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On the coexistence of stellar-mass and intermediate-mass black holes in globular clusters

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ABSTRACT

In this paper, we address the question: what is the probability of stellar-mass black hole (BH) binaries co-existing in a globular cluster with an intermediate-mass black hole (IMBH)? Our results suggest that the detection of one or more BH binaries can strongly constrain the presence of an IMBH in most Galactic globular clusters. More specifically, the detection of one or more BH binaries could strongly indicate against the presence of an IMBH more massive than $\gtrsim 10^3$ $M_\odot$ in roughly 80 per cent of the clusters in our sample. To illustrate this, we use a combination of *N*-body simulations and analytic methods to weigh the rate of formation of BH binaries against their ejection and/or disruption rate via strong gravitational interactions with the central (most) massive BH. The eventual fate of a sub-population of stellar-mass BHs (with or without binary companions) is for all BHs to be ejected from the cluster by the central IMBH, leaving only the most massive stellar-mass BH behind to form a close binary with the IMBH. During each phase of evolution, we discuss the rate of inspiral of the central BH–BH pair as a function of both the properties of the binary and its host cluster.

Key words: scattering – methods: analytical – stars: kinematics and dynamics – globular clusters: general – X-rays: binaries.

1 INTRODUCTION

The topic of whether black holes (BHs) in globular clusters (GCs) exist has long been a topic of debate. Arguments have been made in favour of the presence of not only stellar-mass BHs ($\sim$10–10^2 $M_\odot$), but also intermediate-mass BHs (IMBHs; $\sim 10^3$–10^5 $M_\odot$) in some clusters. The former are descended directly from the evolution of massive ($\gtrsim$25 $M_\odot$) stars, and may or may not experience a kick upon formation due to an asymmetry in the subsequent supernova (SN) explosion (e.g. Fryer et al. 2012). IMBHs, on the other hand, could form from runaway collisions in the cluster core (Portegies Zwart et al. 2004), the direct collapse of gas in a protocluster, or accretion on to a central stellar-mass BH (Leigh et al. 2013b).

The first theoretical suggestions of some kind of connection between GCs and BHs were that the GCs might be quasar remnants (Wyller 1970). This suggestion has long since been abandoned, but the discovery of a disproportionate number of X-ray sources in GCs (Clark 1975) led to the suggestion that these clusters might host BHs of $\sim$1000 $M_\odot$ accreting from the intracluster medium (Bahcall & Ostriker 1975). At nearly the same time, it was found that the velocity dispersion rises towards the centre of M15, one of the X-ray bright clusters (Newell, Da Cost & Norris 1976).

As quickly as the evidence in favour of IMBHs in GCs came together, opposing explanations to account for their signatures were introduced. The X-ray emitting GCs all soon showed Type I X-ray bursts (Grindlay et al. 1976), which were then explained as thermonuclear runaways on the surfaces of neutron stars (Ayasli & Joss 1982). The increase in central mass to light ratio in GCs was argued to be consistent with mass segregation putting the heavy remnants into the very centres of GCs (Illingworth & King 1977). White dwarfs and neutron stars have very large mass-to-light ratios in the optical, and hence they can mimic a central IMBH if enough of them are present. Similar arguments for (Gerssen et al. 2002) and against (Baumgardt et al. 2003) the presence of an IMBH in M15 appeared almost 30 years later.

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More recently, two primary methods of indirect detection have been the focus of attempts to identify IMBHs in GCs. These are the fitting of dynamical models to look for evidence of cusps in the central velocity dispersion profiles (e.g. Gebhardt, Rich & Ho 2005; Noyola, Gebhardt & Bergmann 2008; Pasquato et al. 2009; van der Marel & Anderson 2010; Anderson & van der Marel 2010; Lützgendorf et al. 2013a), and X-ray and especially radio continuum observations to look for evidence of gas accretion on to a central massive BH (e.g. Maccarone & Servillat 2008; Strader et al. 2012a; Haggard et al. 2013). Both kinematic- and accretion-based approaches are subject to caveats when interpreting the observations (e.g. Umbreit & Rasio 2013). For example, although the detection of stellar density and kinematic cusps in GCs could be indicative of an IMBH, cusp stars near the very centre of the cluster could have moderately anisotropic orbits, and this could also mimic the evidence for an IMBH (e.g. Ibata et al. 2009). Similarly, an upper limit can be placed on the Eddington ratio of an accreting IMBH in a GC. But, if the gas content is poorly known (e.g. Bellazzini et al. 2008), a low gas accretion rate could be indicative of a very low-density interstellar medium (ISM), as opposed to a low IMBH mass (e.g. Wrobel, Greene & Ho 2011). Other methods of detecting IMBHs have also been explored. For example, Drukier & Bailyn (2003) proposed that individual fast-moving stars could be created in GCs hosting IMBHs, and that such hypervelocity stars could be observable using the *Hubble Space Telescope*. Gill et al. (2008) also showed that an IMBH can potentially quench mass segregation, and cause the average stellar mass to vary only modestly as a function of the clustercentric radius. Later, Sesana et al. (2012) also suggested that fast-moving millisecond pulsars in the halo of our Galaxy could provide indirect evidence for a substantial population of IMBHs in GCs, since such high velocities are difficult to reproduce via 'standard' formation mechanisms.

Currently, the observational evidence in favour of stellar-mass BHs existing in GCs is more compelling than for IMBHs. To detect these BHs, they must be accreting from a binary companion. For example, Maccarone et al. (2007) first found an accreting BH in a GC associated with the giant elliptical galaxy NGC 4472 in the Virgo Cluster. Although its precise mass is not known, the X-ray luminosity is sufficiently high that it cannot be anything other than a BH in such an old stellar population. This result was expanded upon by Zepf et al. (2007), who also discussed the possible implications for a central IMBH. Soon thereafter, Shih et al. (2010) reported an accreting BH in a GC hosted by the giant elliptical galaxy NGC 1399 located at the centre of the Fornax Cluster. More recently, Strader et al. (2012b) reported the detection of two flat-spectrum radio sources in the Galactic GC M22. The authors presented compelling evidence that these are accreting stellar-mass BHs, the first of their kind to be detected in a Galactic GC. These two detections arguably imply the presence of ∼5–100 stellar-mass BHs in M22, and could be indicative of stellar-mass BHs existing in other Galactic GCs as well. Indeed, Chomiuk et al. (2013) recently reported a candidate BH X-ray binary in the Galactic GC M62.

