ECONOMICS

Long-Term Loan Repayment Methods   no. 3.757

by P.H. Gutierrez and N.L. Dalsted [1] (09/08)

Quick Facts...

Long-term loans can be repaid in a series of annual, semi-annual or monthly payments.

Payments can be equal total payments, equal principal payments or equal payments with a balloon payment.

The Farm Service Agency usually requires equal total payments for intermediate and long-term loans.

Use an amortization table to determine the annual payment when the amount of money borrowed, the interest rate and the length of the loan are known.

Colorado State University

Extension

Money borrowed for long-term capital investments usually is repaid in a series of annual, semi-annual or monthly payments. There are several ways to calculate the amount of these payments:

1. equal total payments per time period (amortization);
2. equal principal payments per time period; or
3. equal payments over a specified time period with a balloon payment due at the end to repay the balance.

When the equal total payment method is used, each payment includes the accrued interest on the unpaid balance, plus some principal. The amount applied toward the principal increases with each payment (Table 1).

The equal principal payment plan also provides for payment of accrued interest on the unpaid balance, plus an equal amount of the principal. The total payment declines over time. As the remaining principal balance declines, the amount of interest accrued also declines (Table 2).

These two plans are the most common methods used to compute loan payments on long-term investments. Lenders also may use a balloon system. The balloon method often is used to reduce the size of periodic payments and to shorten the total time over which the loan is repaid. To do this, a portion of the principal will not be amortized (paid off in a series of payments) but will be due in a lump sum at the end of the loan period. For many borrowers, this means the amount to be repaid in the lump sum must be refinanced, which may be difficult.

Repayment Principles

To calculate the payment amount, all terms of the loan must be known: interest rate, timing of payments (e.g., monthly, quarterly, annually), length of loan and amount of loan. Borrowers should understand how loans are amortized, how to calculate payments and remaining balances as of a particular date, and how to calculate the principal and interest portions of the next payment. This information is valuable for planning purposes before an investment is made, for tax management and planning purposes before the loan statement is received, and for preparation of financial statements.

With calculators or computers, the calculations can be done easily and quickly. The use of printed tables is still common, but they are less flexible because of the limited number of interest rates and time periods for which the tables have been calculated.

Regardless of whether the tables or a calculator is used, work through an example to help apply the concepts and formulas to a specific case.

Lenders Use Different Methods

Different lenders use different methods to calculate loan repayment schedules depending on their needs, borrowers' needs, the institution's interest

Table 1: Example of loan amortization: equal total payment plan.

| | | Loan amount $10,000, annual rate 12%
 8 annual payments | | |
Year	Annual payment	Principal payment	Interest	Unpaid balance
				$10,000.00
1	$2,013.03	$ 813.03	$1,200.00	9,186.87
2	2,013.03	910.59	1,102.44	8,276.38
3	2,013.03	1,019.86	993.17	7,256.52
4	2,013.03	1,142.25	870.78	6,114.27
5	2,013.03	1,279.32	733.71	4,834.95
6	2,013.03	1,432.83	580.20	3,402.12
7	2,013.03	1,604.77	408.26	1,797.35
8	2,013.03	1,797.35	215.68	0
Total	$16,104.24	$10,000.00	$6,104.24	0

Table 2: Example of loan amortization: equal principal plan.

| | | Loan amount $10,000, annual rate 12%
 8 annual payments | | |
Year	Annual payment	Principal payment	Interest	Unpaid balance
				$10,000.00
1	$2,450.00	$1,250.00	$1,200.00	8,750.00
2	2,300.00	1,250.00	1,050.00	7,500.00
3	2,150.00	1,250.00	900.00	6,250.00
4	2,000.00	1,250.00	750.00	5,000.00
5	1,850.00	1,250.00	600.00	3,750.00
6	1,700.00	1,250.00	450.00	2,500.00
7	1,550.00	1,250.00	300.00	1,250.00
8	1,400.00	1,250.00	150.00	0
Total	$15,400.00	$10,000.00	$5,400.00	0

rate policy (fixed or variable), the length of the loan, and the purpose of the borrowed money. Typically, home mortgage loans, automobile and truck loans, and consumer installment loans are amortized using the equal total payment method.

The Farm Service Agency usually requires equal total payments for intermediate and long-term loans.

The Federal Credit Services (FCS) uses the equal total payment method for many loans. Under certain conditions the FCS may require that more principal be repaid earlier in the life of the loan, so they will use the equal principal payment method. For example, in marginal farming areas or for ranches with a high percentage of grazing land in non-deeded permits, FCSs may require equal principal payments.

Production Credit Associations (PCA) usually schedule equal principal payment loans for intermediate term purposes. Operating notes are calculated slightly differently. Other commercial lenders use both methods.

Lenders often try to accommodate the needs of their borrowers and let the borrower choose which loan payment method to use. A comparison of Tables 1 and 2 indicates advantages and disadvantages of each plan. The equal principal payment plan incurs less total interest over the life of the loan because the principal is repaid more rapidly. However, it requires higher annual payments in the earlier years when money to repay the loan is typically scarce. Furthermore, because the principal is repaid more rapidly, interest deductions for tax purposes are slightly lower. Principal payments are not tax deductible, and the choice of repayment plans has no effect on depreciation.

The reason for the difference in amounts of interest due in any time period is simple: Interest is calculated and paid on the amount of money that has been loaned but not repaid. In other words, interest is almost always calculated as a percentage of the unpaid or remaining balance: $I = i \times R$
where:

I = interest payment,

i = interest rate and

R = unpaid balance.

