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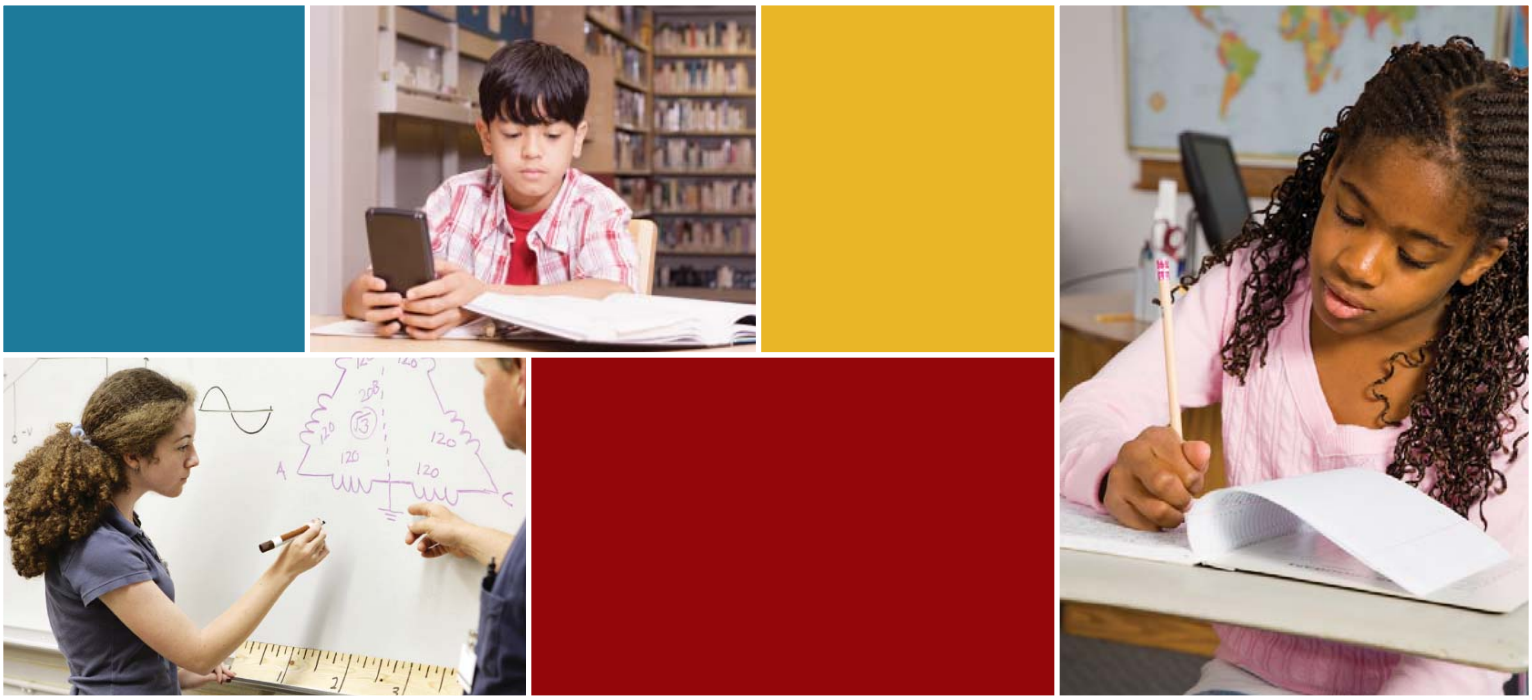
COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Colorado Academic STANDARDS

Third Grade

Mathematics

Mathematics



Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21st century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at www.corestandards.org. While all the expectations of the Common Core State Standards are embedded and **coded with CCSS:** in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

Colorado Academic Standards
Mathematics Standards

"Pure mathematics is, in its way, the poetry of logical ideas."
Albert Einstein

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"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."
Dr. Michael Brown, Nobel Prize Laureate

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In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21st century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The "Continuum of State Standards Definitions" section that follows shows the hierarchical order of the standards components. The "Standards Template" section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

Prepared Graduate Competencies: The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Standard: The topical organization of an academic content area.

