latitude. The solar radiation measurements were construed to be less than or equal to 850 cal cm^{-2} day^{-1}.

5. The crop coefficient tended to underestimate the daily estimates. The length of the period to heading of 85 days suggested by Jensen appeared to be too long. This was reduced to 45 days after consultation with Dr. Cuany of the Agronomy Department. It was reasonable to assume that the leaf area index reaches 3 after 45 days. Another adjustment was also introduced to account for advective losses for the first three days after irrigation or rain.

Another way of comparing the estimated and measured values is by use of the seasonal balances given in table 1. The 1972 data show that the measured water use by treatments 3A-C, 4A-C, and 5 was higher than the potential. This is unreasonable. The estimates of potential ET appear to be very low. The most probable cause may be that the solar radiation measurements were low. The measurements were obviously in error in 1968, since the treatments with the most irrigation result in lower seasonable measures than the dryer ones. The ground water level was high according to the incremental measurements in the root zone.

Below the third foot in the 100 cm. zone, the plots were at field capacity throughout the season. Thus, the 1968 data were not analyzed further. The 1972 data were used in developing yield models.

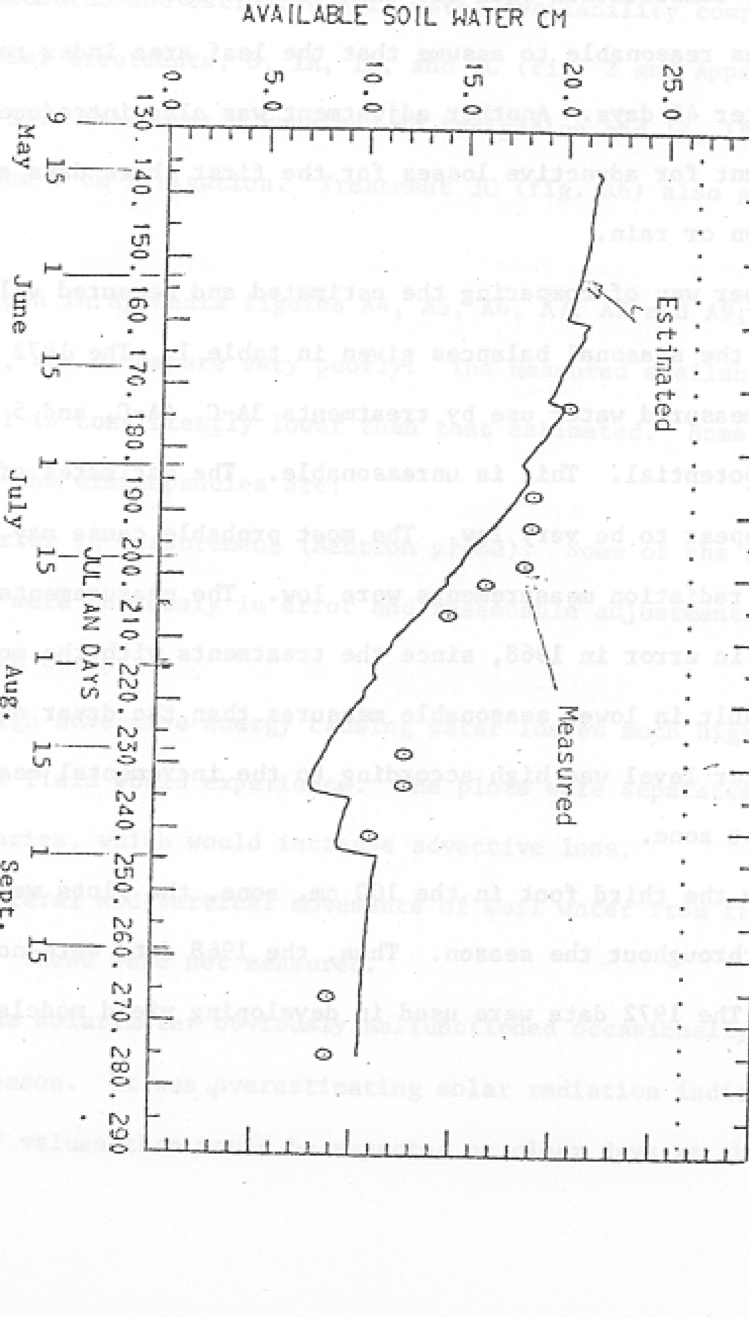


FIG.2--TREATMENT-0--NO IRRIGATION

TABLE 1

SEASONAL ESTIMATES OF WATER USE, RAINFALL AND IRRIGATION FOR CORN IRRIGATION TREATMENTS, AGRONOMY RESEARCH FARM, 1972

Treatment	Water Estimated (cm)	Use Measured (cm)	Potential ET (cm)	Transpiration (cm)	Rain and Irrigation (cm)	No. of Irrigation*
			1972			
0	21.73	22.35	30.34	15.61	9.46	0
1A	22.90	24.76	30.34	15.93	14.46	1
1B	23.10	25.46	30.34	16.22	14.46	1
1C	22.12	23.96	30.34	15.78	14.46	1
3A	23.80	34.16	30.34	17.14	24.46	3
3B	23.85	32.96	30.34	16.96	24.46	3
3C	25.20	33.26	30.34	17.80	24.46	3
4A	24.32	37.08	30.34	17.44	29.45	4
4B	24.00	35.22	30.34	17.40	29.46	4
4C	24.15	30.94	30.34	17.72	29.46	4
5	24.66	30.74	30.34	17.85	34.46	5
			1968			
I	13.75	13.83	16.79	10.63	34.36	4
II	13.88	18.31	16.79	10.41	26.88	3
III	13.86	24.79	16.79	10.39	24.51	2
IV	13.37	20.80	16.79	9.95	17.54	1
V	12.46	27.68	16.79	9.23	11.92	0

*The amount was 5 cm per irrigation in 1972. It was variable in 1968.

