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STATISTICAL METHODS OF ANALYSIS APPLIED TO THE FEEDLOT GAINS OF LAMBS

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The number of lambs necessary in a feeding test to permit statistical analysis of the data obtained is a problem of primary importance.

As the profits of lamb feeding have decreased, individual lamb feeders have studied more carefully the rations they are using, in order to produce more economical gains. Experimentation which will permit more accurate conclusions to be drawn from the results obtained will materially assist lamb feeders because it will increase the reliability of their decisions concerning the rations they use.

Variation in the ability of individual lambs to gain in the feedlot is a recognized factor influencing the interpretation of experimental results. These variations can be readily observed by studying lamb-feeding data, nearly all of which will show greater fluctuation between the lambs fed the same ration than there is between the mean gains of the lots of the comparison.

A study of these variations is reported in this study as they affect the number of lambs needed in a feeding test, to permit a statistical analysis of the data obtained.

LITERATURE REVIEW

Mitchell and Grindley (1), working on a similar problem, found that individuality provides an important influence in the interpretation of sheep-feeding data. They state that the average is at best only an imperfect description of a series of experimental data, and that the average gives very little basis upon which other workers can judge the conclusions. The use of the probable error of the difference in determining the significance of the results is recommended. Their study showed a coefficient of variation of 21 percent for feedlot lambs. They conclude that at least 10 to 14 animals should be fed in each lot and that rations of much similarity should be tested with from 25 to 30 animals, calling attention to the fact that the beneficial effects from increasing the number of animals in the lots does not increase in proportion to the number but in proportion to the square root of the number.

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They showed that physiological selection does not eliminate the poor gainers, that wethers gain faster than ewes, and that good gains invariably mean uniform gains. They maintain that individual feeding is not necessary and not desirable if the practical side of the experiment is to be emphasized, admitting, however, that individual feeding would reduce the experimental error. With regard to the publication of results, they say that there must be a reasonable probability that the practical livestock farmer will benefit by applying the results to his livestock operations. If no probability exists he should be specifically warned.

Crampton (2) states that comparative feeding trials are an attempt to determine the relative value of certain feeds or feed combinations with groups of live animals whose individual response to the same treatment has been shown to be exceedingly variable. The accuracy of an average is dependent upon the number of animals involved and the variability of their response. He states further that in group feeding only the variations in gains can be obtained, while such variations as occur in feed consumption can only be estimated. A close agreement between the variability of hogs group-fed and individually fed is reported. Biologically there should be no less correlation in group feeding than in individually feeding, between feed eaten by an animal and the resultant gains in live weight.

Crampton (3) in another study showed that group-fed animals often do not represent a true cross-section of the populations to which they belong and for this reason statistical treatment may lead to errors in the conclusions. Comparing group feeding with individual feeding, using Students' method of paired comparison, he points out that the latter method permits using the extremes of the sample in the test as long as the members of the pairs are closely alike. In this way, the best, the poorest and the average of the group are possible experimental subjects and random sampling is at least not interfered with. In conclusion he asserts that the adaptation of statistical methods in the analysis of data from feeding trials offers a measure of reliability which, coupled with good common sense, will enable the experimenter to arrive at sound conclusions.

The literature review emphasizes the extent to which variation in ability of the individual lamb to gain affects the reliability of the mean or average results of the lot. The problem then in the choice of material was to secure, insofar as possible, data that would permit analysis of this influence and to draw conclusions as to the best method of giving it due weight.

MATERIAL

The lamb-feeding records of the Colorado Experiment Station were available as a source of material. It was indicated by a preliminary analysis that these records could, in a large part, be made to answer the need of this specific problem.

The following considerations determined the data selected from the station records:

The experimental data of the Colorado Station are based entirely on group feeding. The station records give an individual weight record of each lamb, including initial and final weights and the daily gain and total gain for the period. The feed records being based on lambs group-fed, give the average daily feed per lamb, the average amount of feed consumed per 100 pounds of gain. The feed-replacement value of the feed being studied can be computed from these records.

With variability in the individual lamb the major consideration of the problem, any data that did not permit measurement of this fact were of little value to the study. Therefore, it seemed advisable to confine the work to a study of the weight and gain data.

The next consideration was the limiting of the data used to a point where they would provide ample material for sound conclusions and at the same time eliminate unnecessary volume from the evidence. In this connection it was felt that more would be gained thru making an exhaustive study of a limited amount of data rather than a partial analysis of the extensive accumulation in the station records.

Further examination of the records showed that the data from the corn-alfalfa hay and corn-wet beet pulp-alfalfa hay rations were admirably suited to the purposes of the study.

Six years of comparisons make up the data on these rations. Starting in the feeding season of 1920-1921, a total of 146 lambs have been fed on corn-wet beet pulp-alfalfa hay rations for comparison with a like number of lambs fed on rations of corn-alfalfa hay. The records available are for the seasons of 1920-1921, 1922-1923, 1923-1924, 1924-1925, 1925-1926 and 1926-1927.

EXPERIMENTAL RESULTS

Before proceeding with a discussion of the experimental results, some accepted level of significance will be needed. There is a general agreement among investigators that odds of 20:1 are sufficient to establish results as significant. Such odds are evidenced when the mean difference divided by the probable error equals three or more when the mean difference divided by the standard error equals two or more.

Fisher (5) in a table for "t" values gives a measure of probability in which "t" equals the mean difference divided by the standard error. The table goes further in the determination of odds by recognizing the influence of the number of replications or, for the purpose of this study, the number of animals in the test on the probability. Probability as P. is given for the various "t" values. Because of the precaution offered in recognizing the influence of the number of replications and the convenience of this table, the odds of the calculations are based on standard error with P. = .05 or 20:1 as the level of significance.

ANALYSIS OF ORIGINAL DATA.—As mentioned previously, there were 6 years of data with which to deal. They represented feeding periods of unequal lengths. There was one feeding period of 75 days, two of 90 days, one of 93 days, one of 100 days and one of 120 days. These feeding periods had to be reduced to a uniform length if the data were to be handled satisfactorily. The total gain and daily gain of each lamb were first computed.

In reducing these feeding periods to a uniform length, the question arose as to whether there was any appreciable variation in the daily gains of the lambs due to the difference in the time on feed. To answer this question, a correlation study was made with daily gain as the dependent variable. The corn-alfalfa hay-wet beet pulp lambs were studied first. The data from these lambs were set up in a frequency table and a correlation coefficient calculated. The correlation coefficient obtained was —0.096. The data from the corn-alfalfa hay lambs were analyzed by the same method and a correlation coefficient of 0.0415 was secured. These analyses indicated that the variation in the lengths of the feeding periods from 75 to 120 days could be reduced to a standard without introducing any material error in the results.