High School Expectations: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

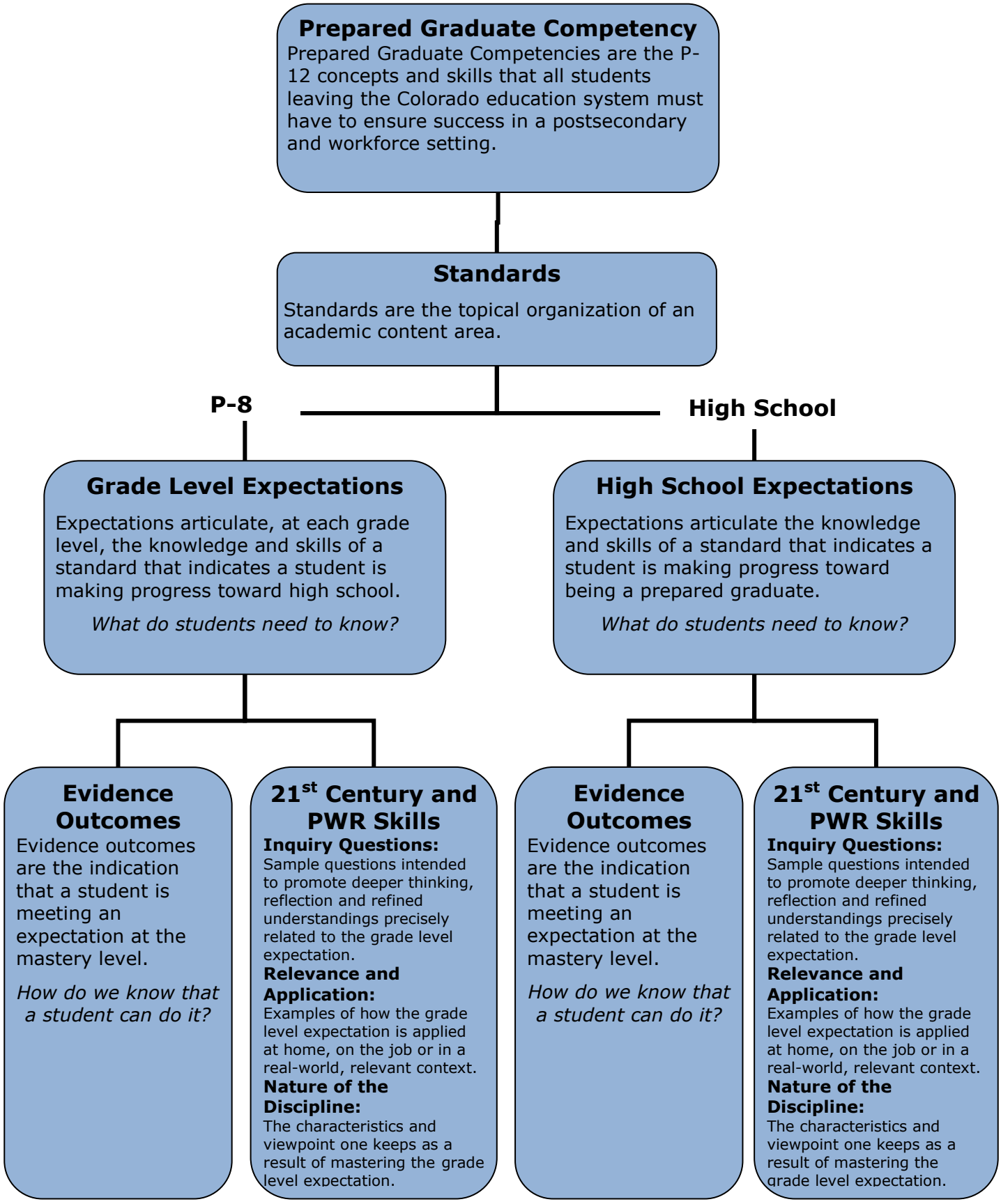
21st Century Skills and Readiness Competencies: Includes the following:

- ***Inquiry Questions:***
  Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

- ***Relevance and Application:***
  Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

- ***Nature of the Discipline:***
  The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Continuum of State Standards Definitions



STANDARDS TEMPLATE

Content Area: NAME OF CONTENT AREA

Standard: The topical organization of an academic content area.

