# INVESTIGATION OF OBJECTIVE FUNCTIONS AND OPERATION RULES FOR STORAGE RESERVOIRS

by

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# ABSTRACT

Water requirements, objective functions and operational rules are important aspects in both the planning and operational stages of a reservoir or a system of reservoirs. When planning the size of a reservoir to meet certain demands, the size depends on the selected objective function and on the type of operational rule. Similarly, when the reservoir capacity is known and the best or optimum operation is to be determined, the operational rule found is only an optimum in regard to the elected objective function. Objective functions, including the probabilities of failures of meeting specified targets or projected demands, should be studied especially when drought impacts are important in design and operation. These functions should be analyzed in conjunction with types of operational rules (either empirically or analytically derived), which provide hedging. It appears that these aspects need more attention and should be studied further.

An aspect of objective functions seems to be nearly always neglected, namely, the continuous or stepwise evolution of objectives, purposes and decision variables with time. This is particularly important in the planning phase, when optimizations and performance measures stretch over the entire project life. It is less important in the operational phase, because the evolution in objectives is taken into account by the change of objective functions (say, the change in objectives, purposes, decision variables, coefficients, forecast, etc.).

The hydrologic forecasting for operational purposes is basically of a short-range type, say of days or up to a couple of weeks, except in case of forecast based on accumulated snow, which extends to the entire snowmelt season.

In measuring the operational forecast benefit, the modeling of time series of supply and demand, and the feasibility of forecast, permit the computation of average maximum possible benefit, average minimum benefit and average benefit of operation with forecast of a water resources system.

The position of forecast benefit between the maximum and minimum on benefits gives two results: (1) how much is the benefit from forecast; and (2) whether there is a significant benefit potential in case of an improved forecast.

# I. INTRODUCTION

The research summarized in this report was directed toward assisting water authorities in the determination of optimal operating policies for reservoir systems.

At present, most reservoir operational decisions are based on "rule curves" and the judgment of the engineer responsible for the day to day operations. In general, the operator is guided by a set of long term "firm-release" decisions for the delivery of sufficient water to meet long-term assured or reliable levels of water supply, hydroelectric energy or releases for other purposes such as low flow augmentation for navigation and environmental control. Normally these are considered to be minimum releases that must be met if feasible to do so at all. These minimums, however, do not provide guidance with respect to the release of water over and above the minimums for beneficial purposes instead of allowing it to spill unused. Likewise, they do not provide guidance with respect to "rationing" the supply in the face of an impending shortage.

To provide some additional guidance, most reservoir operators are provided with an "upper rule curve" which in effect states that, whenever the storage levels in a reservoir are above a certain level, defined as a variable for each point in time during the year, then extra water may be released for additional beneficial purposes. In some cases, a "lower rule curve" is also provided, which in effect states that, if water levels fall below that level, the firm water releases should be curtailed (water rationing).

In an ideal case, a set of "rule curves" can be provided. Three variables may be related by this family of curves: (1) the state of useful water storage, measured either by the water volume above the lowest water level, or by the reservoir level; (2) the time, i.e., the time interval of the year; and (3) the probability of the reservoir state not to be exceeded. These rule curves may be obtained by experience in operation, or by simulation of long "operation" of input and demand-target time series. The operational decisions of releases are then guided by both the state-of-the-storage and the probability of non-exceedence of the state of each decision time. Sometimes the "upper and lower rule curves" guide releases, so that at each new time interval the rates are closer to the 50 percent or the expected rule curve.

These rule curves provide guidance to the operator, indicating when deviations from target releases are justified, but they do not give any guidance regarding the magnitudes of such changes in release, nor do they indicate how rapidly the reservoir should be restored to levels indicated by the rule curve.

The primary effects of possible deviations from the target releases are (1) an increased benefit in the form of additional electric energy and/or water, or in the form of reduced adverse impacts in the case of drought and (2) a change in the risk that there will be future deficits and/or spill losses. The former effects can often be expressed in economic terms. However, the latter are exceedingly difficult to characterize with economic measures, hence, at the outset, it is anticipated that the operational problem must be considered in a multi-objective framework.

The objectives of the research were consequently structured around 12 purposes, which would hopefully be served to an extent that would allow the application of the results to practical operational problems. The 12 purposes were:

(1) Development of a methodology consisting of policy criteria, principles, methods, techniques and procedures, for designing the objective functions for planning and operation of water storage reservoirs.

(2) Introduce the time factor into the objective functions to represent the continuous evolution of the objective functions, because of various changes with time that affect the operation of storage reservoirs.

(3) Test this new methodology, including the time evolution of objective functions, on the three pilot reservoir cases: (i) Bonny Reservoir (single operated), (ii) Green Mountain Reservoir (single operated) and (iii) Green Mountain-Dillon Reservoir System (two-reservoir operation).

(4) Investigate the historical and present state of operational rules and classify the types of existing operational rules for reservoirs, indicating their advantages and disadvantages, the criteria and indices for measuring their performance and the corresponding objective functions.

(5) Find a reasonable bridge between the objective functions and the resulting operational rules of reservoirs.

(6) Compare the results obtained by using the existing operational rules with the results obtained by using various methods and algorithms of optimization in reservoir operation.

