0.986	0.982
62.6	0.935	0.917
71.3	0.824	0.774
81.4	0.711	0.630
94.3	0.605	0.493
104.1	0.559	0.434
116.4	0.506	0.366
131.6	0.462	0.310
149.1	0.428	0.267
169.7	0.389	0.217
201.7	0.346	0.162
236.2	0.324	0.134
271.0	0.300	0.103

\*  $\rho_b = 1.40$  gms/cc,  $\rho_s = 2.60$  gms/cc,  $\phi = 0.463$ ,  $S_r = 0.22$ ,  
 $p_b = 59.2$  mb,  $\lambda = 1.47$

TABLE C-3. CAPILLARY PRESSURE - DESATURATION DATA FOR TOUCHET SILT LOAM (GE3)\*

$P_c$ mb	S	$S_e$
6.8	1.000	1.000
16.5	0.998	0.998
34.9	0.982	0.978
41.6	0.979	0.974
48.4	0.946	0.934
58.2	0.794	0.745
64.6	0.724	0.660
78.2	0.601	0.507
96.2	0.516	0.402
116.5	0.446	0.317
135.1	0.402	0.262
152.5	0.372	0.225
174.7	0.348	0.195
211.8	0.308	0.146
248.0	0.290	0.124
274.5	0.281	0.112

\*  $\rho_b = 1.32$  gms/cc,  $\rho_s = 2.60$  gms/cc,  $\phi = 0.493$ ,  $S_r = 0.19$ ,  
 $p_b = 45.8$  mb,  $\lambda = 1.23$

TABLE C-4. CAPILLARY PRESSURE - DESATURATION DATA FOR COLUMBIA SANDY LOAM\*

$P_c$ mb	S	$S_e$
4.6	1.000	1.000
9.8	0.997	0.996
13.5	0.997	0.996
16.9	0.996	0.995
25.6	0.988	0.985
31.8	0.986	0.981
37.6	0.984	0.979
42.0	0.983	0.978
45.6	0.976	0.969
49.8	0.966	0.957
54.4	0.957	0.944
58.2	0.915	0.891
62.0	0.865	0.827
70.9	0.724	0.646
79.1	0.640	0.538
90.2	0.552	0.426

\*  $\rho_b = 1.44$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.458$ ,  $S_r = 0.22$ ,  
 $p_b = 55.6$  mb,  $\lambda = 1.76$

TABLE C-5. CAPILLARY PRESSURE - DESATURATION DATA FOR COLUMBIA SANDY LOAM\*

$P_c$ mb	S	$S_e$
6.7	1.000	1.000
12.4	0.998	0.997
15.9	0.990	0.987
24.0	0.978	0.972
27.7	0.973	0.966
31.1	0.970	0.962
35.4	0.966	0.956
39.8	0.955	0.942
43.8	0.916	0.892
46.6	0.873	0.837
50.5	0.808	0.754
57.6	0.702	0.618
65.6	0.608	0.497
76.1	0.524	0.389
85.1	0.471	0.322
95.2	0.427	0.266
102.4	0.407	0.240
116.2	0.374	0.197
127.1	0.350	0.167
137.8	0.336	0.148
152.7	0.316	0.123
167.2	0.302	0.105

\*  $\rho_b = 1.34$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.496$ ,  $S_r = 0.22$ ,  
 $p_b = 42.6$  mb,  $\lambda = 1.63$

TABLE C-6. CAPILLARY PRESSURE - DESATURATION DATA FOR COLUMBIA SANDY LOAM\*

$p_c$ mb	S	$S_e$
2.8	1.000	1.000
6.4	0.999	0.999
13.6	0.997	0.996
21.0	0.996	0.995
28.1	0.995	0.994
32.4	0.991	0.989
39.1	0.853	0.811
43.0	0.775	0.712
44.8	0.734	0.659
46.3	0.708	0.626
49.6	0.657	0.561
52.9	0.608	0.487
57.9	0.562	0.438
64.7	0.508	0.370
70.1	0.480	0.333
85.6	0.410	0.243
103.1	0.359	0.179
120.3	0.328	0.139
131.2	0.318	0.126
143.0	0.303	0.106
153.4	0.294	0.0949
167.7	0.280	0.0765