The exact numbers and masses of stellar-mass BHs in GCs are still being heavily debated in the literature. Most, if not all, BHs should be located in the cluster core at the present-day ages of Galactic GCs. This is because they should drift into the core due to mass segregation on the shortest time-scales, if they are not already born there. However, when it comes to BHs, what happens in the core rarely stays in the core. Strong gravitational interactions between BHs can result in the dynamical ejection of all but one or two BHs (Kulkarni, Hut & McMillan 1993; Sigurdsson & Hernquist 1993).

More specifically, the Spitzer (1969) instability shows that a heavy stellar component will dynamically decouple into a central 'sub-cluster' if it has a critical combination of both total mass and ratio of masses of individual objects in the heavy component to the light component. Since the crossing time, and especially the relaxation time, is shorter for the sub-cluster than for a typical GC, this sub-cluster can be expected to evaporate itself on much less than a Hubble time, ejecting the BHs dynamically (Sigurdsson & Hernquist 1993). Furthermore, binary interactions may accelerate the process (Portegies Zwart & McMillan 2000). Given that all the bright ($L_X > 10^{36}$ erg s^{-1}) Galactic GC X-ray sources have shown strong evidence for being neutron stars, a lore developed that the *Spitzer* instability really did lead to the evacuation of BHs from GCs. Only with the discoveries of objects with strong evidence for being GC BHs both in other galaxies (Maccarone et al. 2007, 2011) and in the Milky Way's clusters (Strader et al. 2012b; Chomiuk et al. 2013) was this topic carefully revisited. An important point was raised, namely, that the *Spitzer* instability should never drive the complete evacuation of a cluster's BHs (e.g. Moody & Sigurdsson 2009), since the criterion for the *Spitzer* instability to work includes a total mass requirement on the heavy component (Maccarone et al. 2011).

More recent numerical work has shown that the BH retention fraction is similar to the neutron star retention fraction (e.g. Moody & Sigurdsson 2009; Morscher et al. 2013). For example, Sippel & Hurley (2013) performed *N*-body simulations to match the absolute and dynamical age of M22. The authors argue that multiple BHs are retained at a cluster age of 12 Gyr provided the parent cluster has an extended core radius. These results were quickly expanded upon by Breen & Heggie (2013) and Heggie & Giersz (2014), who argue that it should take on the order of 10 relaxation times for the entire BH sub-population to be ejected, and that clusters with present-day half-mass relaxation times above ∼1 Gyr are the ones that should retain an appreciable population of BHs at 12 Gyr (Downing et al. 2010).

In this paper, we consider the co-existence of a population of stellar-mass BHs and a central IMBH in a GC. In Section 2, we use *N*-body simulations to demonstrate that the most massive stellar-mass BH becomes bound to the IMBH on a relatively short time-scale. We then discuss the evolution of the IMBH–BH binary in Section 3. Using analytic methods, we weigh the probability of actually detecting BH X-ray binaries in GCs hosting an IMBH in Section 4. Finally, our main conclusions are summarized in Section 5.

2 THE FATE OF A POPULATION OF STELLAR-MASS BHS IN A GC HOSTING AN IMBH

Because of their relatively large masses, populations of stellar-mass BHs in GCs rapidly sink to the cluster centre due to mass segregation (Vishniac 1978; Larson 1984; Breen & Heggie 2013; Heggie & Giersz 2014). Here, they can undergo strong gravitational interactions with other BHs, possibly leading to their ejection from the cluster and hence a progressive depletion of the BH sub-population (Spitzer 1969; Sigurdsson & Hernquist 1993; Morscher et al. 2013). If a central IMBH is present, this process is accelerated (Lützgendorf, Baumgardt & Kruijssen 2013b). In this case, the stellar-mass BHs are ejected upon undergoing strong gravitational interactions with the IMBH.

The above scenario is illustrated in Fig. 1, which depicts the results of an *N*-body simulation for cluster evolution, performed