(7) Investigate how the existing operational rules may be improved, by using advantages the optimization and computer techniques provide.

(8) Generalize the concepts of operational rules as applied to reservoirs, by using any available analytical method or other method suitable to accomplish a good degree of generalization.

(9) Test the results of the above four objectives, (4) through (8), on the three pilot reservoir cases as indicated in point (3).

(10) Investigate the concept of equivalent reserviors as an alternative to the multireservoir operational schemes and algorithms.

(11) Design a methodology consisting of criteria, principles, methods, techniques, constraints and procedures for the application of the concept of equivalent reservoirs, and the determination of equivalent reservoir characteritics.

(12) Test the results of the objectives (10) and (11) on the pilot case of a potential joint operation of the Green Mountain-Dillon two-reservoir system.

These objectives were not achieved uniformly. In particular, because of impending litigation, it was not feasible to test the procedures as suggested in objectives (3), (9) and (12).

In addition, as the project evolved, certain aspects were found to be more critical than others, hence these have been given relatively greater attention. Additional objectives were encountered which clearly needed to be resolved before we could continue. These were incorporated into the original objectives. These primarily concerned the development of a rational time basis for the commensuration of the incompatible time scales of immediate project benefits from operations and the consequent risk of deficits in planned target levels of project outputs.

Because of these additional objectives, it was necessary to defer action on objectives (4), (11), and (12). The first (classification of current system of operating rules), was then found to be relatively less important to the project as it evolved. The most generally used class of operational rules consists of month by month target reservoir levels. Our approach was directed toward determination of operational equations which would assist in defining operational policies when the reservoir is not currently on the rule curve or is expected in the future to be above or below. Thus a classification did not appear to be a high-order priority.

In addition, discussion with officials associated with Dillon and Green Mountain Reservoirs, indicated that because of impending litigation it would be preferable not to use these reservoirs as the case studies. Consequently, Bonny Reservoir is the only one that was used. In doing so some modifications had to be hypothesized for its long-term purposes in order to introduce both conservation and flood control aspects.

The conceptualization of an equivalent reservoir to represent a system of reservoirs (objective 10) has been accomplished. After reviewing a number of approaches, we have turned to the "optimal state reservoir" concept originally proposed by W. A. Hall for TVA and CVP and developed further by Prof. W. G. Yeh, UCLA and the U.S. Bureau of Reclamation, Sacramento. However, some substantial changes are recommended, primarily in the definition of the criteria for the optimal states, based on the results of our studies under objectives (1) and (2). Because of the additional problems and related objectives encountered as noted above, objectives (11) and (12) have had to be deferred. However, the basic concept, together with the work on the "Anticipated Decision Influence Period" and risk quantification described in the next section, is very promising indeed.

## II. DEVELOPMENT OF RATIONAL OBJECTIVE FUNCTIONS FOR OPERATIONAL DECISIONS

Operational decisions of water resources systems in general and of storage reservoirs in particular are typically determined based on simulation and/or optimization techniques. In either case an objective function is usually assumed in terms of say economic returns or other performance measures which ultimately determine the decisions and rules for reservoir operation. Thus, the output from simulations and/or optimizations are conditioned on the type of objective function selected for the analysis.

This part of the research was oriented to develop a methodology for selecting and constructing objective functions which accurately reflect the objectives and values of reservoir operators and users. The details are presented in Reference 3 (the M.S. thesis by John Westgate). For the general approach on design of objective functions see Reference 7 (by V. Yevjevich).

To accomplish the above objective a thorough review of literature was made in order to collect the various performance measures currently used in practice and research. The specific criteria for selecting the type of measure which should be used must come from an analysis of the specific decision process and context in which they will be used, since their function is to aid decision making.

Three categories of <u>performance measures</u> were identified in the thesis of Reference 3. The primary grouping of performance measures was by purpose. Purposes represent groupings of users or interests with similar demands upon a reservoir system and often have different legal priorities, pay back requirements, and advocate agencies. The purposes identified are: flood control, irrigation, municipal and industrial water supply, recreation, navigation, hydropower, pollution control, silt control, fisheries, wildlife, integrity of facilities. A second classification of performance measures was based on type; e.g., economic, physical assurance (risk), expected value. Each of these types has advantages, disadvantages, and assumptions associated with it. A third category is that of hierarchies; goal hierarchies, time frame hierarchies, constraint hierarchies. These hierarchies are inherent in the structure of the total water resources decision making process.

There really is no single point of view, common to all decision makers in the heirarchy. From the point of view of the operator, however, performance measures would appear to be best defined by taking the point of view of the ultimate user. The goal of reservoir operations is to benefit the relevant society by adjusting the spatial and temporal distribution of water. The immediate results of reservoir operating decisions are the physical outputs and states of the reservoir. The benefits to society require a user to generate them. The user subsystem has its own decision makers, investment requirements, and processes. Performance measures for reservoir operations should provide the information required by the user for his decision process.

Viewed in this light, reservoir operations for conservation must be based primarily on the implications of the long-term target requirements. These are usually stated in terms of "firm requirements" for water, energy or other major product.