Using the Formulas

Because of the infinite number of interest rate and time period combinations, it is easier to calculate payments with a calculator or computer than a table. This is especially true when fractional interest rates are charged and when the length of the loan is not standard. Variable interest rates and rates carried to two or three decimal places also make the use of printed tables difficult.

Equal Total Payments

For equal total payment loans, calculate the total amount of the periodic payment using the following formula: $B = (i \times A) / [1 - (1 + i)^{-N}]$
where:

A = amount of loan,

B = periodic total payment, and

N = total number of periods in the loan.

The principal portion due in period n is: $C_n = B \times (1 + i)^{-(1 + N - n)}$

where:

C = principal portion due and

n = period under consideration.

The interest due in period n is: $I_n = B - C_n$.

The remaining principal balance due after period n is: $R_n = (I_n / i) - C_n$.

Equal Principal Payments

For equal principal payment loans, the principal portion of the total payment is calculated as: $C = A / N$.

The interest due in period n is: $I_n = [A - C_{(n-1)}] \times i$.

The remaining principal balance due after period n is: $R_n = (I_n / i) - C$.

Calculating Payments with Variable Interest Rates

Many lenders (especially the Farm Credit System) now use variable interest rates, which greatly complicates calculating the payment. The most common way to amortize a loan under a variable interest rate is to calculate the amount of principal due, based on the interest rate in effect on the payment due date. The interest payment is then calculated in the normal fashion.

To illustrate, assume the same loan terms used in Tables 1 and 2: a $10,000 loan at 12 percent interest and an 8-year repayment schedule using the equal total payment method. Assume the interest rate is variable; it remains at 12 percent for the first six months of the year and then changes to 13 percent for the last six months. Instead of calculating the principal due at the end of the first year on the basis of 12 percent, it is calculated using 13 percent. Apply the formulas of the previous section to get:

$C_1 = i \times A / [1 - (1 + i)^{-N}] \times (1 + i)^{-(1 + N - n)} = \$783.87,$

using $i = 0.13$. Consequently, the principal payment is $783.87 instead of $813.03. The interest payment is calculated at 12 percent for six months and at 13 percent for six months:

$I_1 = [\$10,000 \times 0.12 \times (6 / 12)] + [\$10,000 \times 0.13 \times (6 / 12)] = \$1,250$

Thus the total payment for the first year is:

$B_1 = \$783.87 + \$1,250 = \$2,033.87$ and

$R_1 = \$10,000 - \$783.87 = \$9,216.13$

To carry this example one step further, assume the interest rate in the second year of the note remains at 13 percent for two months and then moves to 14 percent and stays there for 10 months. The same formula is used, but now C is calculated using $i = 0.14$ and $n = 2$. Thus,

$C_2 = \$861.50$

and interest is:

$I_2 = [\$9,162.13 \times 0.13 \times (2 / 12)] + [\$9,216.13 \times 0.14 \times (10 / 12)]$

$= \$199.68 + \$1,075.22 = \$1,274.90$

$R_2 = \$9,216.13 - \$861.50 = \$8,354.63$

$B_2 = \$861.50 + \$1,274.90 = \$2,136.40$

This method computes the amount of principal and total payments and is used only for equal total payment loans. If the loan schedule was originally specified as the equal principal payment plan, the calculations are much easier because C (principal payments) remains the same for each period. Interest is calculated in the same manner as in the example above.

[1]*P.H.Gutierrez, former Colorado State University Extension farm/ranch management economist and associate professor, and N.L. Dalsted, Extension farm/ranch management specialist and professor; agricultural and resource economics.*

Table 3: Amortization table. Annual principal and interest paid per $1 borrowed by length of loan and interest rate.

No. of annual payments	Annual Interest Rate									
	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
3	.3535	.3603	.3672	.3741	.3811	.3880	.3951	.4021	.4092	.4163
4	.2690	.2755	.2820	.2886	.2952	.3019	.3087	.3155	.3223	.3292
5	.2184	.2246	.2310	.2374	.2439	.2505	.2571	.2638	.2706	.2774
6	.1846	.1908	.1970	.2034	.2098	.2163	.2229	.2296	.2364	.2432
7	.1605	.1666	.1728	.1791	.1856	.1921	.1987	.2054	.2122	.2191
8	.1425	.1485	.1547	.1610	.1675	.1740	.1807	.1874	.1943	.2013
9	.1284	.1345	.1407	.1470	.1535	.1601	.1668	.1736	.1806	.1877
10	.1172	.1233	.1295	.1359	.1424	.1490	.1558	.1627	.1698	.1770
11	.1081	.1141	.1204	.1268	.1334	.1401	.1469	.1540	.1611	.1684
12	.1005	.1066	.1128	.1193	.1259	.1327	.1397	.1468	.1540	.1614
13	.0940	.1001	.1065	.1130	.1197	.1265	.1336	.1408	.1482	.1557
14	.0885	.0947	.1010	.1076	.1143	.1213	.1284	.1357	.1432	.1509
15	.0838	.0899	.0963	.1030	.1098	.1168	.1241	.1315	.1391	.1468
20	.0672	.0736	.0802	.0872	.0944	.1019	.1095	.1175	.1256	.1339
25	.0574	.0640	.0710	.0782	.0858	.0937	.1018	.1102	.1187	.1275
30	.0510	.0578	.0651	.0726	.0806	.0888	.0973	.1061	.1150	.1241
35	.0465	.0536	.0611	.0690	.0772	.0858	.0946	.1037	.1129	.1223
40	.0433	.0505	.0583	.0665	.0750	.0839	.0930	.1023	.1117	.1213