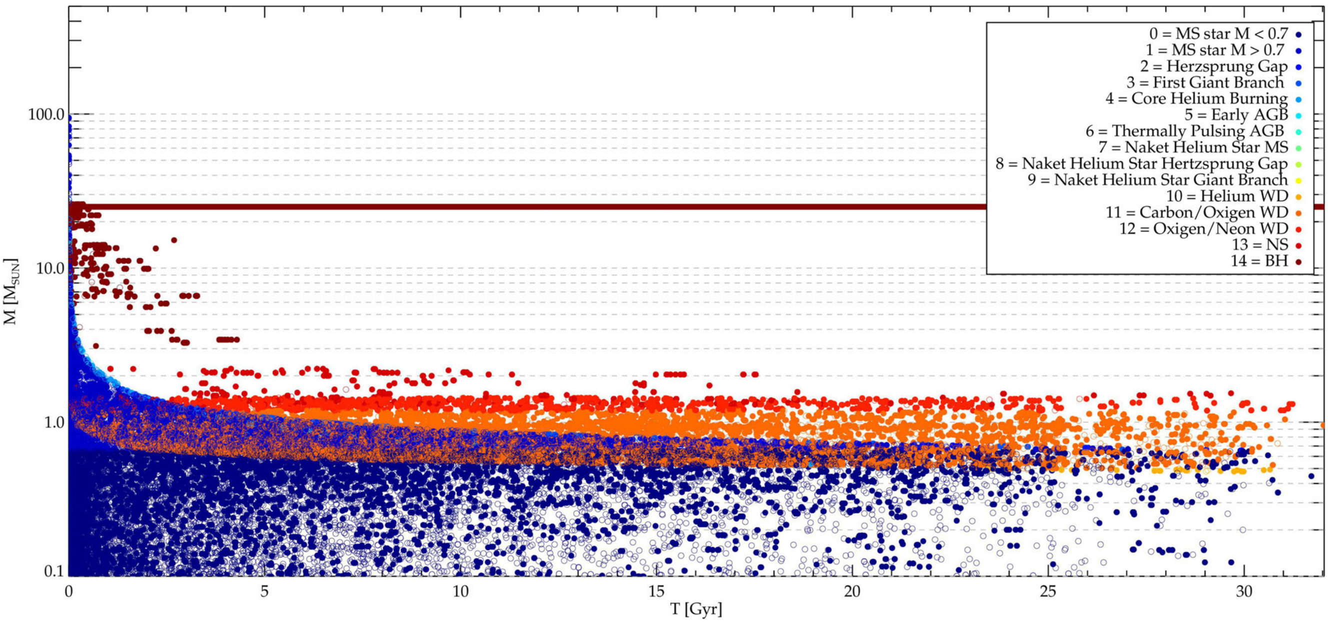

Figure 1. Plot showing every bound (solid circles) and unbound (open circles) encounter that occurs with the central IMBH over the course of the cluster evolution. Time is shown in Gyr on the *x*-axis, whereas the mass (in $M_{\odot}$) of the object that interacts with the IMBH is shown on the *y*-axis. The colour-coding of the circles corresponds to the different stellar types, and are shown in the inset at the top right. The dark red circles correspond to stellar-mass BHs, the last of which (ignoring the BH bound to the IMBH in a close binary at the cluster centre) is ejected from the cluster at ~4 Gyr.

using NBODY6 (Aarseth 1999). We use the same initial conditions and model setup as in Lützgendorf et al. (2013b) with $N =$ 131 072 stars initially. Additional simulations are also performed for $N \sim 64\,000$ and $N \sim 32\,000$ stars (not shown in Fig. 1), adopting again the initial conditions in Lützgendorf et al. (2013b). In all simulations, the IMBH mass, binary and remnant retention fractions are $M_{\bullet} = 0.01 M_{cl}$, $f_{\rm bin} = 0.0$, $f_{\rm BH} = 0.3$, $f_{\rm NS} = 0.1$, $f_{\rm WD} = 1.0$ (Lützgendorf et al. 2013b). Importantly, the IMBH mass is chosen to be 1 per cent of the total cluster mass at 12 Gyr. For lower IMBH masses, the IMBH is typically ejected from the cluster due to strong gravitational interactions with other stellar remnants and/or massive stars (if any remain). Specifically, the IMBH masses are 833 $M_{\odot}$, 415 $M_{\odot}$ and 207 $M_{\odot}$ for the 128 k, 64 k and 32 k models, respectively. This is larger than predicted by extending the M–σ relation observed for super-massive BHs in galactic nuclei (e.g. Ferrarese & Merritt 2000), but more reasonable if the *initial* GC masses were much larger than their present-day values (Kruijssen & Lützgendorf 2013).[1]

Note that we do not include any primordial binaries in these simulations to minimize the computational cost, but do allow for the dynamical formation of (typically wide) binaries.[2] Here, we are primarily concerned with interactions between stellar-mass BHs and the central IMBH. Importantly, all BHs are treated as point particles and we do not consider gravitational wave emission in our simulations. Thus, the orbital separation of the central IMBH–BH binary only evolves due to encounters with other objects, and it never undergoes a merger. We will return to this issue in the subsequent section.

Since we are interested in direct encounters with the central IMBH we modified the code to output every close encounter with the IMBH. For every time-step the distance of each object to the IMBH is calculated and, if smaller than the encounter radius $r_{\rm enc} = 10^{-3}$ pc, their orbital parameters obtained and saved. Bound objects can be identified by their orbital energy or their recurrence in the following time-steps. Specifically, an encounter is classified as bound (unbound) if, at the first time-step after the object has passed its point of closest approach, its kinetic energy is less (greater) than the absolute value of its orbital energy (with the IMBH) at that distance from the IMBH. We follow the cluster evolution for $\gtrsim$12 Gyr in all our simulations.

Fig. 1 depicts all encounters that occurred during the cluster lifetime. Open symbols mark unbound encounters and filled symbols bound encounters. As is clear from Fig. 1, the most massive stellar-mass BH in the cluster becomes closely bound to the central IMBH within the first $\lesssim$100 Myr of cluster evolution. The remaining BHs then undergo strong encounters with this central BH–IMBH binary, until eventually all BHs but the central pair are ejected from the cluster. This characteristic behaviour is seen in all our simulations; however there is some stochasticity to the time required for all BHs to be ejected from the cluster (e.g. Downing et al. 2010; Heggie & Giersz 2014), and it can occur that another BH is exchanged into the IMBH–BH binary, ejecting the first companion from the cluster in the process. The final BH, not bound to the IMBH, is ejected after only 4 Gyr in the simulation shown in Fig. 1. This time-scale ranges from ~a few 100 Myr in our simulations with 32 k particles to several Gyr in our simulations with 128 k particles (see Fig. 5). In the most massive GCs ($\gtrsim 10^6$ $M_{\odot}$) with the longest relaxation times, however, this time-scale could be much longer and even exceed a Hubble time (e.g. Downing et al. 2010). Importantly, the heaviest BHs tend to be ejected first, since they have the shortest

[1] Recent revisions to black hole mass estimates that include a halo dark matter component in the dynamical models have led to roughly a factor of four upward revision in the canonical black hole to host mass ratio (Kormendy & Ho 2013). This suggests that the quoted 1 per cent estimate is actually within the range observed in classical bulges and elliptical galaxies.
[2] Our *N*-body simulations under-estimate the rate of BH binary formation by not including primordial binaries. This issue is addressed by combining the results of our *N*-body simulations with analytic estimates for the rate of BH binary formation.