What is not always stated is that the value of resources such as water and energy rests in a high degree of assurance that they will be available when needed and where needed. Lightening represents an excellent example of a large quantity of electricity with no direct economic value because of the lack of reliability that it will be available when needed, where needed, and in a quality suitable for the intended use.

A major implicit objective of the operation of a reservoir is to maintain the necessary reliability of the availability of the target or firm requirements into the future. In general, this operational objective will dominate all others.

When this implicit objective has been satisfactorily met, the secondary objectives of utilizing any excess water for beneficial purposes can be addressed. "Satisfactorily met" is a somewhat nebulous term, every decision made to utilize excess water which is not actually spilling at the time will tend to increase the risk of a future deficit. Thus it would seem obvious that the primary objectives of the reservoir operation for conservation of water must involve a trade-off analysis between increased risk of future deficits and the immediate economic and social benefits which might be obtained from deviations from the scheduled requirements.

In the same way, when there is an apparent deficit of water or energy, the objectives of reservoir management will be to allocate the shortage over time in such a manner as to minimize the adverse effects of the deficit. However, there is always a nonzero probability that the deficit will be larger (or smaller) than that anticipated. Since these deviations produce (different) adverse effects, once again risk aversion becomes a primary objective, with trade-off analysis required between this risk and the anticipated beneficial effects of allocating the deficit over a period of time.

Finally, even in flood control operations, there is always a risk of underestimation of the incoming flood or overestimating it. In the case of actual floods less than the design flood, the standard operating rules will minimize downstream damages. As the design flood is approached or exceeded, the operational objectives must reflect the trade-off between risk of floods higher or lower than the forecast against the minimization of downstream adverse effects.

Thus from the hierarchy of objectives it would seem that the operator is always in the position of addressing the fundamental trade-off between the effects of his decisions on the modifications of the risks and the potential benefits (or loss mitigation) which might accrue therefrom.

For a more complete discussion of the details of the analysis of objective functions for reservoir operations see Appendix 3.

## III. OPERATIONAL TRADE-OFF BETWEEN RISK-AVOIDANCE AND IMMEDIATE PROJECT BENEFITS FOR WATER AND ENERGY CONSERVATION

### A. The Anticipated Decision Influence Period

One of the difficult problems of water management is that of determining the optimal reservoir operating policies to be followed on a day-to-day basis. Reservoirs are usually planned and designed on the basis of providing certain target levels of service (water supply, hydropower, flood storage space, low flow, etc.) for which a comparatively high degree of reliability over a planning horizon is essential. In effect, the targets are based on a "worst likely" sequence of inflows, using either the historical records or generated "equally likely" sequences.

In actual operation of conservation reservoirs, these worst likely sequences will occur infrequently, hence considerable amounts of extra water, hydroelectric energy, etc., would be lost if operating policies were required to conform exactly to the planned targets without regard to the then current conditions. These lost benefits are generally associated with spill (water unintentionally released without going through the turbins or are released for serving a water supply purpose). One objective of reservoir management for conservation might thus be considered to be "minimize spill losses" during operations by releasing water through the turbins or to beneficial supply purposes, such as groundwater replenishment before an uncontrolled release becomes necessary.

There is a countervailing objective, however, which is negated by overzealous pursuit of the objective of minimizing uncontrolled spills. The inflows to the system are stochastic and cannot be predicted very long in advance. Thus it is virtually certain that if spill is consistently minimized, deficits will occur with increased frequency and magnitude. Because the most important characteristic of both electrical energy and water supply is the level of assurance or reliability with which it will be available when and where needed, an operational policy which only minimizes the spill will defeat this essential characteristic. Thus there are at least two objectives to the conservation operational management problem: (1) minimize losses due to spill and (2) minimize the frequency and magnitude of deficits in water supply and/ or energy production with respect to the planned levels.

At this point it is important to distinguish the operational problem described above from the post-construction planning problem of deliberately changing the target output levels (and the corresponding level of planned reliability) of an existing system as a relatively long-term decision. If the latter is to be done, all activities which depend on the use of energy, water and other levels of benefit can and presumably will be adjusted to accommodate the changes in target levels and the corresponding levels of reliability. The operational problem treated here is not concerned with planned modification of targets, but rather with actions which may inadvertently and unexpectedly modify reliability in the short run without the benefit of advanced planning on the part of the users.

This does not mean that any utilization of water otherwise spilled will in fact decrease the reliability of the supply of water, power, etc. In the operational problem there is considerably more information available than can be considered in planning long-range targets. Actual current storage levels are known. Current inflows are known. In addition there are streamflow forecasts which, although still stochastic, are substantially better than chance, particularly for the immediate future. This information can and should be used to improve the overall benefits from the system.

The classical procedure under these circumstances is to introduce a "rule curve" to guide the operator. For example, in any particular month, if the quantity of water in the reservoir is above that prescribed by the "upper rule curve" for that month, then extra water and/or energy can usually be produced in that month so long as the reservoir volume does not fall below the level specified by the rule curve. A similar "lower rule curve" is used to indicate when releases should be curtailed below the target levels in order to minimize the severity of the expected deficits in the subsequent periods of time.