One hundred days was chosen as a standard length for the feeding period because it more nearly represents the average used by the experiment station and the practical feeder and because it materially simplifies the manipulation of the data.

Using the data for the 100-day gain as the basis of comparison, the results of the 6 years' trials were now analyzed to determine their significance. For the purpose of this analysis the standard method of determining standard error (4) was chosen. The results of these analyses are shown in Table 1.

From Table 1 it will be seen that in no case were the data of the 6 years' trials reversed and all showed significant differences with the exception of No. 2. In this trial the odds of 1.66:1 in favor of the ration where wet beet pulp was added failed to be significant. This fact and the variations found in the mean gains of the six trials emphasize the need of an experimental set-up

Table 1.—Analysis of 6 Years' Original Data of Comparative Rations With Sheep.

	Mean Gain Pounds	S.D.	S.E.M.
Trial No. 1			
Corn-Alfalfa Hay, 16 lambs	36.4	7.13	1.84
Corn-Alfalfa Hay-Wet Beet Pulp, 16 lambs	42.0	7.00	1.80
Mean Difference	5.60		
Standard Error of Mean Difference	2.57		
"t" Value	2.171		
Odds for Significance	20:1		

	Mean Gain Pounds	S.D.	S.E.M.
Trial No. 2			
Corn-Alfalfa Hay, 23 lambs	35.7	9.99	2.13
Corn-Alfalfa Hay-Wet Beet Pulp, 23 lambs	37.2	8.80	1.87
Mean Difference	1.48		
Standard Error of Mean Difference	2.83		
"t" Value	0.522		
Odds for Significance	1.66:1		

	Mean Gain Pounds	S.D.	S.E.M.
Trial No. 3			
Corn-Alfalfa Hay, 24 lambs	33.1	5.62	1.17
Corn-Alfalfa Hay-Wet Beet Pulp, 24 lambs	38.7	7.10	1.48
Mean Difference	5.59		
Standard Error of Mean Difference	1.88		
"t" Value	2.968		
Odds for Significance	100:1		

	Mean Gain Pounds	S.D.	S.E.M.
Trial No. 4			
Corn-Alfalfa Hay, 25 lambs	35.2	7.61	1.55
Corn-Alfalfa Hay-Wet Beet Pulp, 25 lambs	40.0	7.59	1.54
Mean Difference	4.80		
Standard Error of Mean Difference	2.18		
"t" Value	2.201		
Odds for Significance	35:1		

	Mean Gain Pounds	S.D.	S.E.M.
Trial No. 5			
Corn-Alfalfa Hay, 26 lambs	37.5	7.72	1.54
Corn-Alfalfa Hay-Wet Beet Pulp, 26 lambs	44.7	9.50	1.90
Mean Difference	7.25		
Standard Error of Mean Difference	2.44		
"t" Value	2.971		
Odds for Significance	100:1		

	Mean Gain Pounds	S.D.	S.E.M.
Trial No. 6			
Corn-Alfalfa Hay, 32 lambs	32.2	9.01	1.62
Corn-Alfalfa Hay-Wet Beet Pulp, 32 lambs	36.8	7.60	1.34
Mean Difference	4.65		
Standard Error of Mean Difference	2.10		
"t" Value	2.214		
Odds for Significance	50:1		

Average Mean Difference in Gains of All Trials	= 4.80 lbs.
Standard Error of the Mean Difference, All Trials	= 0.95
"t" Value All Trials	= 5.052
Odds for Significance—Very Large.	

that will permit leveling out of such extreme fluctuations. It will be noted further that two of the six trials gave odds of 100:1, one gave odds of 50:1, one gave odds of 35:1 and one gave odds of 20:1. Combining the results of these 6 years' trials by computing the standard error of an average of averages (8) gives the highly significant "t" value of 5.052.

A study of the table shows the extreme variation in gains due to factors influencing individual lambs. In trial No. 2 this variation reaches its high point for the corn-alfalfa hay lambs with a standard deviation of 9.99 and in trial No. 3 the other extreme is reached with a standard deviation of 5.62.

Another point worthy of mention at this time, because it will be developed at a greater length in another part of the study, is the reduction in the value of the standard error of the mean due to the increase in the number of lambs on trial. In the corn-wet beet pulp-alfalfa hay lot of trial No. 1 the standard deviation is 7.00 and for this lot, $N = 16$. In the corn-wet beet pulp-alfalfa hay lot of trial No. 6 the standard deviation is 7.60 and for this lot, $N = 32$. The standard error of the mean for trial No. 1 is 1.80, while in trial No. 6 it is 1.34. Thus from a greater standard deviation a lesser standard error is secured, emphasizing the fact that the standard error decreases in ratio to the increase of the square root of the number in the population.

In view of the variations shown in the standard deviations, a further study of variations was next decided upon. The results of this study are shown in Table 2.

This table gives the coefficients of variation for the 12 lots of lambs compared in the 6 years of trials. The coefficients of variation for the lambs fed the corn-alfalfa hay ration showed greater extremes of variation than did the corn-alfalfa hay-wet beet pulp lambs. In the former the range of variation was from 16.94 to 27.92 or 10.98 percent, while in the case of the latter ration the range was from 16.68 to 23.61 or 6.93 percent. The

Table 2.—Showing the Variation in the Ability of Lambs to Gain, by Use of the Coefficient of Variation.

Corn-Alfalfa Hay				Corn-Wet Beet Pulp-Alfalfa Hay			
Trial No.	M. Gain Lbs.	S.D.	C.V.	Trial No.	M. Gain Lbs.	S.D.	C.V.
1	36.4	7.13	19.59	1	42.0	7.00	16.68
2	35.7	9.99	27.92	2	37.2	8.80	23.61
3	33.1	5.62	16.94	3	38.7	7.10	18.32
4	35.2	7.61	21.60	4	40.0	7.59	18.95
5	37.5	7.72	20.57	5	44.7	9.50	21.21
6	32.2	9.01	27.95	6	36.8	7.60	20.61
Average			= 22.43	Average			= 19.73
Average both lots					= 21.08		

average coefficient of variation for the corn-alfalfa hay lambs was 22.43 percent, while in the case of the corn-alfalfa hay-wet beet pulp lambs the average coefficient of variation was 19.73 percent, indicating a lesser variation where better gains are made.