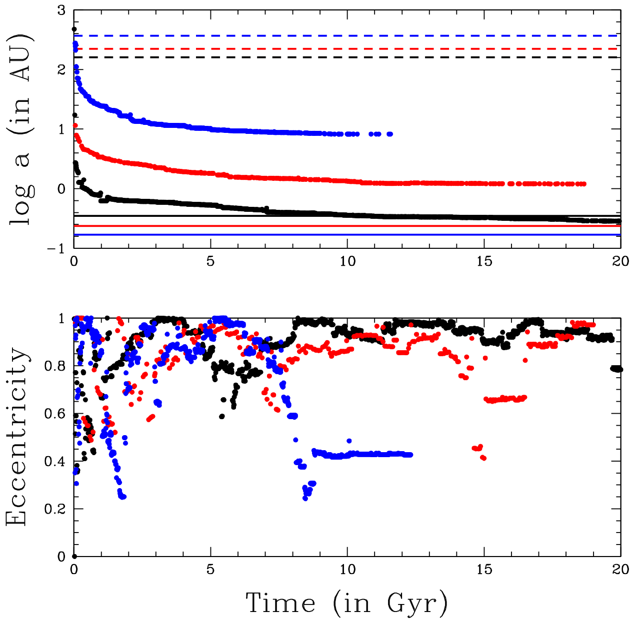

Figure 2. Plot showing the time evolution (in Gyr) of the orbital separation (top panel) and eccentricity (bottom panel) of the central IMBH–BH binary. The results are shown for three different N-body simulations, each beginning with a different number of particles, namely $N \sim 32$ k (blue), 64 k (red) and 128 k (black). The dashed and solid lines correspond to $a = a_{\rm h}$ and $a = a_{\rm GW}$, respectively, calculated for each of our three N-body simulations assuming an eccentricity of 0.5 for the IMBH–BH binary.

relaxation times. Hence, they drift close to the cluster centre on the shortest time-scales, where they undergo a strong interaction with the central IMBH–BH pair. Thus, we expect the ejection times for the heaviest BHs to typically be the shortest.

3 EVOLVING THE CENTRAL IMBH–BH BINARY

Next, we turn our attention to the evolution of the orbital properties of the central IMBH–BH binary. Using the N-body simulations described in Section 2, we begin with a qualitative discussion of the evolution of the IMBH–BH binary prior to the final inspiral, at which point GW emission takes over as the dominant mechanism for orbital decay. Here, however, we also show the results for simulations that begin with $N \sim 32$ k and $N \sim 64$ k particles initially, in addition to the case with $N \sim 128$ k shown in Fig. 1. The time evolution of the semi-major axes and eccentricities of the IMBH–BH binaries formed in these simulations are shown in Fig. 2. The key features to note are the steady decrease in orbital separation due to hardening encounters with the surrounding stellar population, and that the orbital eccentricity is close to unity for much of the binary's lifetime (we will come back to this below).

Initially, it is dynamical friction from the surrounding population acting on the individual binary components that drives the reduction in orbital separation (Milosavljevic & Merritt 2001). Eventually, scattering encounters take over as the dominant hardening mechanism. The critical semi-major axis at which this transition occurs is called the hard binary separation $a_{\rm h}$, and is approximately given by the boundary for the IMBH–BH binary to be classified as dynamically hard in the core (Heggie 1975), or: (Quinlan 1996; Merritt 2013):

$$a_{\rm h} = \frac{M_2}{M_{12}} \frac{r_{\rm m}}{4}, \tag{1}$$

where $M_{12} = M_1 + M_2$ is the total mass of the IMBH–BH binary, with $M_2 < M_1$. The distance $r_{\rm m} = GM_1/\sigma^2$ is the influence radius of the more massive IMBH. Thus, the hard binary separation is inversely proportional to the square of the central velocity dispersion σ^2, which tends to increase with increasing cluster mass due to the virial theorem (along with the expected IMBH mass). This is illustrated in Fig. 2 via the dashed lines, as well as our N-body simulations since, at any given time, the semi-major axis of the central IMBH–BH binary is larger for smaller cluster masses, and hence IMBH masses.

From equation (1), the semi-major axis $a_{\rm h}$ of the IMBH–BH binary at which scattering interactions take over as the dominant mechanism of orbital decay could range anywhere from a few au in the most massive GC cores (with the highest velocity dispersions and the most massive IMBHs) to on the order of 100 au in the least massive GC cores. For example, adopting component masses $M_1 = 10^3$ M$_\odot$ and $M_2 = 10$ M$_\odot$ and assuming a central velocity dispersion of 10 km s^{-1}, equation (1) gives $\sim$22 au roughly independent of M_1 if $M_1 \sim M_{12} \gg M_2$.

We expect the eccentricity of the IMBH–BH binary to evolve over time, maintaining a relatively high eccentricity (ignoring GW emission). Indeed, an initially non-circular orbit is the most likely scenario when the IMBH–BH binary first forms (Milosavljevic & Merritt 2001; Valtonen & Karttunen 2006).

Stellar encounters are not the only physical mechanism affecting the evolution of the IMBH–BH binary. Compact binaries lose orbital energy to gravitational waves. The time-scale for a binary to merge from gravitational wave emission is:

$$\tau_{\rm GW} = 3.3 \times 10^8 \frac{(1+q)^2}{q} \left(\frac{a}{1\,{\rm au}}\right)^4 \times \left(\frac{M_1 + M_2}{10^3\,{\rm M}_\odot}\right)^{-3} (1-e^2)^{7/2}\ {\rm yr}, \tag{2}$$

where a and e are the initial semi-major axis and eccentricity of the IMBH–BH binary, and $q = M_1/M_2$ is the binary mass ratio. This allows us to introduce the concept of a critical semi-major axis a at which a binary of mass ratio q and eccentricity $e_{\rm GW}$ coalesces within a specified time. This is illustrated in Fig. 3, where we show curves of constant $\tau_{\rm GW}$ in the semi-major axis–eccentricity plane. The evolution of the IMBH–BH binaries in our N-body simulations is also shown by the different points. Specifically, the solid black circles, open red squares and blue crosses correspond to the results of our 128 k, 64 k and 32 k simulations, respectively. In all simulations, the time-scale for coalescence due to GW emission drops well below 1 Myr, due primarily to the very high eccentricities reached by the IMBH–BH binary (we will come back to this below).

Eventually, a critical semi-major axis $a_{\rm GW}$ is reached, at which point the time-scale for coalescence due to GW emission is comparable to the (instantaneous) time-scale for coalescence due to scattering interactions. The dependences of the critical semi-major axes $a_{\rm h}$ and $a_{\rm GW}$ on the mass of the central IMBH are shown in Fig. 4 by the dotted and solid lines, respectively, for different eccentricities and assuming a 10 M$_\odot$ BH companion. For very low eccentricities, $a_{\rm GW}$ is roughly two orders of magnitude lower than $a_{\rm h}$ ($\sim$0.1 and $\sim$10 au, respectively) for IMBH masses $\sim 10^3$ M$_\odot$. However, for very high eccentricities approaching unity, we find $a_{\rm GW} \sim a_{\rm h}$ at the same IMBH mass.