Prepared Graduates:

- The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

High School and Grade Level Expectations

Concepts and skills students master:

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

What do students need to know?

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: Evidence outcomes are the indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*	**Inquiry Questions:** Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
	Relevance and Application: Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
	Nature of the Discipline: The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Standards for Mathematical Practice
from
The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions,
explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x^2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or

as being composed of several objects. For example, they can see 5 – 3(x – y)^2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x^2 + x + 1), and (x – 1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Third Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships and forms the foundation for efficient algorithms 2. Parts of a whole can be modeled and represented in different ways 3. Multiplication and division are inverse operations and can be modeled in a variety of ways
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to describe data
4. Shape, Dimension, and Geometric Relationships	1. Geometric figures are described by their attributes 2. Linear and area measurement are fundamentally different and require different units of measure 3. Time and attributes of objects can be measured with appropriate tools

From the Common State Standards for Mathematics, Page 21.

Mathematics | Grade 3
In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students

understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

21st Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of 21st century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21st century skills, as follows:

Critical Thinking and Reasoning
Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

Information Literacy
The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration
Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-Direction
Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

Invention
Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Colorado's Description for School Readiness

(Adopted by the State Board of Education, December 2008)

School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

Colorado's Description of Postsecondary and Workforce Readiness

(Adopted by the State Board of Education, June 2009)

Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

How These Skills and Competencies are Embedded in the Revised Standards

Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline.* These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

Inquiry Questions – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

Relevance and Application – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

Nature of Discipline – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind's eye.

1.Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. The whole number system describes place value relationships and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use place value and properties of operations to perform multi-digit arithmetic. (CCSS: 3.NBT) i. Use place value to round whole numbers to the nearest 10 or 100. (CCSS: 3.NBT.1) ii. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 3.NBT.2) iii. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations. [1] (CCSS: 3.NBT.3)	**Inquiry Questions:** 1. How do patterns in our place value system assist in comparing whole numbers? 2. How might the most commonly used number system be different if humans had twenty fingers instead of ten? **Relevance and Application:** 1. Knowledge and use of place value for large numbers provides context for distance in outer space, prehistoric timelines, and ants in a colony. 2. The building and taking apart of numbers provide a deep understanding of the base 10 number system. **Nature of Mathematics:** 1. Mathematicians use numbers like writers use letters to express ideas. 2. Mathematicians look for and make use of structure. (MP) 3. Mathematicians look for and express regularity in repeated reasoning. (MP)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Parts of a whole can be modeled and represented in different ways

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Develop understanding of fractions as numbers. (CCSS: 3.NF) i. Describe a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; describe a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. (CCSS: 3.NF.1) ii. Describe a fraction as a number on the number line; represent fractions on a number line diagram.[2] (CCSS: 3.NF.2) iii. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (CCSS: 3.NF.3) 1. Identify two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (CCSS: 3.NF.3a) 2. Identify and generate simple equivalent fractions. Explain[3] why the fractions are equivalent.[4] (CCSS: 3.NF.3b) 3. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.[5] (CCSS: 3.NF.3c) 4. Compare two fractions with the same numerator or the same denominator by reasoning about their size. (CCSS: 3.NF.3d) 5. Explain why comparisons are valid only when the two fractions refer to the same whole. (CCSS: 3.NF.3d) 6. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.[6] (CCSS: 3.NF.3d)	**Inquiry Questions:** 1. How many ways can a whole number be represented? 2. How can a fraction be represented in different, equivalent forms? 3. How do we show part of unit? **Relevance and Application:** 1. Fractions are used to share fairly with friends and family such as sharing an apple with a sibling, and splitting the cost of lunch. 2. Equivalent fractions demonstrate equal quantities even when they are presented differently such as knowing that 1/2 of a box of crayons is the same as 2/4, or that 2/6 of the class is the same as 1/3. **Nature of Mathematics:** 1. Mathematicians use visual models to solve problems. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Third Grade