The average coefficient of variation of all lambs shows remarkable agreement with the work reported by Mitchell and Grindley (1), the coefficient of variation secured in this analysis being 21.08 percent, while their results gave 21.0 percent.

In the light of the coefficient of variation secured in the analysis of the six trials, the next question was, "What measurable factors in the data caused these variations?" The individuality of the lambs as to breeding and conformation is not on record, but there is a record of the sex and the initial weight of the lambs and the effects of these factors were next analyzed.

One hundred and three wether lambs and 43 ewe lambs were fed on rations of corn-alfalfa hay and 106 wether lambs and 40 ewe lambs were fed on rations of corn-wet beet pulp-alfalfa hay. Table 3 shows the results of the analyses of the differences caused by sex.

The results of the analyses gives a "t" value of 1.355, with odds of 5:1 in favor of the wether lambs fed on corn-alfalfa hay rations; a "t" value of 2.525, with odds of 75:1 in favor of the wether lambs fed on corn-wet beet pulp-alfalfa hay, and a "t" value of 2.806, with odds of 100:1 for all wether lambs compared with all ewe lambs. With the level of significance for odds 20:1, the difference of wether and ewe lambs fed corn and alfalfa hay is not sufficiently positive. In the case of the corn-wet beet pulp-alfalfa hay lambs the odds of 50:1 are significant and this fact combined with the odds of 100:1 that were secured when all lambs were compared, makes it apparent that sex does materially influence the gain.

Table 3.—A Study of Differences Caused by Sex.

	Ration of Corn-Alfalfa Hay	Ration of Corn-Alf. Hay-W. Beet Pulp
Mean Gain, pounds, Wether Lambs	35.51	40.37
Mean Gain, pounds, Ewe Lambs	33.29	36.63
Mean Difference, pounds	2.22	3.74
Standard Error of Mean Difference	1.638	1.481
"t" Value	1.355	2.525
Odds for Significance	5:1	75:1

All Wethers Compared All Ewes

Mean Gain, All Wethers	= 38.01
Mean Gain, All Ewes	= 34.90
Mean Difference, Pounds	= 3.11
Standard Error of Mean Difference	= 1.108
"t" Value	= 2.806
Odds for Significance	= 100:1

One hundred three wether lambs and 43 ewe lambs were fed a ration of corn-alfalfa hay and 106 wether lambs and 40 ewe lambs were fed a ration of corn-alfalfa hay-wet beet pulp.

To determine the effect of the initial weights on daily gains, the lambs of the two rations were studied in a correlation table set up with initial weight as the independent variable and daily gain as the dependent variable. In these calculations the data of the corn-alfalfa hay rations gave a correlation coefficient between initial weight and daily gain of 0.090 and the corn-wet beet pulp-alfalfa hay data gave a correlation coefficient of 0.066.

These calculations complete the work with the original data as such. They have been made in an effort to determine the effect of variation on the reliability of experimental results and this effect has been increasingly apparent in every calculation thus far.

ANALYSIS OF DATA FROM RANDOMIZED GROUPS.—The data on each lamb was now recorded on an individual index card and the lambs of each year's trials were paired (3). In this pairing process it seemed good procedure to pair lambs of as nearly the same initial weights as possible; however, where variation between the initial weights was necessary, no particular concern was felt in the light of the correlation results. As mentioned before, sex was recognized as an important factor in the pairing process and, since it was not possible to pair lambs of the same sex at all times, it was recognized that this discrepancy might constitute a systematic error in the procedure. However, the two rations are almost evenly represented as to sex and, while there may be some minor discrepancies due to this error, they will in the final analysis prove to be compensating.

The object in pairing the data from the lambs of the original lots was to permit a more extensive application of statistical methods of analysis. By random selection new lots of lambs were now set up, which for the purpose of this study, constitute years of experimental work made to order. To recognize the seasonal influences on gains, the distribution of the lambs in the 6 years of trials was determined on a percentage basis and, while selection for the new trials was by random within the years, between the years the percentage governed the allotment.

With these restrictions in mind, the following new groups were drawn: 3 groups 2 pairs, 3 groups 3 pairs, 3 groups 4 pairs, 30 groups 5 pairs, 30 groups 10 pairs, 3 groups 20 pairs, 3 groups 30 pairs, 3 groups 40 pairs, 3 groups 60 pairs and 3 groups 80 pairs.

The first analysis to which any of the randomized groups were subjected was for the purpose of comparing three statistical methods. For this comparison the statistical methods chosen were: Students' method of paired comparison (6), standard method of determining standard error, and deviation-of-the-mean method (7). To make this comparison 30 groups of 10 pairs were analyzed by each of the methods. In order to facilitate comparison, the results were all computed on the basis of standard errors.

The results of the Students' method of paired comparison and standard method of determining standard error analyses are shown in Table 4.

From these analyses it will be seen that 19 of the 30 trials showed greater significance when analyzed by Students' method of paired comparison. In further testing these results, they themselves were subjected to statistical analysis to see if the difference had remained within the realm of chance. The analysis gave a "t" value of 3.131, with odds better than 100:1 that the difference is due to the method.

The results obtained by use of the deviation-of-the-mean method are compared with the results of Students' method of paired comparison in Table 5.

Here again analysis by Students' method of paired comparison gave more significance than did the other method. Subjecting the results to statistical analysis gave a "t" value of 3.145, with odds better than 100:1 that the difference is due to the method rather than chance.

The results of these two analyses indicate that thru the use of Students' method of paired comparison, significance may be demonstrated in a smaller mean difference, and for this reason the method was used in the remaining calculations of the paper.

It is probably well at this point to include a short discussion on the differences of the three methods of analysis that contrib-

Table 4.—Showing "t" Values Secured When 30 Trials of 10 Random Pairs Were Analyzed by Students' Method of Paired Comparison and the Standard Method of Determining Standard Error with an Analysis of the "t" Values to Determine Whether the Differences Are Significant.