Crop Response to Selected Yield Indicators

A. Models

Selected seasonal and intra-seasonal yield indicators were used to evaluate parameters for production functions for corn. The following functions were formulated:

$$Y = Y_{max}(B_1)^{X_1} B_2^{X_5} \tag{10}$$

$$Y = Y_{max} B_2^{X_2} B_2^{X_5} \tag{11}$$

and

$$Y = Y_{max}(1 - B_1 e^{-B_2 X_3}) B_3^{X_5} \tag{12}$$

In the above expressions, Y is grain yield in kg ha^{-1}, Y_{max} is the maximum yield, X_1 is the number of days soil moisture depletion was greater than 12 cm between June 22 and October 2. X_2 is the number of three-day periods that soil moisture depletion was above 12 cm, X_3 is the number of days soil moisture depletion was below 12 cm during the season, X_4 is the same as X_1 but applies to the silking period only. X_5 is the number of days irrigation has been delayed after silking began, and the B's are parameters. Note that Y_{max} is also a parameter.

Equations 10 - 11 are power (exponential) models. Equation 12 interacts the Mitscherlich (Heady and Dillon, 1961) and the power functions. Definitely both are non-linear. Note that the form of the power models require yield decreasing measures, whereas the form of Mitscherlich model requires a yield increasing measure.

Attempts were made to estimate the parameters in the above equations using STAT 31R (CSU) which is a computer algorithm for non-linear least squares estimation. The algorithm is based on the Marquardt's compromise (Marquardt, 1963) which represents a compromise between the steepest descent and linearization methods.

B. Convergence problem

The iterative process involved in deriving the parameter in Model 13 leads to non-convergence. Constraining the parameters B_1 and B, such that both live between 0.0 and 1.0 lead to estimates of Y_{max} and B_2 that were extremely large and non-realistic. The power functions converged at relatively few iterations, the maximum being 6, which is very efficient. The speed of convergence depends on the closeness of the initial guesses to the solution values.

C. Results

The data basis for fitting the models is presented in table 2. Estimates of the parameters in the power (exponential) models 10-11 are summarized in table 3.

The estimates of B_1 and B_2 fell between 0 and 1.0 as would be expected. An increase in the variables X_1, X_2, and X_5 reduces yield. The rate of reduction is:

$$\frac{\partial Y}{\partial x_i} = Y \ln B_j \qquad (13)$$

i = 1, 2, 5 and j = 1,2

The coefficients of determination apply to linear transformation of the models. This procedure was applied to get an idea of the explained variation in terms of standard linear analysis. The non-linear technique uses other criteria for convergence and R^2 is not given. However, it should be greater than the indicated value.

TABLE 2

CORN GRAIN YIELD AND RELATED YIELD INDICATORS FOR WATER PRODUCTION FUNCTIONS
AGRONOMY RESEARCH FARM, COLORADO STATE UNIVERSITY, FORT COLLINS, COLORADO
1972

Treatment	Grain Yield for Indicated Population Densities L (kg/ha)	M (kg/ha)	H (kg/ha)	Soil Water Related Yield Indicators* Seasonal DSM** - 12cm 1 Day Period X_1	Seasonal DSM** - 12cm 3 Day Period X_2	Seasonal DSM - 12cm 1 Day Period X_3	Silking DSM - 12cm 1 Day Period X_4	Irrigation Delay Days X_5
0	5140	4138	4030	-	-	-	-	-
1A	7823	7128		72	24	30	8	0
1B	7080	7142		61	14	41	5	5
1C	6873	6817		70	23	32	1	10
3A	9164	9450	8696	61	16	41	5	0
3B	8834	9037	9028	58	17	49	3	2
3C	8267	8906	8777	4	1	99	4	7
4A	10363	10130	9973	31	8	71	0	0
4B	9537	9972	9611	21	7	81	0	3
4C	8879	9553	10107	0	0	102	0	6
5	10119	9799	10035	0	0	102	0	2

* Obtained from corrected plot of soil water depletion over time.

** DSM = Depleted soil moisture.

TABLE 3

Non-linear Least Squares Estimate of Parameters in Power Function (Models 10 - 11) for Corn, Agronomy Research Farm, Colorado State University, Fort Collins, Colorado, 1972

	Model	Parameters Related to Indicated Variables[1/] Y_{max}	X_1	X_2	X_3	X_5	Standard Error of Estimate	R^2
	10	10959.2	0.996			0.973	551.1	0.76[2/]
S.E.[3/]		275.1	0.0005			0.004		
N.C.I.	Upper	1140.7	0.997			0.983		
	Lower	1051.1	0.995			0.963		
	11	10838.7		.989		.974	584.8	0.74
S.E.		280.5		0.002		.005		
N.C.I.	Upper	11309.4		0.992		.984		
	Lower	10368.1		0.985		.964		
	12		Did not converge					

1/ For definitions, see Table 2.