The position of the rule curves is usually determined in advance by analysis, experience and judgment at levels considered to be reasonably safe. Beyond this the operator must use his judgment, taking into account not only the current reservoir level but any other pertinent information. For both the rule curve and the additional judgment, introduction of an increase in risk of failure to meet "firm" supply levels is an implicit objective to be minimized in the judgmental optimization by the engineer. It is implicit because there is an inherent difficulty in defining the risk to be minimized in precise quantitative terms. Risk of what? In how long a time period? These questions are considered implicitly in the judgmental optimization. If an analytical optimization is to be useful it must also treat these questions as objectively as possible.

In this report, these questions are developed for quantitative analysis. There are two general classes of answers to the first or "risk of what," question. There is the risk that water in some quantity $V_s$ will spill from the reservoir without serving a useful purpose. There is also the risk that a deficit $V_d$ may occur in the quantity of water needed to meet the firm levels of service specified in the current long range plan. Both can be quantified.

If these risks are to be stated in terms of time quantitative probabilities (e.g., exceedence probabilities), then the question concerning the time periods of spill or deficit must be answered, since the numerical magnitude of these probabilities is dependent on the period of time. Thus a question to be resolved is the time period to be used in the statement of the risk index. With this information, risk can then be quantified.

Obviously this time period cannot be entirely arbitrary. In most cases, this period has been selected as one year, as a matter of judgment, since most of the reservoir storage volume is usually used to regulate the seasonal distribution. In this paper the time period is defined by an analysis which attempts to identify the "anticipated decision influence period" (ADIP) (see Reference 1). The basic concept is that there is a limiting period of time beyond which the current operational decision would have no further effect on the probability of either future spill or deficit. These periods can be expected to depend on the current state of the system, for example, current reservoir storage levels, projected inflows, projected demands, etc. In actual use, however, it has been found that for most water conservation reservoir systems the mean ADIP will be relatively constant. This relative constancy permits it to be specifically and quantitatively defined a priori as a function of the state of the system.

The concept of the ADIP can be illustrated as follows: Suppose, as an exaggeration, that the reservoir is full at a point in time where, for the next several months, the inflow is normally substantially greater than the demand targets would require. Obviously some of the excess water will be spilled with something close to virtual certainty. Instead of letting it spill uselessly, it could be sent through the turbines up to the limit of their capacity. This high probability of spill will continue until the inflow falls below the target demand. Once this occurs, although the reservoir is full, the probability of spill suddenly drops from the previous high value, to a very low value whether or not the earlier excess volumes were spilled, sent through the turbines or otherwise beneficially used. Thus in this case, the prior decisions have no influence on the reservoir level at this point in time, hence can have no impact on any future deficit. Furthermore, nothing done after this time can possibly influence the amount of water spilled prior to this time. If the potential spill is to be used beneficially, the decisions to do so must be taken before this time. Later we will refer to this as an ADIP Type I. This type is always associated with the spill minimization problem.

A similar analysis, again exaggerated, can be made for the deficit problem. If the reservoir is empty at some point in time for which demand will exceed the inflow, it is obvious that a deficit will occur with a very high level of probability. The problem, of course, is to distribute the deficit over the preceding period of time in such a way as to minimize the adverse impacts of the anticipated deficit $V_d$. At some subsequent point in time, the inflow will once again exceed the target demand and storage can be increased above zero. When this occurs, none of the excess water reaching the reservoir can be used to mitigate the previous deficit. Any decision to mitigate the effects of the deficit must be made and executed prior to this point in time. Thus this point in time defines an ADIP Type II, distinguished by its relationship to the problem of mitigation of adverse effects of shortage.

The above analysis was deliberately exaggerated to illustrate the concepts. In the usual case, storage levels are rather less than maximum at the beginning of the filling period. Instead of a virtual certainty of spill described above, there is a probability of spill on or before the date on which demand will exceed the inflow. This probability will depend upon the actual reservoir levels, the predicted inflows and demands, the random components of these two factors, and any other factors involved. If the reservoir level is low enough the probability of spill before that date may approach zero for the ADIP Type I. Likewise, if the reservoir is full at the beginning of the drawdown period, the probability of a deficit during the ADIP Type II period will be relatively small. Any intermediate storage level will have an intermediate probability of a deficit prior to the end of the ADIP Type II.

Next suppose that we are entering a fill period, but with a reservoir level so low that it is quite unlikely that there will be any spill. Decisions made during the period nominally defined as ADIP Type I will now in fact have an influence on the magnitudes and probability of a deficit during the nominal Type II period. Thus an ADIP Type III needs to be defined for such cases when the influence of decisions will extend beyond the subsequent filling or drawdown periods.

The ADIP Type I is thus seen to correspond in general concept to storage levels above the usual upper rule curve, ADIP Type II corresponds to storage levels below the usual lower rule curve. ADIP Type III corresponds to storage levels between the two curves. However, by combining these concepts with probability analysis it is possible to identify the anticipated decision influence period with the basic operational decisions and objectives, and to replace each of the two rule curves with a set of optimal decision functions which not only depend on the current level of storage but also on predictive information and probability information, and potential beneficial effects from utilizing extra water and minimizing impacts of deficits.