"t" Values Standard Method	"t" Values Students' Method	Dif.	(Dif.)^2
2.237	2.325	.088	.007744
1.218	1.114	—.104	.010816
.708	.971	.263	.069169
1.039	1.201	.162	.026244
3.825	4.610	.785	.616265
1.762	1.519	—.243	.059049
.529	.970	.441	.194481
1.193	1.088	—.105	.011025
1.628	2.339	.711	.505521
2.103	2.736	.633	.400689
2.102	2.703	.601	.361201
2.637	3.422	.785	.616225
1.664	1.193	—.471	.221841
1.917	1.557	—.360	.129600
2.012	2.580	.568	.322624
.908	1.912	1.004	1.008016
1.755	1.917	.162	.026244
1.507	1.784	.277	.076729
1.458	1.810	.352	.123904
2.092	2.574	.482	.232324
1.317	1.244	—.073	.005329
2.459	2.563	.104	.010816
.217	.203	—.014	.000196
.664	.830	.166	.027556
1.628	1.510	—.118	.013924
1.400	1.302	—.098	.009604
.847	.896	.049	.002401
2.084	1.977	—.107	.011449
1.552	1.876	.324	.104976
1.953	1.849	—.104	.010814

Mean Difference = .2089
Standard Deviation = .359
Standard Error of Mean Difference = .0667
"t" Value = 3.131
Odds for Significance—Very Large.

uted to the differences in the "t" values secured. Since the mean differences remain the same in every instance of comparison, the difference in the "t" value must be due to a difference in the methods of weighing the variation. Examination of the methods proves this true.

With the standard method of determining standard error, the mean gain of each of the lots compared is first determined. The differences between the mean gain and the individual gains are then secured and squared. These values are summated and from the product the standard deviation and standard error are computed. Securing the standard error in this manner for both lots, the standard error for the mean difference is then secured by squaring the standard errors of the two lots, adding the results and taking the square root of the sum.

Table 5.—Showing "t" Values Secured When 30 Trials of 10 Random Pairs Were Analyzed by Students' Method of Paired Comparison and the Deviation-of-the-Mean Method with an Analysis of the "t" Values to Determine Whether the Differences Are Significant.

"t" Values Dev.-of-Mean Method	"t" Values Students' Method	Dif.	(Dif.)^2
2.237	2.325	.088	.007744
1.258	1.114	—.144	.020736
.709	.971	.262	.068644
1.083	1.201	.118	.013924
3.875	4.610	.735	.540225
1.762	1.519	—.243	.059049
.529	.970	.441	.194481
1.193	1.088	—.105	.011025
1.623	2.339	.716	.512656
1.943	2.736	.793	.628849
2.096	2.703	.607	.368449
2.631	3.422	.791	.625681
1.660	1.193	—.467	.218089
1.890	1.557	—.333	.110889
2.007	2.580	.573	.139129
.908	1.912	1.004	1.008016
1.751	1.917	.166	.027556
1.507	1.784	.277	.076729
1.458	1.810	.352	.123904
2.081	2.574	.493	.243049
1.317	1.244	—.073	.005329
2.459	2.563	.104	.010816
.218	.203	—.015	.010816
.664	.830	.166	.027556
1.628	1.510	—.118	.013924
1.396	1.302	—.094	.008836
.845	.896	.051	.002601
2.078	1.977	—.101	.010201
1.546	1.876	.330	.108900
1.944	1.849	—.095	.009025

Mean Difference = 0.2098
Standard Deviation = 0.359
Standard Error of Mean Difference = 0.0667
"t" Value = 3.145
Odds for Significance—Very Large.

The deviation-of-the-mean method changes this procedure at the point where the standard deviation is determined. The differences between the mean gain and individual gains are secured the same as with the standard method of determining standard error. But instead of computing the standard deviation for each lot the squared differences in both lots are summed and the standard deviation of the experiment is determined. From this the standard deviation in percentage is determined for the mean of all individuals. The standard error in percentage is then secured for the number of individuals in each lot. This percentage times the mean of the lot gives the standard error for the mean of the lot. The standard error for the difference is then secured by squaring the standard errors of the two lots, summing the results and securing the square root of this value.

In Students' method of paired comparison the variation from the mean of each lot is not determined. Here the standard deviation is computed from the differences between the individuals of the pairs. These differences are squared and summed and from the value the standard deviation is secured. Then the standard error of the mean difference is computed.

From these discussions it will be seen that the standard error of the standard method is based on the variation of the individual within the lots separately; the standard error by the deviation-of-the-mean method is based on the variation of the individuals of the entire experiment; and the standard error for Students' method of paired comparison is based on the variation between the individuals of each pair in the comparison. Thus the results shown in Tables 4 and 5 were secured because Students' method of paired comparison takes into account any correlation between the animals of the pairs.

In the discussion of the original data attention was called to the fact that standard error of the difference decreased in ratio to the square root of the number. In order to demonstrate this fact further and to establish information on which to base calculations for the number of pairs needed for experimentation, two sets of pairs were analyzed.

The set-up for the first of these analyses was taken from the 30 groups of 5 pairs. With the influence of N on the "t" value as the main objective, these groups were combined and analyzed so that the mean difference and standard deviation could be held as nearly constant as possible. To do this the groups were combined as follows: First trial groups 1, 2 and 3; second trial groups 1, 2, 3, 4, 5 and 6, etc., adding three new groups each time until the analyses represented 5 pairs taken 3 times to 5 pairs taken 21 times, or an increase in the effect of N from 15 to 105. The results are shown in Table 6.

Table 6.—Showing Results Secured When Trials of Five Pairs Were Analyzed by Students' Method of Paired Comparison in Combinations of 3, 6, 9, 12, 15, 18 and 21. Used to Demonstrate the Effect on the Significance of the "t" Value When the Number of Pairs Is Increased.

Combinations	Number Pairs	"t" Value for odds 20:1	"t" Value Secured	Standard Deviaton	Mean Difference, Pounds Gain	Standard Error of Mean Dif.
Trials 1, 2, 3	15	2.145	2.602	8.97	6.22	2.39
Trials 1 to 6, inc.	30	2.045	2.760	11.49	5.88	2.13
Trials 1 to 9, inc.	45	1.959	3.292	10.90	5.40	1.64
Trials 1 to 12, inc.	60	1.959	4.295	11.00	6.10	1 42
Trials 1 to 15, inc.	75	1.959	4.808	10.94	6.01	1.25
Trials 1 to 18, inc.	90	1.959	5.330	11.82	6.61	1.24
Trials 1 to 21, inc.	105	1.959	5.822	12.15	6.87	1.18