2/ R^2 was estimated by linearization. The non-linear R^2 must be greater than indicated.

3/ SE = Standard error of parameter estimates.

N.C.I. = Non-linear confidence limits.

Conclusion

These models are suitable for simulation of crop responses if accurate measurements of the soil water status can be made. The parameters given here need to be reevaluated in light of the fact that the soil water measurements employed here appeared to have errors.

EMPIRICAL ESTIMATES OF RESPONSE FUNCTIONS OF CORN TO SOIL MOISTURE STRESS

(Dr. Dan Yaron)

Introduction

This study uses empirical estimates of response functions of corn to soil moisture stress. These estimates are based on two corn irrigation experiments at the Agronomy Research Station, Colorado State University, Fort Collins, in the years 1972 and 1968.

In the expression of the soil moisture variables the concept of "stress days" or "critical days" was applied. A "critical day" was defined as one in which the soil moisture in the root zone was depleted below a certain level (45-55 percent of the available soil moisture, AMS). The number of "critical days" thus defined, or, alternatively the number of "noncritical" or "growth" days were used as explanatory variables in the response functions.

Two general formulations of the response function were applied: (a) the Mitscherlich function; and (b) the exponential function. The specific forms of these functions, which were applied, and their interpretation are discussed, along with the empirical estimates, in the following sections.

Response Function of Corn to Soil Moisture, Ft. Collins, 1972

The Experimental Data

Irrigation experiments which provided the data for the analysis were conducted in 1972 at the Colorado State University Agronomy Research Station, and were studied by Twyford (1973). Corn was planted on May 11 and harvested on October 26. 2/ There were 11 treatments in the experiment varying with time and number of irrigation applications. Each irrigation

---

2/ Final harvest.

applied water to a depth of 2 inches (5 cm.). The sequence of irrigation treatments is shown in table 4. Yields from the zero quota plots, those that received no water, were excluded from the analysis due to the possible contribution of ground water to soil moisture in the experimental plots of this treatment.

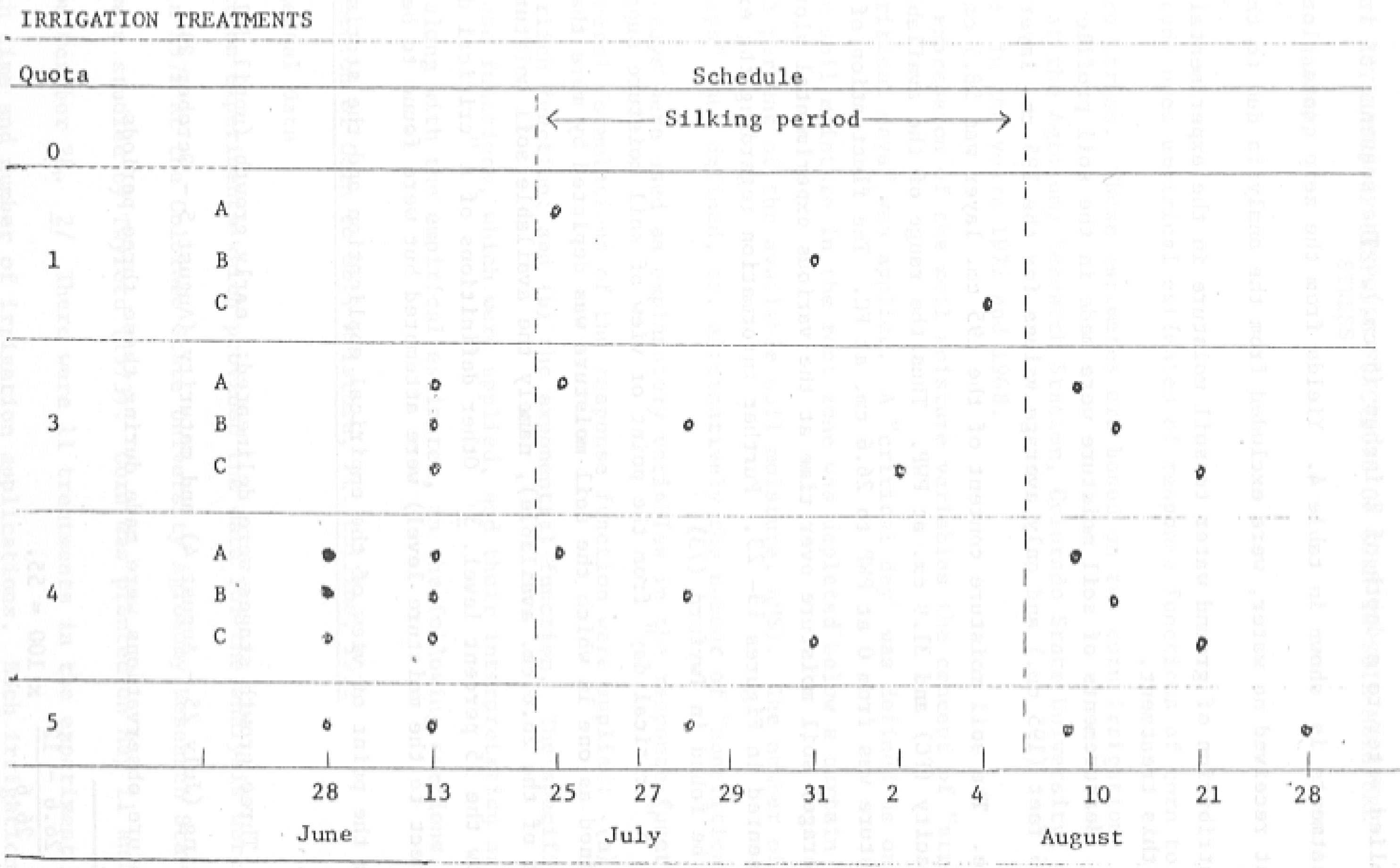
Measurements of soil moisture were made in the soil profile to a depth of 6 feet (195 cm.) and only average values for the 195 cm. layer are available. The soil moisture content of the 195 cm. layer was 58.5 cm. at field capacity (FC) and 31.9 cm. at PWP. Thus the range of the available soil moisture was from 0 at PWP to 26.6 cm. at FC. The fluctuation of the (average) soil moisture over time at the various experimental plots are presented in Figures 13- 23. Further information regarding the experiment can be found in Twyford (73).