### B. Use of the ADIP to Develop Indices of Impact of Operational Decisions on Risks Avoidance and Operational Benefits

Consider a conservation reservoir at any time $t_o$ with $q_o$ units of water currently in storage. We will assume that adequate streamflow data is available to define the statistical chracteristics of the flow, e.g., $\bar{y}(t)$, $\sigma(t)$, $\delta(t)$, $\rho(t-\tau)$, etc., sufficient to allow the generation of equally likely sequences of $y(t)$, using any desired method. Knowing the current storage level, it is then possible, using the mass balance equation, to generate a fairly large number of "equally likely" storage volumes for future points in time $(t_o + \tau)$. Note that in this simplified explanatory analysis, no adjustments are made for current knowledge other than initial storage level, the long term statistical characteristics of streamflow and actual target demand, including any expected random fluctuations about the target levels that would be expected. The latter might be caused, for example, by random rainfall events on an area irrigated by the project. If the latter random events are correlated with the fluctuations in streamflow, this can and should be reflected in the sequence generating techniques.

The length of each sequence should be at least long enough for the reservoir volume simulated to "change phase" from filling to drawdown or vice versa. Where a brief change of phase would occur, simulation should be continued until a new phase is encountered. The usual equally likely sequence would be from four to eight months (seasonal regulation), but it could be several years (interannual regulation). The historical record will usually provide a good guide.

The mass balance equations used for discrete time periods $i = 1\Delta t$, $2\Delta t$, ect., are:

$$B_i = q_i + y_i - x_i - \bar{e}_i(q_i, q_{i+1}) - s_i(q_i, q_{i+1}) \quad . \qquad (1)$$

$$\begin{aligned} q_{i+1} &= B_i \quad \text{if} \quad B_i \leq q_{i+1\max} \\ &= q_{i+1\max} \quad \text{if} \quad B_i > q_{i+1\max} \\ &= q_{i+1\min} \quad \text{if} \quad B_i \leq q_{i+1\min} \\ L_i &= B_i - q_{i+1\max} \quad \text{if} \quad B_i > q_{i+1\max} \end{aligned}$$

where $q_i$ is the volume of water in storage at the beginning of period $i$ (e.g., MCM),

$y_i$ is the simulated inflow for time period $i$ (e.g., MCM/unit period),

$x_i$ is the simulated demand for time period $i$ (e.g., MCM/unit period,

$e_i$ is the evaporation loss during period $i$ (e.g., MCM/unit period),

$s_i$ is the seepage or other losses not returned to the system, occurring in time period $i$ (e.g., MCM/unit period),

$L_i$ is the spill loss occurring when $B_i > q_{i\max}$,

$d_i$ is the deficit in $x_i$ when $B_i < q_{i\min}$

$q_{i+1\max}$ is the maximum allowable storage at the beginning of time period $i + 1$,

$q_{i+1\min}$ is the minimum allowable storage at the beginning of time period $i + 1$, and

$B_i$ is the "volume balance function," used to determine the value of $q_{i+1}$ according to its magnitude

Note that if $B_i \leq q_{i+1\max}$ $\quad L_i = 0$

$B_i > q_{i+1\max}$ $\quad L_i = B_i - q_{i+1\max}$

$B_i > q_{i+1\min}$ $\quad d_i = 0$

$B_i < q_{i+1\min}$ $\quad d_i = q_{i+1} - B_i$

In addition we define the accumulated spill volume

$$V_k = \sum_{1}^{k} L_i \tag{2}$$

and the accumulated deficit volume

$$D_j = \sum_{1}^{j} d_i \tag{3}$$

From the set of m equally likely reservoir inflow-outflow sequences for any given initial storage $q_1$, it is possible to identify a set of periods of duration k, i = 1,2,3...k, where k is the time period in which, for that sequence $B_i < q_{k+1\max}$ after at least one $B_i > q_{i+1\max}$, and for which $B_i < q_{i+1\max}$ for several additional periods. The looseness of the latter statement is simply to assure that a persistent change of phase has in fact occurred.

The values of k for the sequences thus terminated will have a mean, standard deviation, etc., just as any statistical quantity. Note that some of the sequences will never encounter $B_i > q_{i+1}$. These do not have a value for k (by the definition of k), hence they are not included in the ADIP Type I set. They may be in either the Type II or Type III set (the j set or the quasi-infinite set). These will be treated later.

Continuing with the ADIP Type I analysis, define $\bar{k}$ the mean duration of the time length of the set of k. Also determine $\sigma(k)$. This will permit the definition of $T_1$, the termination period of the ADIP Type I by an expression of the form

$$T_1 = \bar{k} + a\sigma(k) \tag{4}$$

Where $T_1$ is the termination time index for ADIP Type I, $\bar{k}$ is the mean duration of the Type I sequences, $\sigma$ is the standard deviation of the set of $k$ and $a$ is a judgmentally selected constant. Normally $a = 0$ will be fully satisfactory when $\sigma$ will come out to be small. The purpose of $a\sigma$ is to allow a somewhat longer period so that the probability of significant spill after $T_1$ is made small.

All of the above calculations can be made a priori from general knowledge, given the initial storage level $q_1$ at the beginning of time period No. 1.