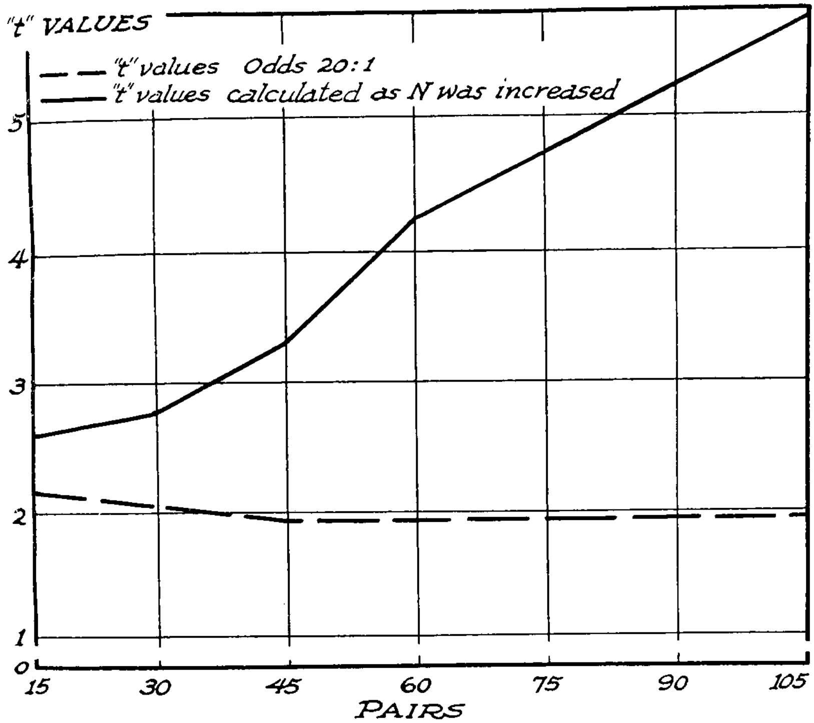

Figure No. 1.

An examination of the table shows that the method used in the set-up maintained a fairly constant effect on the mean difference and standard deviation. Both of these values showed a tendency to increase as N increased, which was to be expected because the nature of the set-up introduced a cumulative effect on these sums. These variations in the mean differences and standard deviations were reflected in the "t" values secured, but the fact that the "t" values increased in each instance that N was increased, regardless of the fact that the mean difference and standard deviation changed their relationship, served to emphasize the importance of the number of pairs of animals used in determining the significance of a difference.

This effect of increasing N is better shown in Figure 1, where the "t" values secured in the calculations and the "t" values for odds of 20:1 have been plotted. In this graph it will be noted that when N=60 a straight-line curve was secured.

The second series of calculations were made to demonstrate the effect of N on the "t" value and to give some indication as to the point where the increase of N was no longer necessary for satisfactory experimental procedure. For this purpose three

Table 7.—Showing Results Obtained When Single Groups and Combinations of Three Groups of 2, 3, 4, 5, 10, 20, 30, 40, 60 and 80 Pairs Were Analyzed by Students' Method of Paired Comparison.

N-1 used for numbers less than 50.

	"t" Value Single Trial	"t" Value for Odds 20:1	"t" Value 3 Trials Comb.	"t" Value for Odds 20:1
2 Pairs	4.166			
	7.371			
	2.677*	12.706	0.970	2.571
3 Pairs	1.234			
	.571			
	.864	4.303	1.172	2.306
4 Pairs	1.354			
	1.350			
	0.643	3.182	3.123	2.201
5 Pairs	.728			
	2.129			
	1.547	2.776	2.602 Sig	2.145
10 Pairs	2.325 Sig			
	1.114			
	0.971	2.262	2.465 Sig	2.045
20 Pairs	2.230 Sig			
	2.208 Sig			
	1.513	2.093	3.737 Sig	1.959
30 Pairs	1.938			
	2.746 Sig			
	3.438 Sig	2.045	4.339 Sig	1.959
40 Pairs	3.086 Sig			
	2.319 Sig			
	2.095 Sig	1.959	4.400 Sig	1.959
60 Pairs	4.578 Sig			
	2.985 Sig			
	3.849 Sig	1.959	6.464 Sig	1.959
80 Pairs	4.222 Sig			
	3.349 Sig			
	4.113 Sig	1.959	7.398 Sig	1.959

*Reversal favoring Corn-Alfalfa Hay Ration.

groups of 2 pairs, 3 pairs, 4 pairs, 5 pairs, 10 pairs, 20 pairs, 30 pairs, 40 pairs, 60 pairs and 80 pairs were used. The results from the single groups and from the combinations are shown in Table 7.

In the first column of the table the value of "t" is given for the individual trials. It will be noted that no significant value for "t" was secured in this column until N=10 pairs. At this point one value of significance was secured in the three trials. With N=20 pairs, two "t" values of significance were secured, and with N=30 pairs, two "t" values of significance were secured. Beginning with N=40 pairs, the "t" values of these single analyses were all significant for odds of 20:1.

In column two of the table are given "t" values for odds of 20:1 taken from Fisher's table for "t" values.

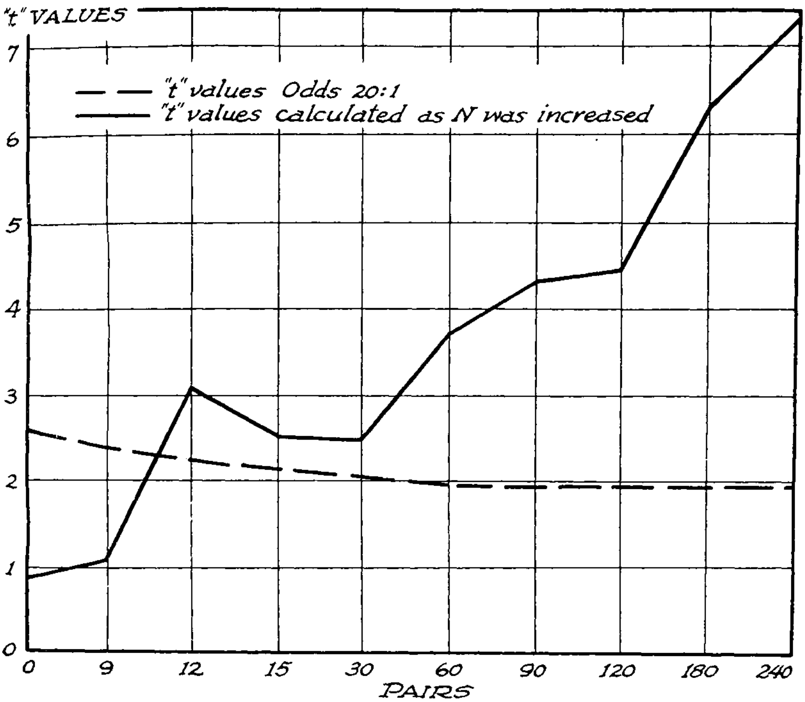

Figure No. 2.