The key point to take away from Figs 3 and 4 is that, as the eccentricity increases, the efficiency of GW emission increases, acting to circularize the IMBH–BH binary on potentially shorter time-scales than scattering interactions can increase it. Thus, if the

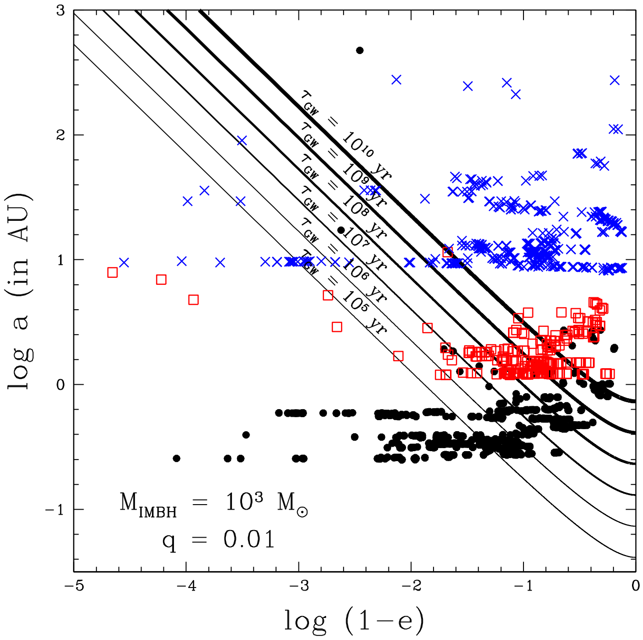

Figure 3. Relation between the initial eccentricity and the initial semi-major axis for an IMBH–BH binary that will merge due to gravitational wave emission within the indicated time τ_{GW}, calculated using equation (2). The IMBH and BH masses are set to $M_{IMBH} = 10^3$ M$_\odot$ and $M_{BH} = 10$ M$_\odot$, respectively, giving a mass ratio $q = 0.01$. The different linewidths correspond to different time-scales (10^5, 10^6, 10^7, 10^8, 10^9 and 10^{10} yr) for a merger to occur due to GW emission, such that the time-scale increases with increasing linewidth. The evolution of the IMBH–BH binaries in our *N*-body simulations is shown in the log (1 − *e*) − log (a)-plane by the different points. Specifically, the black solid circles, red open squares and blue crosses correspond to the results from our 128 k, 64 k and 32 k simulations, respectively. Note that, in all simulations, the semi-major axis of the IMBH–BH binary decreases monotonically with time, and the time between points varies but is on the order of 10 Myr.

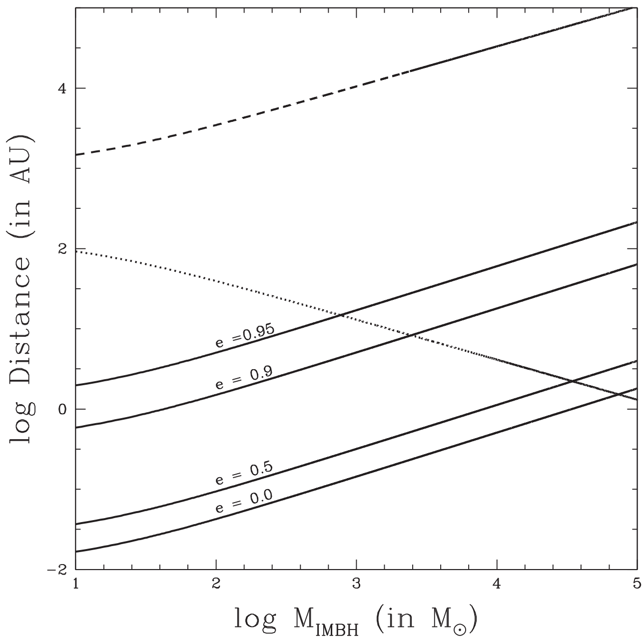

Figure 4. The parameters a_h (dotted line) and a_{GW} (solid lines) denote the critical semi-major axes at which hardening interactions and gravitational wave emission, respectively, take over as the dominant source of orbital decay. These are shown in au as a function of IMBH mass (in M$_\odot$), calculated from equations (19) and (20) in Sesana (2010) assuming $\gamma = 1.0$ (appropriate for clusters with cores, as opposed to cusps). The companion BH mass is set to 10 M$_\odot$, and we show the results for orbital eccentricities $e = 0$, 0.5, 0.9, 0.95. Also shown by the dashed line is the influence radius of the central IMBH, as given by equation (3) in Sesana (2010).

eccentricity becomes sufficiently high, the rate of orbital decay due to GW emission could dominate over scattering interactions, even at large semi-major axes.

To summarize, the orbital eccentricity of the central IMBH–BH binary reaches very high values at relatively small semi-major axes in as little as a few Gyr in all our simulations, due to scattering interactions with the surrounding stellar population. This significantly reduces the time-scale for the IMBH–BH binary to merge due to GW emission to ∼a few to a few hundred Myr (depending on the exact semi-major axis and eccentricity), which is not accounted for in our *N*-body simulations. Thus, we expect many IMBH–BH binaries in GC cores to merge well within a Hubble time, in particular those with large masses and located in dense GCs. Naively, this could reduce the time-scale for the depletion of the stellar-mass BH population. These issues should be properly addressed in future *N*-body models that incorporate a proper treatment of the General Relativistic evolution of the central IMBH–BH binary.

4 CAN A BH BINARY CO-EXIST WITH AN IMBH?

In the subsequent sections, we address in more detail whether or not any BHs should remain in a cluster hosting an IMBH at the present-day and, if so, whether or not they might harbour binary companions.