Concepts and skills students master:

3. Multiplication and division are inverse operations and can be modeled in a variety of ways

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and solve problems involving multiplication and division. (CCSS: 3.OA) i. Interpret products of whole numbers.[7] (CCSS: 3.OA.1) ii. Interpret whole-number quotients of whole numbers.[8] (CCSS: 3.OA.2) iii. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.[9] (CCSS: 3.OA.3) iv. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.[10] (CCSS: 3.OA.4) v. Model strategies to achieve a personal financial goal using arithmetic operations (PFL) b. Apply properties of multiplication and the relationship between multiplication and division. (CCSS: 3.OA) i. Apply properties of operations as strategies to multiply and divide.[11] (CCSS: 3.OA.5) ii. Interpret division as an unknown-factor problem.[12] (CCSS: 3.OA.6) c. Multiply and divide within 100. (CCSS: 3.OA) i. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division[13] or properties of operations. (CCSS: 3.OA.7) ii. Recall from memory all products of two one-digit numbers. (CCSS: 3.OA.7) d. Solve problems involving the four operations, and identify and explain patterns in arithmetic. (CCSS: 3.OA) i. Solve two-step word problems using the four operations. (CCSS: 3.OA.8) ii. Represent two-step word problems using equations with a letter standing for the unknown quantity. (CCSS: 3.OA.8) iii. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 3.OA.8) iv. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.[14] (CCSS: 3.OA.9)	**Inquiry Questions:** 1. How are multiplication and division related? 2. How can you use a multiplication or division fact to find a related fact? 3. Why was multiplication invented? Why not just add? 4. Why was division invented? Why not just subtract? **Relevance and Application:** 1. Many situations in daily life can be modeled with multiplication and division such as how many tables to set up for a party, how much food to purchase for the family, or how many teams can be created. 2. Use of multiplication and division helps to make decisions about spending allowance or gifts of money such as how many weeks of saving an allowance of $5 per week to buy a soccer ball that costs $32?. **Nature of Mathematics:** 1. Mathematicians often learn concepts on a smaller scale before applying them to a larger situation. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP) 4. Mathematicians look for and make use of structure. (MP)

[1] e.g., 9 × 80, 5 × 60. (CCSS: 3.NBT.3)
[2] Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. (CCSS: 3.NF.2a)
Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. (CCSS: 3.NF.2b)
[3] e.g., 1/2 = 2/4, 4/6 = 2/3). (CCSS: 3.NF.3b)
[4] e.g., by using a visual fraction model.(CCSS: 3.NF.3b)
[5] Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. (CCSS: 3.NF.3c)
[6] e.g., by using a visual fraction model. (CCSS: 3.NF.3d)
[7] e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. (CCSS: 3.OA.1)
For example, describe a context in which a total number of objects can be expressed as 5 × 7. (CCSS: 3.OA.1)
[8] e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. (CCSS: 3.OA.2)
For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. (CCSS: 3.OA.2)
[9] e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS: 3.OA.3)
[10] For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ▪ ÷ 3, 6 × 6 = ?. (CCSS: 3.OA.4)
[11] Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) (CCSS: 3.OA.5)
[12] For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. (CCSS: 3.OA.6)
[13] e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8. (CCSS: 3.OA.7)
[14] For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. (CCSS: 3.OA.9)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

Grade Level Expectation: PRESCHOOL THROUGH THIRD GRADE

Concepts and skills students master:

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: **Expectations for this standard are integrated into the other standards at preschool through third grade.**	**Inquiry Questions:**
	Relevance and Application:
	Nature of Mathematics:

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Visual displays are used to describe data

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and interpret data. (CCSS: 3.MD) i. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. (CCSS: 3.MD.3) ii. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.[1] (CCSS: 3.MD.3) iii. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. (CCSS: 3.MD.4)	**Inquiry Questions:** 1. What can data tell you about your class or school? 2. How do data displays help us understand information? **Relevance and Application:** 1. The collection and use of data provides better understanding of people and the world such as knowing what games classmates like to play, how many siblings friends have, or personal progress made in sports. **Nature of Mathematics:** 1. Mathematical data can be represented in both static and animated displays. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)