A "critical day" from the point of view of soil moisture supply was defined as one in which the soil moisture was depleted by more than 12 cm. (out of the 26.6 cm. available), namely the available soil moisture dropped below the 55 percent level. 3/ Other definitions of a "critical day" (with respect to the moisture level) were attempted but were found to be inferior from the point of view of the empirical application and the statistical fit.

Three growth stages were delineated: early growth (until July 24), silking (July 25 - August 4) and maturity (August 5 - October 2). Soil moisture observations were made during these three periods.

3/ $\frac{26.6 - 12}{26.6} \times 100 = 55.$

Table 4--Irrigation treatments, corn irrigation experiment, Fort Collins, Colorado, 1972



The Mitscherlich (modified) function

One specification of the response function attempted (a combination of the Mitscherlich and the exponential forms) was the following:

(1) $y = A(1 - Be^{-kx_1^*})C^{x_2}$

where

y = grain yield of corn /kg/ha/,

x_1^* = relative number of "non-critical" growth days in the non-reproductive period (early growth and maturity stages) with soil moisture above 55 percent of ASM, expressed as the percentage of the total number of days in this period.

x_2 = the number of critical days during the silking stage.

A, B, k and C = the parameters estimated.

The estimates of (1) were obtained using a steepest descent (computerized) search method intended to minimize, or rather to obtain low values of $\Sigma(y_i - \hat{y}_i)^2$. The "best" estimates in terms of $\Sigma(y_i - \hat{y}_i)^2$ and general soundness of the results are presented in table 5.

The estimated values of $\hat{y}$ versus the actual yields using estimate No. 1 are presented in table 6, and those using estimate No. 2 are presented in the Appendix in table A1.

The interpretation of Estimate 1 suggests that:

(a) The asymptotical yield ($\hat{A}$) is 10,000 kg/ha approximately (161 bu./acre).

(b) Each critical day in the silking period reduces the yield by 2 percent approximately ($\hat{C}$ = .98).

(c) Under conditions of $x_1^* = 0$ the maximal yield will be reduced by 90 percent. Note that the value of $\hat{B}$ from which this result is derived was imposed. In Estimate 2 with $\hat{B} = 1$, for $x_1^* = 0$, the yield is reduced by 100 percent.

Table 5--Empirical estimates of the Mitscherlich type response function for corn, Ft. Collins, 1972

Estimate number	Estimated parameters: $\hat{A}$	$\hat{B}$	$\hat{k}$	$\hat{C}$	$\bar{R}^2$ 2/
1	9,957	1/ 0.90	5.529786	0.98163	0.75
2	9,946	1.0	5.820091	0.98184	0.75

1/ With the value of $\hat{B}$ being imposed to equal 0.90.
2/ Computed as $\bar{R} = \frac{\Sigma(y_1 - \hat{y}_1)^2}{\Sigma(y_1 - \bar{\bar{y}}_{yi})^2}$ with the conventional notation.