4.1 Will any BHs remain at the present-day?

In general, for the same initial BH retention fractions, the time-scale for all BHs to be ejected increases with increasing cluster mass, since the relaxation time increases with increasing cluster mass (and the number of initial BHs likely increases with cluster mass). This is illustrated in Fig. 5. Here, T_{BH} indicates the time until all but the last three stellar-mass BHs have been ejected from the cluster, since there is considerably more stochasticity between simulations in the time until all but *two* BHs have been ejected. Three different fits to the data are shown; however more models will be needed to better constrain the fits at higher initial *N*-values. To first order, the linear-*N* and logarithmic-*N* fits can be regarded as upper and lower limits, respectively, for T_{BH}. All three fits show an increase in T_{BH} with increasing *N*. Assuming $T_{BH} \propto N^x$, where *N* is the total number of particles initially and *x* is a free parameter, Fig. 5 shows that more than three BHs should remain at 12 Gyr in a cluster with $N = 10^6$, provided $x \lesssim 0.6$.

If some BHs remain in the cluster when the first stellar-mass BH companion merges with the IMBH due to gravitational wave emission, the IMBH will capture another BH companion on a time-scale that is comparable to the time-scale for BH ejection. As illustrated in Fig. 1, the most massive BHs tend to be ejected first. Hence, each time the IMBH captures an additional BH companion after undergoing a merger, the new BH is likely less massive than its predecessor(s). The lower companion mass increases the time-scale for orbital decay due to GW emission at a_{GW}, which contributes to decreasing the rate at which the IMBH merges with other stellar-mass BHs over time. Apart from this, we do not expect mergers of BHs

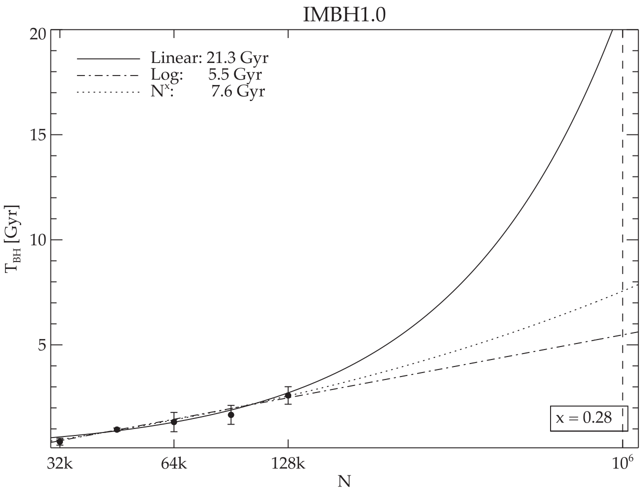

Figure 5. The time until all but the last three BHs have been ejected from the cluster ($T_{\rm BH}$) is shown as a function of the initial total number of stars in the cluster (*N*) for all three *N*-body models. Three different fits to the data are shown by the solid, dotted and dot–dashed lines, as described in the inset. The indicated times in the inset correspond to the time at which all (but three) BHs have been ejected in a cluster with $N = 10^6$ stars initially. Additional *N*-body simulations taken from Lützgendorf et al. (2013b) are also shown.

with the central IMBH to significantly change the presented picture for the dynamical depletion of a cluster's BH sub-population.

4.2 Will any primordial BH binaries remain in the cluster at the present-day?

In this section, we discuss only BHs born with a binary companion, which we call primordial BH binaries, and post pone the discussion of their dynamical formation to the subsequent section.

Most, and probably all, primordial BH binaries born in the cluster core should be disrupted by the central IMBH by the present-day, or ejected from the cluster. This is because the time-scale for two-body relaxation in the core is less than a Hubble time in even the most massive Galactic GCs, but is <1 Gyr in a typical GC (Harris 1996). The central two-body relaxation time provides a rough guide (we will return to this in the next section, where we calculate a more accurate time-scale) for the time-scale on which massive objects already in the core sink sufficiently deep within the cluster potential to enter the sphere of influence of the IMBH–BH binary, where the force of gravity from the IMBH becomes significant. For example, using equation (2.12) in Merritt (2013) and assuming a central velocity dispersion of 10 km s^{-1}, the influence radius is ∼0.04 pc for a 10^3 M$_\odot$ IMBH, or ∼0.4 pc for a 10^4 M$_\odot$ IMBH. Even if a BH binary is able to survive a direct encounter with an IMBH or, more likely, an IMBH–BH binary (in the subsequent section, we quantify this probability), there is usually ample time for subsequent interactions to occur, since the core and even half-mass relaxation times are typically much shorter than the cluster age. This significantly increases the probability that any primordial BH binaries will be disrupted by the present-day cluster age, or even ejected from the cluster. While it is possible that some BH binaries could survive an interaction with the central IMBH–BH binary and survive to the present-day by being ejected to the cluster outskirts where the time-scale for dynamical friction to operate is very long, such BH binaries should be rare and should most likely still be located in the cluster outskirts at the present-day.

Can primordial BH binaries born outside of the core survive to the present-day? To address this question, we consider two different types of BHs, motivated by the most recent theoretical work and defined according to the mass of their progenitor. The first type (Type I) of BH consists of the lowest mass BHs in the cluster (with a progenitor mass $\lesssim$40 M$_\odot$, although the exact limit is highly uncertain), and experiences a significant kick upon formation (Fryer et al. 2012). We therefore assume that most BHs of Type I are formed as isolated objects, and do not have a binary companion at birth. This is because, even if the BH progenitor had a binary companion when it underwent a SN explosion, the imparted kick is highly likely to dissociate the binary.[3] Thus, if BHs of Type I are to have binary companions, they must typically capture them via some dynamical mechanism. Importantly, we expect most BHs to be of Type I (before considering ejection from the cluster due to natal kicks) given that the initial mass function (IMF) observed in the field and young star-forming regions rises towards lower masses (e.g. Kroupa 2002). With that said, it is difficult to quantify this difference considering the considerable uncertainties in the IMF in massive clusters at the high-mass end, as well as the magnitude of their natal kicks.