Standard: 3. Data Analysis, Statistics, and Probability
Third Grade

[1] For example, draw a bar graph in which each square in the bar graph might represent 5 pets. (CCSS: 3.MD.3)

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Geometric figures are described by their attributes

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: a. Reason with shapes and their attributes. (CCSS: 3.G) i. Explain that shapes in different categories[1] may share attributes[2] and that the shared attributes can define a larger category.[3] (CCSS: 3.G.1) 1. Identify rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS: 3.G.1) ii. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.[4] (CCSS: 3.G.2)	**Inquiry Questions:** 1. What words in geometry are also used in daily life? 2. Why can different geometric terms be used to name the same shape? **Relevance and Application:** 1. Recognition of geometric shapes allows people to describe and change their surroundings such as creating a work of art using geometric shapes, or design a pattern to decorate. **Nature of Mathematics:** 1. Mathematicians use clear definitions in discussions with others and in their own reasoning. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians look for and make use of structure. (MP)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Linear and area measurement are fundamentally different and require different units of measure

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use concepts of area and relate area to multiplication and to addition. (CCSS: 3.MD) i. Recognize area as an attribute of plane figures and apply concepts of area measurement.[5] (CCSS: 3.MD.5) ii. Find area of rectangles with whole number side lengths using a variety of methods[6] (CCSS: 3.MD.7a) iii. Relate area to the operations of multiplication and addition and recognize area as additive.[7] (CSSS: 3.MD.7) b. Describe perimeter as an attribute of plane figures and distinguish between linear and area measures. (CCSS: 3.MD) c. Solve real world and mathematical problems involving perimeters of polygons. (CCSS: 3.MD.8) i. Find the perimeter given the side lengths. (CCSS: 3.MD.8) ii. Find an unknown side length given the perimeter. (CCSS: 3.MD.8) iii. Find rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS: 3.MD.8)	**Inquiry Questions:** 1. What kinds of questions can be answered by measuring? 2. What are the ways to describe the size of an object or shape? 3. How does what we measure influence how we measure? 4. What would the world be like without a common system of measurement? **Relevance and Application:** 1. The use of measurement tools allows people to gather, organize, and share data with others such as sharing results from science experiments, or showing the growth rates of different types of seeds. 2. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods. **Nature of Mathematics:** 1. Mathematicians use tools and techniques to accurately determine measurement. 2. People use measurement systems to specify attributes of objects with enough precision to allow collaboration in production and trade. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians model with mathematics. (MP)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Third Grade

Concepts and skills students master:

3. Time and attributes of objects can be measured with appropriate tools

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (CCSS: 3.MD) i. Tell and write time to the nearest minute. (CCSS: 3.MD.1) ii. Measure time intervals in minutes. (CCSS: 3.MD.1) iii. Solve word problems involving addition and subtraction of time intervals in minutes[8] using a number line diagram. (CCSS: 3.MD.1) iv. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (CCSS: 3.MD.2) v. Use models to add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units.[9] (CCSS: 3.MD.2)	**Inquiry Questions:** 1. Why do we need standard units of measure? 2. Why do we measure time? **Relevance and Application:** 1. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods. **Nature of Mathematics:** 1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade. 2. Mathematicians use appropriate tools strategically. (MP) 3. Mathematicians attend to precision. (MP)

[1] e.g., rhombuses, rectangles, and others. (CCSS: 3.G.1)
[2] e.g., having four sides. (CCSS: 3.G.1)
[3] e.g., quadrilaterals. (CCSS: 3.G.1)
[4] For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape. (CCSS: 3.G.2)
[5] A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.5a)
A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (CCSS: 3.MD.5b)
[6] A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.5a)
A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (CCSS: 3.MD.5b)
Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (CCSS: 3.MD.6)
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 3.MD.7a)
Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS: 3.MD.7b)
[7] Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (CCSS: 3.MD.7d)
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (CCSS: 3.MD.7c)
[8] e.g., by representing the problem on a number line diagram. (CCSS: 3.MD.1)
[9] e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (CCSS: 3.MD.2)

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