* * *

Table 6--Actual and estimated values of corn yield using estimate No. 1, Ft. Collins, 1972 irrigation experiment

Treatment	Variables' values 1/: x_1	x_2	y_i	$\hat{y}_1$	$(y_i-\hat{y}_i)$	Relative deviation, percent 2/
1A	.27	6	7,128	7,107	21	0
3A	.42	7	9,450	7,974	1,476	16
4A	.65	0	10,130	9,711	419	4
1B	.36	5	7,142	7,960	-818	-11
3B	.47	2	9,037	8,953	84	1
4B	.84	0	9,972	9,871	101	1
1C	.38	9	6,817	7,499	-682	-10
3C	1.00	5	8,906	9,043	-137	- 2
4C	1.00	0	9,553	9,922	-369	- 4
5	1.00	0	9,799	9,922	-122	- 1

1/ See text for the definition of the variables.
2/ Computed as $\frac{(y_i - \hat{y}_i)}{y_i} \times 100.$

(d) In order to obtain the marginal productivity of x_1 we define

(2) $\hat{A}^* = \hat{A}\,\hat{C}^{x2}$

and

(1)' $\hat{Y} = \hat{A}^* \, (1 - \hat{B}e^{-kx_1^*})$

and take the partial derivative of (1)' with respect to x*, to obtain:

(3) $\frac{\partial y}{\partial x_1^*} = \hat{k}\,(\hat{A}^* - \hat{y})$.

Values of the marginal productivity of x_1^* based on Estimate No. 1 for selected situations are presented in table 7, and those based on Estimate No. 2 are given in table A2 in the Appendix. Note that the range of observations was $.27 \leq x_1^* \leq 1.00$, $0 \leq x_2 \leq 9$, and $6{,}817 \leq y \leq 10{,}130$.

Examination of table 4 suggests that the marginal contribution of x_1^* is in the range of between 0.2 to 9 percent of the maximal yield or between 2 to 90 kg/ha. The results expressed on the basis of the actual rather than relative number of non-critical days are much alike since there were 92 days in the non-silking period. The marginal contribution of x_1^* at the mean of x_1^* is 1.4, 1.3 and 1.2 percent of the maximal yield for $x_2 = 0$, $x_2 = 5$ and $x_2 = 10$, respectively.

<u>The exponential function</u>

Another specification of the response function was the following:

(4) $y = A\, b_1^{x_1}\, b_2^{x_2}$

with

x_1 = the number of critical days in the non-reproductive period (soil moisture below 55 percent of AMS);

y, x_2 = as previously defined;

A, B_1, b_2 = parameters ($0 < b_1, b_2 \leq 1$).

Table 7--Marginal product of x_1^* for selected combinations of x_1^* and x_2^*, based on Estimate 1

Variables' values[1/]				
x_1^*	x_2	$\hat{y}$	$MP_{x_1^*}$ [2/]	Relative marginal product [3/]
.30	0	8,251	94	9.4
.30	55	7,521	86	8.6
.30	10	6,855	78	7.8
.50	0	9,393	31	3.1
.50	5	8,561	28	2.8
.50	10	7,803	26	2.6
.75	0	9,816	7.8	0.78
.75	5	8,946	7.1	0.71
.75	10	8.154	6.5	0.65
1.00	0	9,922	1.96	0.196
1.00	5	9,043	11.79	0.179
1.00	10	8,242	1.63	0.163

1/ See text for the definition of the variables.
2/ Computed according to (3) in text as $\frac{\partial \hat{y}}{\partial x_1^*}$ =
3/ Computed as $(MP_{x_1^*}/1\hat{A}) \times 100$.

The estimate obtained by regression techniques was:

$$(5)\ \hat{y} = \underset{**}{10,070} \cdot (0.9983)^{x_1} \underset{**}{(0.975)}^{x_2} \quad R^2 = 0.65$$

with ** denoting significance of the estimates parameter at 1 percent, and § denoting nonsignificance at an acceptable probability level.

The interpretation of the above estimate is as follows:

(a) The asymptotic yield is 10,000 kg/ha approximately ($\hat{A} = 10,070$);

(b) Each "critical day" in the silking stage reduces the yield by 2.5 percent ($b_2 = .975$);

(c) Each "critical day" in the nonreproductive period reduces the yield by 0.27 percent approximately ($\hat{b}_1 = .9983$).

Note that the latter result was derived from the estimated value of b which was found to be "non-significant", i.e., subject to a considerable error of estimate.

In another attempt to evaluate the effect of x_1 Figure 11 was drawn, showing the relationship between y and x_1. The figure suggests that for each of the three groups of treatments A, B, C (early, medium and late irrigation during the silking stage) there exists a response function (free-hand drawn on the graph), with only one observation in the C group (3C) diverging from the otherwise quite regular pattern. In view of this observation two separate functions of the form

$$(6)\ y = A\, b_1^{x_1}$$

were estimated for the A and B groups. Note that there were only three observations in each group and the formal statistical significance of the estimates is rather dubious. Nevertheless, the estimates obtained provide a notion of the effect of soil moisture on the yield in the non-productive period.

The estimates were:

(7) Group A: $\hat{y} = 14,250\ (0.9904)^{x_1}$

Group B: $\hat{y} = 11,210\ (0.9936)^{x_1}$

The above estimates suggest that each critical day in the non-reproductive period reduces the yield by 0.7 - 1.0 percent.

Estimates of (6) with imposition of $\hat{A} = 10,000$ were

(8) Group A: $\hat{y} = 10,000\ (0.9960)^{x_1}$

Group B: $\hat{y} = 10,000\ (0.9968)^{x_1}$

indicating that each critical day in the non-reproductive period reduced the yield by 0.3 - 0.4 percent.