In column three the "t" values of the three trials combined are given and in this column it is noticeable that there is no consistency in the relation of the "t" values until N=30 or 3 trials of 10 pairs combined. At this point apparently the difference due to the ration begins to supercede the differences due to chance and the "t" values increase each time N is increased. The leveling out process of these two influences is better shown in Figure 2, which gives a line showing the "t" values of three trial combinations and the "t" values for odds of 20:1 taken from Fisher's table of "t" values. The erratic behavior of the curve continues until it reaches 10 pairs 3 trials, when it shows a gradual increase toward "t" values of greater significance.

From these calculations evidence was secured that between the point where N=30 and N=40 the effect of chance on the gain became less and the difference due to the ration became more apparent.

The results of the calculations shown in Table 7 indicated that chance was largely responsible for the significance of the "t" value secured where N was less than 30. In order to demonstrate further this indication 30 random groups of 5 pairs

Table 8.—Showing "t" Values Secured from Analysis by Students' Method of Paired Comparison of Single Trials of 5 Pairs, Combination of 3 Trials 5 Pairs, Single Trials 10 Pairs and Combinations of 3 Trials 10 Pairs.

Trial No.	"t" Values Single Trial 5 Pairs	"t" Values Com. 3 Trials 5 Pairs	"t" Values Single Trial 10 Pairs	"t" Values Com. 3 Trials 10 Pairs
1	.728		2.325 Sig	
2	2.129		1.114	
3	1.547	2.602 Sig	0.971	2.465 Sig
4	1.489		1.201	
5	1.198		4.610 Sig	
6	.450	1.530	1.519	3.301 Sig
7	.933		.970	
8	2.270		1.088	
9	1.051	1.769	2.339 Sig	2.440 Sig
10	4.325 Sig		2.736 Sig	
11	1.340		2.703 Sig	
12	.480	2.772 Sig	3.422 Sig	5.233 Sig
13	.751		1.193	
14	1.318		1.557	
15	1.172	1.945	2.580 Sig	2.875 Sig
16	7.181 Sig		1.912	
17	.096		1.917	
18	1.217	2.382 Sig	1.784	2.709 Sig
19	.011		1.810	
20	1.297		2.574 Sig	
21	1.986	2.239 Sig	1.244	3.310 Sig
22	1.661		2.563 Sig	
23	.761		.203	
24	.909	2.047	.830	1.826
25	.708		1.510	
26	.296		1.302	
27	2.044	1.383	.896	2.126 Sig
28	.419		1.977	
29	1.076		1.876	
30	1.797	1.975	1.849	3.205 Sig

"t" Value for Odds 20:1 N= 4=2.776
"t" Value for Odds 20:1 N=14=2.145
"t" Value for Odds 20:1 N= 9=2.262
"t" Value for Odds 20:1 N=29=2.045

each were analyzed singly and in combinations of three. The results are shown in Table 8 and compared with the results of the random groups of 10 pairs analyzed in the same manner.

The first column of Table 8 shows the results obtained when groups of 5 pairs were analyzed singly. It is noticeable that in the analysis of single groups two "t" values of significance were secured. In the second column, showing the results when groups of 5 pairs in combinations of three were analyzed, four of the ten "t" values were significant. In the third column, showing the analysis of groups of 10 pairs singly, there are 9 out of 30 "t" values that are significant, and in the fourth column, which shows the results for the analysis of the 10 pair groups in combinations of 3, 9 of the 10 "t" values are significant. In all cases except two the "t" values of N=30 were greater than those secured

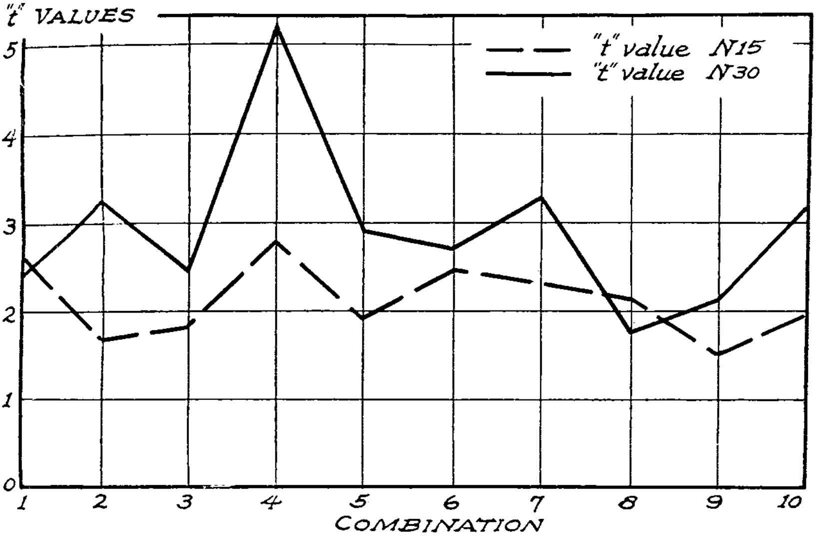

Figure No. 3.

when N=15. As mentioned previously, these calculations were made in an effort to answer the question of the influence of chance on the value of "t" when N was less than 30. The results shown in this table further emphasize the erratic behavior of chance in this connection and indicate that the number of pairs should be 30 or larger.

The relationship of the "t" values secured from these analyses is better shown in Figure 3.

Referring to Table 7 again, it will be noticed that in the first and third columns significant "t" values begin to appear in the zone N=30 and N=40.

To test this indication further, random groups of 10 pairs were assembled to secure 10 groups of 10 pairs in combinations of three, 10 groups of 10 pairs in combinations of four, and 10 groups of 10 pairs in combinations of five. The results of these calculations are shown in Table 9.