For this same initial storage and initial date, the termination date $T_1$, it is now possible to utilize the set of values of the accumulated spill to that date (Eq. 2) for each of the "equally likely" sequences of inflow and outflow to define an exceedence probability for total spill during the ADIP Type I. For this calculation all equally likely sequences are used to define the probability, $P_1(q_{T1max}|q_o)$ that $q_{T1} < q_{T1max}$, i.e., that the system will be less than full at time $T_1$. Again use all sequences and the probability $P_1(q_T|q_o)$ that any other value of $q_{T1}$ will be exceeded $q_{T1min} \leq q_{T1} \leq q_{Timax}$, if one starts at $t_o$ with a reservoir storage level $q_o$.

Also note that the same exceedence probability statements can also be derived for any other time $i$ between $t_o$ and $T_1$ from the same set of equally likely sequences, should this be desired for any reason. For example, it may be desirable to know the mean date of initial spill.

The result of the foregoing calculations is a set of conditional exceedence probabilities, which answer the questions, (1) what is the probability that the reservoir level will be at or above any given storage level, $q_{T1}$, given the initial storage $q_o$ at time $t_o$, $(i = 1)$,

(2) what is the probability that the accumulated volume of spill will exceed any given level $V_{T1}$, given the initial storage level, (3) the mean date $T_1$ at which decisions regarding utilization of that spill must be taken, again given the initial storage level, and (4) the probability $P_f(q_{T1}|q_1)$ that on this mean date, the reservoir volume will be at or above any given volume $q_{T1}$.

What has not yet been determined is the probability that a deficit will occur in the subsequent Type II period. This will be accomplished by utilizing the conditional probabilities (computed in a similar manner) of the accumulated deficit exceeding $D_{T_2}$, given the storage level at the end of period $T_1$. The joint probability can then be determined directly. This will be discussed further under ADIP Type II.

Next presume that the above calculations have been accomplished for the entire range of $q_1$, the initial storage, still using only the long term statistical parameters. What if, at time $i = 1$, information is available which indicates that the streamflow in the next few time periods will be greater by some total amount Y over and above that which would be expected from the long term sequences. That is, presume that the mean $y_i$ from $i = 1$ to $i = T_1$ for good reason is expected to exceed that predicted from the long term statistics. If the standard deviation, serial correlations, etc., are not expected to change because of this, then the magnitude of $V_{T1}$ associated with any probability $p_v(V_{T1})$ would simply be increased by this magnitude Y. That is, $p_v(V_{T1})_a = p(V_{T1} + y)_b$ where a represent~~e~~s the original sequences without the additional knowledge and b represents the sequences with the additional knowledge.

Now we can state that, with probability $p_v(V_{T1})$ we will have a total surplus, potentially spilling, of $(V_{T1} + Y)$ or more. Conversely we have the probability that the surplus will be less than $(V_{T1} + Y)$. This potential surplus, together with its probability statement, represents the resource which might be allocated for beneficial use prior to the termination of the ADIP Type I period, i.e., $T_1$.

Suppose it is decided to utilize some volume X of this potentially available surplus for beneficial use (also yet to be determined). What will be the impact of such a decision on the probability of a subsequent deficit? This will be analyzed in two steps, again using conditional probabilities as the linkage.

If X units of water are released in addition to the target amounts sometime during the period $T_1$, the spill volume would be reduced to $V_{T1} + Y - X$. So long as this quantity is positive for any $p(V_{T1})$, the decision will have no influence on events subsequent to $T_1$ with probability $p(V_{T1})$. The discharge in this case would have occurred in any event. Thus for $V_{T1} + Y - X < 0$, the probability that the decision will have no subsequent effect is simply $p_v(0)$.

For values of $V_{T1} + Y - X < 0$, the probability increases that $q_T < q_{Tmax}$. These are all represented by the exceedence probabilities of $q_T$, i.e., the $p_q(q_T)$. Without the excessive release $(X - Y)$, the storage level would have been $p_q(q_T)$ previously determined.

The exceedence probability of any $q'_{T1}$ where the prime indicates decision modified storage is thus reduced from that calculated with the undisturbed sequences to $p(q_T + Y - X)$.

$$p'(q'_T) = p(q_T + Y - X)$$

In the above simplified form, it is assumed that the introduction of $Y$ and $X$ produced no change in the standard deviations of the $q_T$. Since $Y$, particularly, will have an important random component of its own the assumption may not be valid. On the other hand, $X$ usually has the capability of being modified accordingly as $Y$ actually accrues to a greater or lesser value than that anticipated. To the extent that the decisions on $X$ can thus be adjusted, the random component of $Y$ is of no consequence until and unless the actual accumulated $X$ releases exceed the accumulated $Y$ increment to flow, since it can be assumed that the optimum net excess release is a function of $V_T + Y$ and would in fact be adjusted to maintain the integrity of subsequent probabilities of deficit.

When this is not the case, for example where short term but "firm" contracts for excess water or power are let, then the impact of the variability of $Y$ must be incorporated in the probability $p'(q'_T)$ as well.

## C. Risk-Based Operational Rules

Past efforts of reservoir operation optimization have been mathematically well defined. Indeed, the last decade has seen the inclusion of risk constraints as integral mechanisms for mathematical programming techniques. Unfortunately, the time horizon in previous studies has not been well defined, if at all realistically practiced. Conventional approaches treat reservoir operations over the expected economic life or some similar lengthy measure. Also, typically the measure for optimality is placed in purely economic terms.