<u>Conclusions</u>

It seems that the following conclusions can be derived from the analysis of the 1972 Ft. Collins corn irrigation experiment:

(a) The concept of "critical days" (or non-critical ones) provides a valid basis for the definition of explanatory variables in the specification of response of corn to soil moisture.

(b) Any critical day in the silking period, here defined as a day with soil moisture lower than 55 percent of ASM in the root zone reduces the yield by 2 - 2.5 percent.

(c) Any critical day in the non-reproductive stages of growth reduces the yield by a fraction which has not been uniquely estimated on the basis of the experimental data available. The estimates derived using the Mitscherlich function indicate a reduction factor varying between 9 percent and 0.2 percent of the maximal yield per each critical day. For the mean value of x^* (= .645) the reduction per day is 1.2 - 1.4 percent. The estimates derived using the "nonrestricted" exponential function (7) indicate a reduction of 0.7 - 1 percent, and with the imposition of A = 10,000 as in (8) a reduction of 0.3 - 0.4 percent. Additional information is needed to improve these estimates. 4/

Finally, it should be noted that the definition of a "critical day" applied in the above analysis is somewhat arbitrary. No significant difference between the 55 percent of AMS as the critical level versus, say 50 percent of AMS, could be claimed. Note that the number of days with soil moisture below .55 AMS is highly correlated with that below .50 AMS. Further information from plant physiologists and soil scientists is needed in order to define the critical level more precisely.

Response Function of Corn to Soil Moisture, Ft. Collins, 1968

The Experimental Data

The experiment analyzed in the following was conducted in 1968 at the Agronomy Research Station, Colorado State University on Nunn clay loam soil. The field was planted with Kitely K4-17 (105 day season) hybrid seed on May 9. The experiment involved two factors, namely soil moisture (irrigation) and nitrogen fertilizer. The experimental design and treatments, the irrigation schedule, the rainfall records, and the grain yield are shown in tables 8 through 11 respectively.

Measurements of soil moisture tension were taken throughout the major part of the season (July 3- August 28) for the three upper soil layers of one foot depth each. Soil moisture tension of ½ bar, equivalent to 26.1 percent of soil moisture (on gravimetric basis) was considered as field

---

4/ Note that (7) and (8) yield a compounded reduction rate. For the mean number of 64 critical days compounded reduction rate of 0.5 percent is equivalent to a non-compounded rate of 1.1 percent ($0.995^{64} = 0.011 \times 64$).

Table 8--Experimental design and treatments, corn, Fort Collins, Colorado, 1968[1/]

Nitrogen fertilizer treatments (lbs./acre)	Irrigation treatments				
	I	II	III	IV	V
200	2		2		2
150		1		1	
100	2		2		2
50		1		1	
0	2		2		2
	Maximum soil water tension (in Bars)[2/]				
	0.7	1.0	3.0	6.0	9.0

1/ Figures represent the number of replications in each of the blocks.
2/ Maximum soil water tension in Bars allowed at 12-inch soil depth.

* * *

Table 9--Irrigation schedule, corn, Fort Collins, Colorado, 1968

Date	Irrigation treatment				
	I	II	III	IV	V
	Inches of water/irriation				
July 5	2.57				
July 11		2.02			
July 15			2.94		
July 18	2.21				
July 23		2.02			
July 29	2.21				
Aug. 2				2.76	
Aug. 6			2.21		
Aug. 7		1.84			
Aug. 23	1.84				
TOTAL	8.83	5.88	5.15	2.76	0.00

Table 10--Rainfall records, Fort Collins, Colorado, 1968

Week	Precipitation Inches	Week	Precipitation Inches
May 5 - May 11	0.07	July 14 - July 20	0.10
May 12 - May 18	0.43	July 21 - July 27	0.19
May 19 - May 25	1.92	July 28 - Aug. 3	0.00
May 26 - June 1	0.06	Aug. 4 - Aug. 10	1.05
June 2 - June 8	0.20	Aug. 11 - Aug. 17	1.25
June 9 - June 15	0.12	Aug. 18 - Aug. 24	0.00
June 16 - June 22	0.00	Aug. 25 - Aug. 31	0.00
June 23 - June 29	0.48	Sept. 1 - Sept. 7	0.00
June 30 - July 6	0.02		
July 7 - July 13	0.01	Total for season	5.90

* * *

Table 11--Grain yield in bushels per acre, corn, Fort Collins, Colorado, 1968

Nitrogen fertilizer treatments (lbs./acre)	Irrigation treatments I	II	III	IV	V
	Bushels/acre				
200	1/ 143.7		141.9		96.2
150		136.7		115.1	
100	140.4		128.7		101.4
50		150.3		100.0	
0	126.2		116.5		82.0

1/ One bushel = 56 lbs. One lb. = 0.45 kg.

capacity (FC), and soil moisture tension of 15 bars equivalent to 12.4 percent of soil moisture was considered as permanent wilting point (PWP). The levels of gravimetric soil moisture (percent), and volumetric soil moisture (percent), (averaged over four soil layers, of one foot each) corresponding to selected values of soil moisture tension over the relevant range of moisture situations are presented in table 12.