In the first column of this table the "t" values of the single trials are shown. They are included because they serve to emphasize the uniform behavior in the "t" values when N is increased. In the second column of the table are shown the "t" values for 10 pairs 3 combinations N=30. In this column 9 of the 10 "t" values are significant for odds of 20:1. The effect of chance is in evidence in the fourth trial of this group with a "t" value of 5.233, which exceeds even the "t" value of N=50. It is noticeable that this extreme variation is not overcome when the number is

Table 9.—Showing "t" Values Secured When Single Trials of 10 Pairs, Combinations of 3 Trials 10 Pairs, Combinations of 4 Trials 10 Pairs, and Combinations of 5 Trials 10 Pairs Were Analyzed by Students' Method of Paired Comparison.

Trial No.	"t" Value Single Trial 10 Pairs	"t" Value Comb. 3 Trials 10 Pairs	"t" Value Comb. 4 Trials 10 Pairs	"t" Value Comb. 5 Trials 10 Pairs
1	2.325			
2	1.114			
3	.971	2.465		
31	3.855		3.711	
32	1.335			3.958
4	1.201			
5	4.610			
6	1.519	3.301		
33	2.070		3.928	
34	0.664			3.709
7	0.970			
8	1.088			
9	2.339	2.440		
35	0.360		2.305	
36	1.374			2.582
10	2.736			
11	2.703			
12	3.422	5.233		
37	0.747		4.045	
38	2.095			4.589
13	1.193			
14	1.557			
15	2 580	2.875		
39	0.175		2.600	
40	1.793			3.069
16	1.912			
17	1.917			
18	1.784	2.709		
41	1.440		3.107	
42	1.030			3.209
19	1.810			
20	2.574			
21	1.244	3.310		
43	1.437		3.558	
44	1.813			4.062
22	2.563			
23	.203			
24	.830	1.826		
45	1.807		2.478	
46	0.246			2.331
25	1.510			
26	1.302			
27	.896	2.126		
47	2.000		2.752	
48	.869			2.887
28	1.977			
29	1.876			
30	1.849	3.205		
49	1.192		3.412	
50	1.735			3.873

"t" Value for Odds of 20:1 10 Pairs =2.262
"t" Value for Odds of 20:1 10 Pairs 3 times or 30=2.045
"t" Value for Odds of 20:1 10 Pairs 4 times or 40=1.959
"t" Value for Odds of 20:1 10 Pairs 5 times or 50=1.959

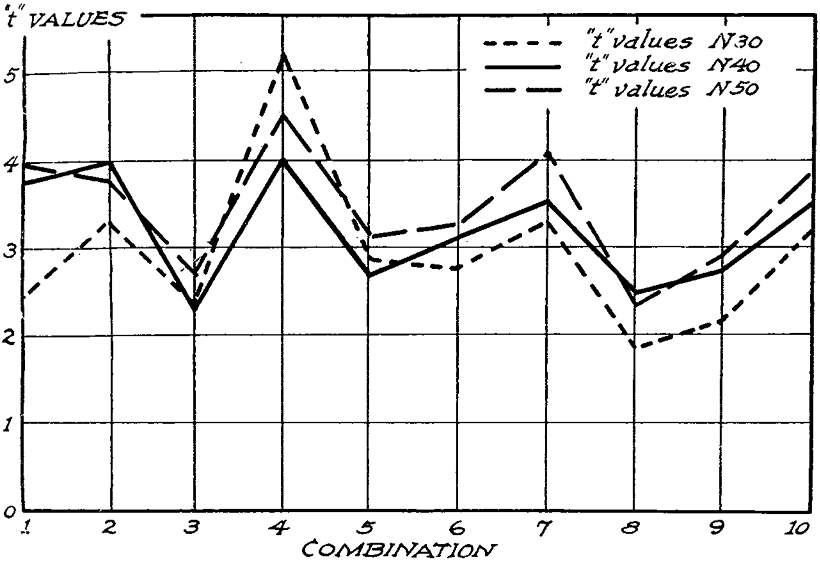

Figure No. 4.

increased to 50, altho it is leveled out somewhat. In the third column of the table N has been increased by 10 pairs and now equals 40. The "t" values for this column are all significant for odds of 20:1 or more, and in all cases except three they exceed those of the first groups.

The "t" values of the fourth column, N=50, can go without much comment. They are larger than those where N=40, with two exceptions, and all are significant.

Figure 4 shows curves representing the "t" values secured in these calculations and the relationship they maintain is further indicative of the influence of N on the element of chance.

The results of the analyses thus far have indicated that rations showing a mean difference equal to or in excess of that obtained from the corn-alfalfa hay and corn-alfalfa hay-wet beet pulp rations of the Colorado Experiment Station can be demonstrated by paired comparison in not less than three trials of ten pairs each.

DISCUSSION

In the light of the above statement, the question arises as to whether it is possible to compute the number of years a comparison giving a "t" value not significant would have to be repeated in order to establish the significance of the difference. The point prompting the question is the fact that in actual experimental procedure occasion will often arise when a comparison carried 3 or 4 years will have failed in proving a significant difference and the investigator will want to know whether he can continue the trial a given number of years and prove the significance of the results.

One of the primary concepts of statistical methods (8) is the fact that a sample of a population if it is a true cross-section will give statistical measures representative of the entire mass. Insofar as the comparison under consideration will meet the limitations set forth in the foregoing statement, the "t" value to be expected from continuing the trial a given number of years can be computed by the following formula:

$$\frac{t_2}{t_1} = \frac{\sqrt{N_2}}{\sqrt{N_1}}$$

In the formula

t_1 = "t" value obtained in the comparison to date.

t_2 = Expected "t" value.

N_1 = Number of pairs of animals in the comparison to date.

N_2 = Number of pairs of animals involved in the comparison to date plus the proposed extension.

The formula is derived in the following manner:

Fisher (5) gives the following formula for "t" value:

$$\text{"t"} = \frac{\overline{X}\sqrt{N}}{\text{S.D.}} \qquad \overline{X} = \text{Mean difference}$$

From this formula then:

$$\frac{\text{"t"}}{\sqrt{N}} = \frac{\overline{X}}{\text{S.D.}}$$

Then

$$\frac{t_1}{\sqrt{N_1}} = \frac{\overline{X}_1}{\text{S.D.}_1}$$

and

$$\frac{t_2}{\sqrt{N_2}} = \frac{\overline{X}_2}{\text{S.D.}_2}$$

Now, insofar as the pairs represented in N_1 and N_2 are a true cross-section of the population

$$\overline{X}_1 = \overline{X}_2$$

$$S.D._1 = S.D._2$$

And

$$\frac{\overline{X}_1}{S.D._1} = \frac{\overline{X}_2}{S.D._2}$$

Then

$$\frac{t_1}{\sqrt{N_1}} = \frac{t_2}{\sqrt{N_2}} \quad \text{or}$$

$$\frac{t_2}{t_1} = \frac{\sqrt{N_2}}{\sqrt{N_1}}$$

Following this formula thru will quickly demonstrate its weakness, but the reliability of a "t" value so computed will be accurate insofar as the sample of the conditions being compared are accurate. Under no circumstance can such a "t" value replace one computed from actual data.