What the present research has focused upon was to predict the short-term (seasonal) effects of various operation policies on future failure probabilities. In this manner, policy could be set over the season and at the end of the season, reservoir state variables and knowledge of the stochastic nature of reservoir inputs and outputs allows the reassessment of policy for the new season. Thus the infusion of new information at periodic stages aids in setting policy over a very short time horizon. The short-time horizon is necessary due to the dynamic nature of economic benefits as well as the more nonquantifiable elements directing reservoir operation, viz. social, political, etc. considerations.

The steps in deriving the seasonal risk rules for a reservoir are: identification of the end of failure (called deficit), sensitivity to the initial condition, and generation of the cumulative distribution of future failure volumes. The utilization of these three steps was to formulate reservoir policy based on the future failure volume distribution and implementation of the policy if future failure probability criteria were violated. The results of various levels of action, to decrease the future failure probability, was then mapped and left in this form for the decision maker. Thus the future risk of failure, due to prior risk formulated policy, may not only be decreased, but the magnitude of the decrease may be controlled (see Reference 4).

Two reservoirs were analyzed: one with almost insignificant seasonal storage, and the other with considerable overyear storage. The risk formulated rules were then scrutinized for their usefulness by comparing them to standard normal operating policy. The comparative measures were in a form perhaps most suitable to the short-term decision maker: decrease in future failure probability and decrease in the future failure volumes. The method of comparison and testing of the risk rules and standard normal rules was to use them as fixed policy for both reservoirs over ten 20-year horizons; the results being the volume of water spilled and unavailable for each record. Also, for all 200 years of testing, comparisons were made of the cumulative distributions of: runs of spills and deficits of given lengths; the maximum single period excess and deficit; and the total spill and deficit volumes over a continuous failure period.

The conclusions reached were that the risk formulated rules yield considerable improvement over standard normal operations for the small reservoir and the improvements are magnified even more so for the large reservoir.

## III. OPERATIONAL TRADE-OFF BETWEEN RISK AVOIDANCE AND IMMEDIATE BENEFITS FOR FLOOD CONTROL POLICIES

### A. Operational Policy for a Flood Control Reservoir

Reservoir operations for flood control are expected to alleviate flooding problems. Success in reducing flooding problems is affected by the inherent limitations of the reservoir system, the manner in which flood control release decisions are made and the flooding problem which existed before the dam was constructed. Also, success varies according to the objective of a reservoir storage user in a multipurpose system. Conservation, flood control, irrigation and recreation uses are all potential sources of conflict within a reservoir decision system. Inherent limitations of the reservoir system, fixed storage capacity, conflicting and increasing uses, instantaneous and unpredictable inflows, fixed release capacities, uncontrolled spillways and decreasing availability of reservoir sites encourage efficient operation of existing flood control systems.

The objective of this portion of the study was to evaluate an existing flood control operational policy and hydrological record to determine the adequacy of the flood control storage. Also, a comparison between the existing operational policy and an alternate flood control policy for the reservoir system was made (see Reference 2, the thesis of Miss Jan Kimsey).

Bonny Reservoir in eastern Colorado was selected for this study because of the primary importance of flood control in the operation of the system. Bonny is operated in conjunction with downstream reservoirs. The problem is approached by first analyzing storm events from which a set of extreme flood hydrographs are developed. Then an algorithm of the standard operating policy routes each flood through the reservoir system. The alternative policy attempts to minimize the flood stage at the downstream damage center for a set of extreme flood events rather than the single policy developed to minimize the flood stage for a design storm. The downstream stages for each storm hydrograph from both routing algorithms are compared.

From the continuous stage records for inflow to Bonny Reservoir a peak flow $Q_p$, volume V, time to peak $t_p$, and time to centroid $t_c$, were measured and calculated for selected hydrographs. Mean, standard deviation and skewness coefficients were calculated for a volume frequency analysis. (Flood volume was considered the most important variable for flood control.) Five probability distributions were fit to the data and tested for goodness of fit with the chi-square test. The two-parameter log-normal distribution was selected as the "best fit" to generate a set of extreme values.

In performing the volume frequency analysis, the 20 years of gage records for inflow to Bonny Reservoir was insufficient to provide satisfactory results when compared with historic flood events. A regional frequency analysis of flood volumes was undertaken to supplement the previous work. A regional mean and standard deviation based on area-weighted values from three stations were calculated.

Linear regression relations were developed between $V$ and $Q_p$ and $\frac{V}{Q_p}$ and $t_c - t_p$ to develop sets of $Q_p$ and $t_c - t_p$ from generated $V$. With sets of $V$, $Q_p$ and $t_c - t_p$, hydrographs are defined from a gamma-type function,

$$Q(t) = Q_p \exp[-t | t_c - t_p][1 + \frac{t}{t_p}]^{\frac{t_p}{t_c - t_p}}$$

where $t$ is time.