Table 12--Soil water relationships, Fort Collins Agronomy Research Station

Moisture tension, bars 1/	1/3	1/2	1	5	10	15
Gravimetric soil moisture, percent 2/	28.2	26.1	19.2	14.6	12.9	12.4
Volumetric soil moisture, percent 2/	38.1	35.2	25.9	19.7	17.4	16.7

1/ Desorption data from pressure membrane.
2/ Averaged over four layers of one foot each.

More details on the experiment can be found in Technical Bulletin 107, Colorado State University Experiment Station, Fort Collins, Colorado, January, 1970.

On the basis of soil moisture tension measurements, tension - soil moisture relationships (table 12), irrigation and rainfall data (tables 9 and 10), soil moisture values (averaged over the two upper soil layers of one foot each) were computed, and the corresponding soil moisture fluctuation curves were drawn (figures A12-A16)5/, and the number of days with soil moisture below 45 percent of AMS were counted for each treatment. The results of the number

5/ Two layer averages were computed since the moisture variation in the third foot layer was very small. It was at field capacity throughout the season in treatments I, II and III, and only slightly below field capacity during the last part of the season (August) in treatments IV and V.

of days below 45 percent AMS along with the nitrogen fertilizer and yield data (the latter two transformed into the metric system for sake of conformity with the previous section) are shown in table 13.

The Estimated Functions

The following response functions were specified and estimated with reference to the data in table 13:

(9) $\hat{y} = 9{,}195(.9905)^{x_1}_{**}\ (.9993)^{(225-x_2)}_{**}$ $\quad R^2 = 0.90$

(10) $\hat{y} = 10{,}000(.9899)^{x_1}\ (1 - .16156e^{-1.0867x_2 \cdot 10^{-3}})$ $\quad \bar{R} = 0.81$

with

$\hat{y}$ = estimated corn grain yield kg/ha;

x_1 = number of days with soil moisture below 45 percent AMS in two upper feet of soil;

x_2 = nitrogen fertilizer level kg/ha

** - denotes significance of the parameter at 1 percent probability level.

The asymptotic yield in (10) was imposed to be 10,000 kg/ha. $\bar{R}^2$ was computed as $\Sigma(y_i - \hat{y}_i)^2/\Sigma(y_i - \bar{y}_i)^2$.

Table 14 shows the actual (y_i) and the estimated yield ($\hat{y}_i$) for the various treatments using estimate (10), and the deviations between them.

According to both estimates (9) and (10) each "critical day," namely with soil moisture below 0.45 AMS, during the July-August period reduced the yield by one percent. The asymptotic yield was 9,200 kg/ha according to (9). In (10) it was imposed to equal 10,000.

It is unfortunate that the data available from the 1968 and the 1972 experiments were different and no common basis for the comparison of the results could be designed.

Table 13--Number of days with soil moisture below 45 percent of AMS during July-August season, the level of nitrogen fertilizer and the yield of corn grain, Fort Collins, Colorado, 1968[1/]

Observation number	Irrigation treatment	Days with soil moisture below 45% AMS	Nitrogen fertilizer level, kg/ha	Corn grain yield, kg/ha
1	I	0	0	7,951
2		0	112	8,845
3		0	225	9,053
4	II	0	56	9,469
5		0	169	8,612
6	III	1	0	7,340
7		1	112	8,108
8		1	225	8,940
9	IV	14	56	6,300
10		14	169	7,252
11	V	39	0	5,166
12		39	112	6,388
13		39	225	6,061

1/ The data in tables 5 and 8 were transformed into the metric units using the following relationships: one bushel of corn = 56 lbs; one lb. = 0.45 kg.

Table 14--Actual and estimated values of corn yield using estimate (10), Fort Collins, Colorado 1968 experiment

Observation number	Variables' values[1/] x_1	x_2	y_i	$\hat{y}_i$	$y_i - \hat{y}_i$	Relative deviation Percent
1	0	0	7,951	8,384	-433	- 5
2	0	112	8,845	8,570	275	3
3	0	225	9,053	8,735	318	4
4	0	56	9,469	8,480	989	10
5	0	169	8,612	8,655	- 43	0
6	1	0	7,340	8,300	-960	-13
7	1	112	8,108	8,484	-376	- 5
8	1	225	8,940	8,646	294	3
9	14	56	6,300	7,148	-848	-13
10	14	169	7,252	6,781	471	6
11	39	0	5,166	5,640	474	9
12	39	112	6,388	5,765	623	10
13	39	225	6,061	5,876	185	3

1/ See text for the definition of variables.

Two major differences were: In the 1972 experiment only averaged soil moisture measurements to the depth of 195 cm (78 inches) were available, while for the 1968 experiment there were observations only for three top layers of one foot each (91.5 cm). 6/ There was no detailed information on the silking period in 1968, while it was well defined in 1972. In the analysis of the 1972 experiment the number of critical days during the silking period was found to be an important factor.

Summary and Conclusions

In this study empirical estimates of response functions of corn to soil moisture were presented.

The analyses provide evidence that the concept of "critical days" as defined in this study, provides a useful basis for the expression of soil moisture variables in the specification of crop response functions. Under conditions with no extreme weather situations, the reference to soil moisture alone, rather than to a combination of soil moisture and other atmospheric evaporative conditions, is sufficient for the definition of a "critical day," for sake of a statistical analysis and estimation of response functions. Obviously, such a definition would not be satisfactory from the point of view of plant physiologists.