\*  $\rho_b = 1.28$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.518$ ,  $S_r = 0.22$ ,  
 $p_b = 34.1$  mb,  $\lambda = 1.57$

TABLE C-7. CAPILLARY PRESSURE - DESATURATION DATA FOR COLUMBIA SANDY LOAM\*

$p_c$ mb	S	$S_e$
4.6	1.000	1.000
11.6	0.997	0.996
18.9	0.995	0.994
29.2	0.935	0.920
32.5	0.875	0.845
36.8	0.763	0.707
41.9	0.657	0.676
49.9	0.548	0.442
61.0	0.455	0.327
76.0	0.384	0.239
94.2	0.329	0.172
119.3	0.284	0.115

\*  $\rho_b = 1.22$  gms/cc,  $\rho_s = 2.66$  gms/cc,  $\phi = 0.544$ ,  $S_r = 0.19$ ,  
 $p_b = 29.2$  mb,  $\lambda = 1.52$

TABLE C-8. CAPILLARY PRESSURE - DESATURATION DATA FOR AN UNCONSOLIDATED SAND\*

$p_c$ mb	S	$S_e$
3.4	0.969	0.966
5.1	0.902	0.892
5.7	0.843	0.828
6.2	0.729	0.703
6.8	0.514	0.466
7.4	0.353	0.289
8.7	0.235	0.160
12.0	0.126	0.0390
15.3	0.102	0.0132

\*  $\rho_b = 1.56$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.424$ ,  $S_r = 0.090$ ,  
 $p_b = 5.7$  mb,  $\lambda = 4.38$

TABLE C-9. CAPILLARY PRESSURE - DESATURATION DATA FOR AN UNCONSOLIDATED SAND\*

$p_c$ mb	S	$S_e$
5.1	0.951	0.946
6.4	0.541	0.497
7.0	0.402	0.345
8.1	0.252	0.179
9.2	0.180	0.101
11.3	0.135	0.0519
13.5	0.109	0.0230
17.3	0.0940	0.00658

\*  $\rho_b = 1.53$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.435$ ,  $S_r = 0.088$ ,  
 $p_b = 5.4$  mb,  $\lambda = 4.26$

TABLE C-10. CAPILLARY PRESSURE - DESATURATION DATA FOR AN UNCONSOLIDATED SAND\*

$p_c$ mb	S	$S_e$
4.1	0.955	0.951
5.5	0.894	0.884
6.1	0.829	0.813
6.6	0.589	0.550
7.3	0.394	0.337
8.0	0.293	0.225
9.0	0.220	0.145
11.5	0.130	0.0472
15.0	0.102	0.0160

\*  $\rho_b = 1.52$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.439$ ,  $S_r = 0.087$ ,  
 $p_b = 5.7$  mb,  $\lambda = 4.31$

TABLE C-11. CAPILLARY PRESSURE - DESATURATION DATA FOR AN UNCONSOLIDATED SAND\*

$p_c$ mb	S	$S_e$
5.2	0.781	0.760
6.2	0.438	0.385
6.9	0.336	0.273
8.0	0.196	0.121
9.9	0.140	0.0586
12.2	0.109	0.0256
15.1	0.0943	0.00908

\*  $\rho_b = 1.50$  gms/cc,  $\rho_s = 2.71$  gms/cc,  $\phi = 0.445$ ,  $S_r = 0.086$ ,  
 $p_b = 4.8$  mb,  $\lambda = 4.16$

Key Words: Similitude, Permeability, Porosity, Pore-size, Bubbling pressure, Undisturbed media, Scaling criteria, Partially saturated, Soils, Capillary pressure.

Abstract: Some of the problems associated with satisfying the scaling criteria of the Brooks-Corey modeling theory for partially saturated porous media are examined critically. The effect of the porosity of disturbed media on the hydraulic properties which are significant in the modeling theory is determined experimentally by varying porosity. The results indicate that the pore-size distribution index is changed only slightly over a wide range of porosities but permeability and bubbling pressure may be changed several fold over the same range. Evidently, bubbling pressure and permeability may be adjusted to suit the size of the model by changing the porosity without appreciably changing the pore-size distribution. A functional relationship among the hydraulic properties which are significant in the modeling theory is developed, beginning with the fundamental equations used in

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