Frequency analysis of flood volumes indicate the flood control storage capacity of the reservoir is adequately sized. The flood control capacity without surcharge is 128,800 acre-feet and the frequency analysis shows 100,000 acre-feet to be a 500 year event. With this information, an analysis is made of change in risk level with a reduction of storage. It appears that the conservation pool, which lies below the flood pool may feasibly be raised 10 feet which would increase the storage by 25,000 acre-feet, with little or no impact on flood control results.

Flood control operations for reservoirs are also evaluated by comparing existing policies and those found by optimization and simulation techniques. A case study for Bonny Reservoir, Colorado, routes five generated storm hydrographs through the reservoir considering the influence at the downstream reservoir (Swanson). The storms are routed by a computer algorithm which simulates the standard operating rule and by a dynamic programming optimization algorithm. The resulting releases were lower for the more extreme events and higher for the less extreme storm events with the dynamic programming algorithm than the simulated standard operating rule.

### B. Use of the ADIP in Flood Operational Policies

A similar analysis of concepts as in the case of ADIP for water and energy conservation can be made for other types of operational problems. For example, operation of the flood control space may involve deliberate and assured downstream flooding at one level in order to minimize risk of even more severe flooding. There is a period of time for any particular predicted flood hydrograph in which the deliberate flooding decision must be made and executed. If delayed, the remaining volume for flood storage will have decreased, perhaps to zero before the flood peak arrives at the reservoir. On the other hand, if the predicted magnitude of the flood proves to be considerably larger than the actual flood, unnecessary damage will have been incurred. As an additional risk consideration in this instance, the problem may be complicated by the probability of additional flood events occurring before the control space can be emptied safely. Again a Type I, Type II concept can be utilized effectively to separate and recombine these effects for maximum computational effectiveness.

## IV. MULTIPLE RESERVOIR OPERATION

In previous sections, the operational problems were discussed in terms of a single reservoir. Where a number of reservoirs are involved, the calculations of joint probabilities resulting from decisions at any one reservoir could become hopelessly complex.

The guiding principle used in this case again refers back to be basic purpose of the system, i.e., to provide assured services at some target or "firm" level. That being the case, the decisions to be made will consist only of deviations from those norms.

After reviewing a number of approaches (see Reference 5), the "optimal state" equivalent reservoir concept originally proposed by W. A. Hall for the TVA and CVP systems and further developed by W. W-G Yeh for the CVP system would be readily adaptable for these "deviation" types of decisions since the optimal state would be fairly well defined.

In theory, for every set of reserviors and the corresponding target demands, inflow hydrology, etc., given the total volume of storage $S$ at a given time $t$ there is an optimum distribution of that $S$ to the several reservoirs $S_j^*$ such as to maximize the potential for the system to meet its near term and long range service requirements.

Once such a set of optimal relationships $S_j^*(S)$ are established for each time period in a cyclically varying supply-demand situation, the day to day operational decisions can then be analyzed in terms of the single equivalent reservoir contaning an initial storage $S$ in much the same manner as that described for a single reservoir in the preceeding sections.

The problem comes in the practical application of the concept to the specific case. In particular, it is difficult to establish, a priori, the criteria for any particular state being optimal.

Some of the ramifications of this problem are presented in Reference 5 (thesis of Mr. Ricardo Smith), together with one possible procedure for determination of the optimal state based on the criteria that the optimal state at any point in time is that which maximizes the hydroelectric generation potential of the system under a constraint that requires water supply requirements to be met.

It is to be noted that, with respect to water supply requirements alone, the allocation of $S$ to the $S_j^*$ is a relatively indifferent, or "almost equally optimal" matter. The only advantages of storage in one reservoir over another are (1) the possibility of avoiding spill losses out of the total system and (2) the relative evaporation losses.

Assuming the differences in evaporation losses are negligible, and recognizing that maximum of hydroelectric energy generation will automatically tend to minimize unnecessary spill losses, the use of potential for energy generation to determine the optimal state was believed to be justified under most conditions.

The problem was addressed in Reference 5 using energy flow and its storage as the basis for the calculation. This was not necessary, but it allows for a simpler analysis. To make the analysis it is essential to utilize the expected target outflows (energy and/or water) as a part of the defined "state" of the system, together with the water energy in storage and the forecasted inflows. This is necessary since energy production is a function of both head (storage levels) and quantity of water to be released. Fortunately, this is very compatible with the reservoir operational problem, hence no serious problems should be encountered in most cases.

On this basis a set of criteria for determining the optimal state of a reservoir system were developed in Reference 2. Since these are somewhat complex and subject to error if cited out of context, that Reference should be consulted for details.

With an optimal state defined for each point in time, in the probabilities associated with the equivalent reservoir system, S can then be calculated using the procedures outlined for a single reservoir. In this case, the volume balance function must be properly interpreted. The $q_i$ is of course the equivalent reservoir volume. The $y_i$ represent the total net inflow to the reservoir system rather than streamflow at a specific point. The $x_i$ are the net target release requirements from the entire system. The evaporation term is computed from the losses from the optimal state relationship since $q_i$ determines $q^*_{ij}$.

Owing to a lack of time and funds, the probabilities for an equivalent reservoir could not be accomplished. Although somewhat more complex there does not appear to be any significant problem once the optimal states have been defined.

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