The concept of "critical days" has an advantage of having an operation implication in irrigation management and scheduling. The simplest rule is to irrigate crops before soil moisture falls to the level that would allow a "critical day" to occur, because the occurrence of a "critical day" reduces the yield of the crop. These studies show, however, that at some periods

---

6/ This explains the different levels of critical soil moisture in the two years (.55 AMS in 1972 and .45 AMS in 1968) which were found appropriate from the point of view of statistical fit.

during the growing season a "critical day" will result in a greater decline in yield than other periods. For instance, during the silking, tasseling period for corn each critical day reduces yields an estimated 2 to 2.5 percent while critical days before or after reduce yields .75 percent to 1 percent per day. If it is not possible to irrigate to avoid critical days on all crops, then one should apply the marginal principle and allocate water to those crops in which highest loss would occur due to delay in irrigation. Obviously, a precondition for proper management and timing of irrigation is the knowledge of the variations in soil moisture in the irrigated plots during the season. Methods for relatively easy tracing of these variations should be devised and adapted to the farmers' needs, so that the decline in soil moisture can be followed and irrigations scheduled to avoid the development of soil moisture conditions that cause occurrence of "critical days."

REFERENCES

Jensen, M.E. and D.F. Heerman. "Meterological Approaches to Irrigation Scheduling," National Irrigation Symposium, ASAE, Lincoln, Nebr., 1970.

Heerman, D.F. and M.E. Jensen. "Adapting Meterological Approaches in Irrigation Scheduling to High Rainfall Areas," National Irrigation Symposium, ASAE, Lincoln, Nebr., November 1970.

Kincaid, D.C. and D.F. Heerman. "Scheduling Irrigations Using a Programmable Calculator," U.S. Dept. of Agriculture, Agricultural Research Service, North Central Region, ARS-NC-12, Feb. 1974, p. 55.

Hanks, R.J. "A Model for Predicting Plant Yield as Influenced by Water Use," Agronomy Journal, Vol. 66, Sept. 1974, pp. 660-64.

Hill, R.W., R.J. Hanks, J. Keller, and P.V. Rasmussen. "Predicting Corn Growth as Affected by Water Management: An Example," Utah Agricultural Experiment Station, Sept. 1974.

Yaron, D., et al. "Wheat Response to Soil Moisture and the Optimal Irrigation Policy Under Conditions of Unstable Rainfall," in Water Resources Research, Vol. 9, 1973, pp. 1145-53.

Stewart, J.I., R. Hagan, and W. Pruitt. "Functions to Predict Optimal Irrigation Programs," in Journal of Irrigation and Drainage, ASCE, June, 1974b, pp. 179-99.

Hall, W. and N. Buras. "Optimum Irrigated Practices Under Conditions of Deficient Water Supply," Transactions of ASAE, Vol. 4, 1961, pp. 131-134.

__________ and W.S. Butcher. "Optimal Timing of Irrigation," Journal of Irrigation and Drainage, Division ASCE, June 1968, pp. 267-75.

Aron, G. "Optimal Timing of Irrigation: Discussion," Journal of Irrigation Drainage, Division ASCE, Vol. 95, No. IRI, 1969, pp. 254-57.

Hall, W.A. and J.A. Dracup. Water Resources Systems Engineering. McGraw-Hill, New York, 1970.

Minhas, B.S., et al. "Toward the Structure of a Production Function for Wheat Yields with Dated Inputs of Irrigation Water," Water Resources Research, June 1974, pp. 383-93.

Dudley, N. D. Howell, and W. Musgrave. "Optimal Intraseasonal Irrigation Allocation," Water Resources Research, Aug. 1971a, pp. 770-88.

Dudley, N., D. Howell, and W. Musgrave. "Irrigation Planning 2: Choosing Optimal Acreages Within a Season," Water Resources Research, Oct. 1971b, pp. 1051-63.

Anderson, R. and Maass, A. "A Simulation of Irrigation Systems," Technical Bulletin No. 1431, U.S. Dept. of Agriculture, 1971, Rev. Sept. 1974.

Young, R.A. and J.D. Bredehoeft. "Digital Computer Simulation for Solving Management Problems of Conjunctive Groundwater and Surface Water Systems," in Water Resources Research, Vol. 8, June 1972, pp. 533-56.

Parsons, R.M. Co. "Efficient Water Use and Farm Management Study in India," Report prepared for Government of India and USAID, Contract No. AID/nesa-409, Report Number In-631.7-P269, 1970.

Smith, Douglas V. Stochastic Irrigation Planning Models. Center for Population Studies, Harvard University, Cambridge, Mass., Jan. 1970.

Mustonen, S.E. and J.L. McGuiness. Estimating Evapotranspiration in a Humid Region. Tech. Bul. No. 1389, U.S. Dept. of Agriculture, July 1968.

Flinn, J.C. and W.F. Musgrave. "Development and Analysis of Input-Output Relations for Irrigation Water," Australian Journal of Agronomy and Economics, 11 (1), 1967, pp. 1-9.