The second type (Type II) of BH constitutes the most massive stellar remnants in the cluster (with the possible exception of an IMBH) with a progenitor mass $\gtrsim$40 M$_\odot$, and does not experience a kick upon formation, since the progenitor collapses directly to a BH without a SN explosion, and with little mass-loss (Fryer et al. 2012).[4] Thus, it is entirely possible, and even likely, that BHs of Type II are born with a binary companion, given the high binary fractions among massive stars (e.g. Sana, Gosset & Evans 2009; Sana, James & Gosset 2011). However, given the large masses of these BHs, they have very short dynamical friction time-scales, even at very large clustercentric radii. To check this, we calculate the distance from the cluster centre at which the dynamical friction time-scale is equal to 12 Gyr for a 40 M$_\odot$ object, for 42 non-core collapsed GCs using data taken from the ACS Survey for GCs (Sarajedini et al. 2007). The GC sample, single-mass King model fits and method for our procedure are described in detail in Leigh, Sills & Knigge (2011). We find that the distance from the cluster centre is $\gtrsim$95 per cent of the tidal radius in most GCs. Even for a GC as massive as ω Cen, BHs of Type II must reside well beyond the cluster half-mass radius at birth in order to have mass segregation times that exceed the age of the cluster, and avoid drifting into the core within a Hubble time. Thus, most, if not all, BH binaries of Type II should end up in the core well before the present-day cluster age, at which point they should undergo a strong gravitational interaction with the central IMBH–BH binary within a central relaxation time.

[3] The presence of a binary companion at the time of formation may help to retain BHs in clusters, since it can occur that a large fraction of the imparted kick energy goes into unbinding the binary. Also, the additional mass of the companion reduces the mass segregation time of the object, and natal kicks are less likely to eject BHs if they occur deeper within the gravitational potential of the cluster.

[4] We note that, as far as we know, no observational evidence exists for BHs descended directly from stellar progenitors with masses $\gtrsim$40 M$_\odot$. Moreover, we expect significant mass loss due to stellar winds in such massive star progenitors near the Eddington limit, and this could also significantly lower the final BH mass relative to the initial progenitor mass. We use this division in mass, given originally in Fryer et al. (2012), as a guide to help explore the possible presence of a sub-population of especially massive stellar-mass BHs that do not experience natal kicks.

We conclude that most, if not all, primordial BH binaries should have either been disrupted by the central IMBH–BH binary or ejected from the cluster by the present-day.

4.3 How can isolated BHs acquire a binary companion?

Ignoring kicks imparted by dynamical interactions in the core, most BHs will spend at least some of their lifetimes in the cluster core. The exceptions are Type II BHs born at very large clustercentric radii (well beyond the half-mass radius and, for all but the most massive GCs with the longest relaxation times, likely very near to the tidal radius), and Type I BHs kicked to comparably large clustercentric radii by their natal SN kick or an encounter with the central IMBH. Any Type I BHs born at large clustercentric radii are likely to be ejected from the cluster due to their natal SN kick. The key point that most BHs will potentially spend significant time in the core is important since, as we will explain below, dynamical mechanisms for BH binary formation require a high stellar density to be efficient. Thus, *most BH binaries should form in the core* (see below for a more quantitative estimate). We consider only dynamical formation channels in the core for BH binaries, and do not address their possible formation via binary evolution. Thus, we ask: how long does it take for a single BH to acquire a binary companion via some dynamical mechanism?

First, we consider an exchange interaction between an isolated BH and a binary, which occurs in the cluster core. Hence, we use the single-binary encounter time given in Leigh & Sills (2011), multiplied by the number of single stars in the core:

$$\tau_{\rm sb} = 1.4 \times 10^9 \left(\frac{1}{f_{\rm b}}\right) \left(\frac{10^5\ {\rm pc}^{-3}}{n_0}\right) \times \left(\frac{v_{\rm m}}{5\ {\rm km\,s^{-1}}}\right) \left(\frac{0.5\,{\rm M}_{\odot}}{m_{\rm BH}}\right) \left(\frac{1\ {\rm au}}{a}\right)\ {\rm yr}, \tag{3}$$

where $f_{\rm b}$ is the core binary fraction, n_0 is the core number density, $v_{\rm m}$ is the root-mean-square stellar velocity in the core, $m_{\rm BH}$ is the mass of the interloping BH and a is the average binary semi-major axis in au. This gives the time for a *specific* BH to encounter *any* binary, as opposed to the time for *any* BH-binary pairing to occur. The latter time-scale is shorter than the former by a factor $N_{\rm BH}$, where $N_{\rm BH}$ is the number of BHs remaining in the core at the present-day cluster age. Thus, technically, we should also divide the single-binary time-scale given in Leigh & Sills (2011) by the number of BHs $N_{\rm BH}$ in order to obtain the time for *any* BH to acquire a binary companion via an exchange interaction. We will come back to this below.

Assuming a core radius of 1 pc, a central mass density of 10^5 M$_{\odot}$ pc^{-3}, a central velocity dispersion of 10 km s^{-1}, an average stellar mass of 0.8 M$_{\odot}$, a binary fraction of 10 per cent and an average binary semi-major axis of 1 au,[5] we calculate a time of 0.6 Gyr for a *specific* 10 M$_{\odot}$ BH to encounter *any* binary. This is shorter than the core relaxation time calculated above for our example cluster, for which we obtain 1.5 Gyr (following Binney & Tremaine 1987). For comparison, if for the same example cluster we adopt a more modest binary fraction of 3 per cent, which is observed in the cores of a few massive MW GCs (e.g. Milone et al. 2012), we calculate an encounter time of 1.9 Gyr for a 10 M$_{\odot}$ BH to encounter any binary, which is longer than the central relaxation time. We note that the encounter time is even shorter if divided by a factor $N_{\rm BH}$ to obtain the time for *any* BH to undergo an encounter. On the other hand, the time for an *exchange* to occur is longer than the encounter time by a factor of ∼a few (e.g. Leonard 1989), since not every encounter results in an exchange event. Technically, *this* is the relevant time-scale for comparison to the time-scale on which most stellar-mass BHs (already in the core) will undergo a strong encounter with the central IMBH.[6] This will either dissociate any binary companions the BHs may have acquired, or eject them from the cluster. We note that binaries formed from exchanges (and tidal capture events; see below) typically are not compact, and only very compact binaries are expected to survive an encounter with the central IMBH–BH binary (e.g. Sigurdsson & Phinney 1993).