In order to study the application of the formula, an analysis was made of the data of 21 groups of 5 pairs combined (Table 6.) The results are shown in Table 10.

Table 10.—Testing the Extension of "t" Values by Formula, Using Data from Table 6.

N Equals	"t" Value of Data	"t" Value Ex-tended	"t" Value Extended to N = 105
15	2.602		7.103
30	2.760	3.720*	4.943
45	3.292	3.394	5.072
60	4.295	3.818	5.669
75	4.808	4.767	5.673
90	5.330	5.240	5.756
105	5.822	5.756	

$$* \; \frac{3.720}{2.602} = \frac{\sqrt{30-1}}{\sqrt{15-1}}$$

In this table the first column gives the number of pairs of animals used as the calculations progressed. The second column gives the actual "t" value secured as each 15 pairs of animals were added to the test.

The third column gives the extended "t" value as computed by the formula in an effort to predict what the actual "t" value would be. This "t" value was secured in the following manner: The actual "t" value secured from the data of 15 pairs is 2.602. Computing the expected "t" value by the formula gives,—

$$\frac{t_2}{2.602} = \frac{\sqrt{30-1}}{\sqrt{15-1}}$$

$$t_2 = 3.720$$

In a like manner the expected "t" value for 45 pairs was computed by using the actual "t" value shown in the second column for 30 pairs. Thus each "t" value of the third column represents the results to be expected from a 1-year extension of the test.

Comparison of the "t" values secured from the data as shown in the second column and the computed "t" values as shown in the third column shows the actual "t" value for 30 pairs to be 2.760, while the computed "t" value is 3.720. From this point on the computed "t" values show reasonable agreement with the actual "t" values of the data, with the possible exception of the "t" value for N=60.

The fourth column of the table shows an extension of the actual "t" values for the various N's to an expected "t" value for N=105. Extending the "t" value for N=15 to an expected "t" value for N=105 gives a "t" value of 7.103 compared with an actual "t" value from the data of 5.822. The extended "t" values for N=30 and 45 more nearly approach the actual "t" value for N=105 and at the point where N=60 there is a fair agreement between the extended and actual values.

The results of these analyses indicate very clearly that the reliability of a "t" value extended by the use of the formula is dependent upon the reliability of data secured from the original sample. If the variables of the test have been sufficiently sampled to level out the effect of chance, the "t" value to be expected from repeating the experiment can be computed with reasonable accuracy.

SUMMARY

The data analyzed were taken from 6 years' lamb-feeding comparisons with rations of corn-alfalfa hay and corn-alfalfa hay-wet beet pulp. One hundred forty-six lambs were fed on each ration.

Correlation analyses of length of feeding period and daily gain gave correlation coefficients for the corn-alfalfa hay lambs of 0.0415 and for the corn-alfalfa hay-wet beet pulp lambs of -0.096.

Analysis of the six trials by statistical methods gave no reversals and significant "t" values for all but one trial. The "t" value for the combined trials is 5.052, which is highly significant for the ration with wet beet pulp added.

A study of the variations in gain gave a coefficient of variation of 21.08 percent.

The wether lambs made gains significantly larger than the gains made by the ewe lambs.

A correlation analysis of initial weight and daily gains gave a coefficient for the corn-alfalfa hay lambs of 0.090 and for the corn-alfalfa hay-wet beet pulp lambs of 0.066.

A comparison of analyses by Students' method of paired comparison with those of the standard method for determining standard error and the deviation-of-the-mean method gave odds of 100:1 in favor of Students' method of paired comparisons.

An analysis of groups of 5 pairs in combinations of 3, 6, 9, 12, 15, 18 and 21 shows the effect of the increase of N on the "t" value.

Analyses were made of single groups and in combinations of 3 for 2 pairs, 3 pairs, 4 pairs, 5 pairs, 10 pairs, 20 pairs, 30 pairs, 40 pairs, 50 pairs and 80 pairs, which demonstrates the leveling out of chance by the difference in the ration when N equals 30 or more.

An analysis of random groups of 5 pairs in combinations of 3 demonstrates the effect of chance upon the value of "t" when N is small.

Trials of 10 pairs in combinations of 3 were analyzed and 9 of the 10 analyses show significant differences.

Trials of 10 pairs in combinations of 4 and combinations of 5 were analyzed and all show significant differences.

CONCLUSIONS

The variation in the ability of the individual lamb to gain in the feedlot is an important factor influencing the interpretation of experimental results. This influence, expressed in terms of the coefficient of variation, is equal to 21 percent of the average gain of the lambs.

Of the factors measurable in the data studied, sex contributes the most of the variation of the individual gains. The variation in the length of feeding period and the variation in the initial weights of the lambs do not materially influence the gains for feeding periods from 75 to 120 days in length.

Students' method of paired comparison will demonstrate as significant, smaller mean differences than will the deviation-of-the-mean method or the standard method for determining standard error.

The element of chance materially affects the results of trials where N is less than 30, and N should equal 40 before the results be vested with too much reliability.

It is evident that there is considerable variation due to season and that 10 pairs of lambs fed in 4 different seasons will give more reliability to the results than will 40 pairs of lambs fed in one trial.

From the study it is apparent that reliable significance can be determined for the data of a comparison that gives a mean difference equal to and a standard deviation no larger than that of the corn-alfalfa hay and corn-alfalfa hay-wet beet pulp rations with 4 trials of 10 pairs.

The influence of N in establishing the significance of the difference makes apparent the need of an experimental set-up which will permit the addition of trials when necessary.

In setting up pairs for analysis by paired comparison, sex is more important as a factor in pairing than initial weight.

Weight data secured from lambs paired when allotted, but group-fed, can be satisfactorily analyzed by paired comparison, but this method of feeding does not permit recognition of the variability in individual feed consumption.

Thru the use of lambs paired and individually fed, it would be possible not only to weigh the variation in gains, but also to study the correlation between gains and amounts of feed consumed, a question that has been persistently annoying thruout the analysis of this problem.

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