A second possible formation scenario for BH binaries involves an isolated BH acquiring a binary companion in the cluster core via tidal capture, most likely of a single main-sequence (MS) star.[7] We adopt the tidal capture time-scale given in Kalogera, King & Rasio (2004), with an additional factor $(1 - f_{\rm b})^{-1}$, where $f_{\rm b}$ is the binary fraction in the cluster core. This factor adjusts for the fact that we are not concerned with interactions involving binaries in the tidal capture scenario.[8] This gives

$$\tau_{\rm tc} = 10^9 \left(\frac{1}{1 - f_{\rm b}}\right) \left(\frac{10^5\, 5pc^{-3}}{n_0}\right) \times \left(\frac{v_{\rm m}}{10\,{\rm km\,s^{-1}}}\right) \left(\frac{5\,{\rm R}_{\odot}}{r_{\rm tc}}\right) \left(\frac{10\,{\rm M}_{\odot}}{m_{\rm BH}}\right)\ {\rm yr}, \tag{4}$$

where $r_{\rm TC}$ is the tidal capture radius.

For the example cluster described in the previous paragraph and assuming a core binary fraction $f_{\rm b} = 0.1$, we calculate for a 10 M$_{\odot}$ BH a tidal capture time of 0.3 Gyr (assuming $r_{\rm tc} = 5$ R$_{\odot}$). This is shorter than the time-scale for a BH to acquire a binary companion via an exchange interaction in the core by a factor of ∼2. Thus, in this example cluster, the tidal capture time-scale becomes longer than the binary exchange time-scale for core binary fractions $\gtrsim$20 per cent. Despite these arguments in favour of the tidal capture scenario, it is important to note that tidal capture may very well lead to the complete disruption of a MS star (Kalogera et al. 2004). More detailed modelling of these interactions will be required in future studies to fully address this issue.

4.4 Can the detection of BH binaries constrain the possible presence of an IMBH?

To evaluate whether or not the detection of one or more BH binaries can be used to argue against the simultaneous presence of an IMBH, we first must calculate the time for BHs to be ejected from the cluster

[5] This is slightly shorter than the semi-major axis corresponding to the hard–soft boundary, which is ∼7 au.

[6] More accurately, the exchange time-scale should be compared to the time-scale for diffusion of BHs into the central IMBH–BH binary's loss-cone. The loss-cone can be roughly defined as the collection of orbits with a periastron distance that is roughly equal to the IMBH–BH binary's semi-major axis. However, this time-scale is comparable to, albeit typically slightly shorter than, the central relaxation time. We will return to this important issue in the subsequent section, where the relevant loss-cone time-scale is calculated directly.

[7] A BH could also collide directly with a red giant star, forming an ultra-luminous X-ray binary in the process (Ivanova et al. 2005a). We do not consider this mechanism directly, and instead assume that all single stars lie on the MS.

[8] It is certainly possible, however, that tidal capture occurs during interactions involving binaries. Thus, our derived time-scale can be regarded as an upper limit.

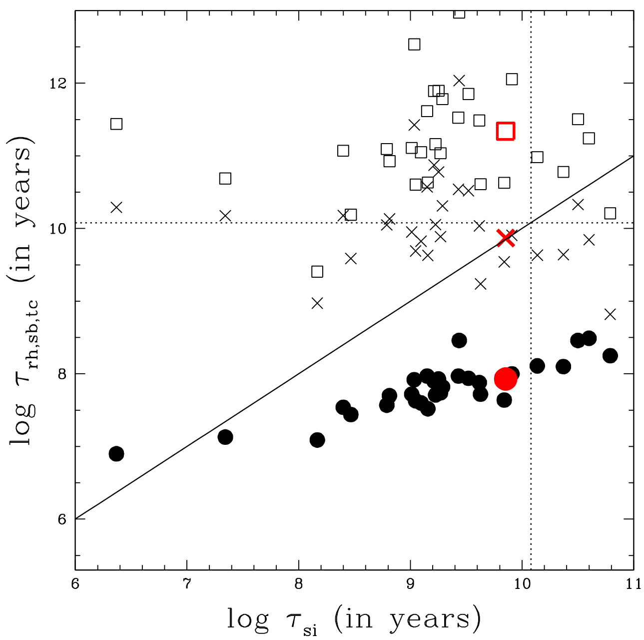

Figure 6. The half-mass relaxation time $\tau_{\rm rh}$ (filled circles), single-binary encounter time $\tau_{\rm sb}$ (open squares) and tidal capture time $\tau_{\rm tc}$ (crosses) are plotted as a function of the strong interaction time $\tau_{\rm si}$. As described in the text, all time-scales are calculated for a *single* 10 $M_\odot$ BH. The half-mass relaxation time is multiplied by the ratio of the average stellar mass to the mass of the BH, and we adopt 0.1 au for the average binary semi-major axis in calculating the binary encounter time. The GC sample and data are described in section 2 of Leigh et al. (2013c). The solid line denotes the one-to-one line, and the dotted lines show the 12 Gyr mark on both axes. The red enlarged points correspond to M22 (NGC 6656).

due to strong gravitational interactions with the central IMBH–BH binary. This depends on the efficiency of diffusion of BHs into the loss-cone of the IMBH–BH binary, which we define as the ensemble of orbits with a periastron distance that is on the order of the semi-major axis of the IMBH–BH binary. Objects on loss-cone orbits are expected to undergo sufficiently strong encounters with the IMBH–BH binary that they are typically ejected from the cluster within a crossing time. We assume that the binary loss-cone is always full, such that objects on intersecting orbits with the IMBH–BH binary are continually re-supplied as they are ejected. This is a reasonable assumption in GCs, since the time for BHs to be ejected (see below) is typically comparable to the half-mass relaxation time (see Fig. 6), or:

$$\tau_{\rm rh} = 1.7 \times 10^5\, N^{1/2} \left(\frac{r_{\rm h}}{1{\rm pc}}\right)^{3/2} \left(\frac{1\,{\rm M}_\odot}{\bar{m}}\right)^{1/2} {\rm yr}, \qquad (5)$$

where N is the total number of objects in the cluster, $r_{\rm h}$ is the half-mass radius and $\bar{m}$ is the average stellar mass.

In this approximation, the relevant time-scale corresponds to that for all BHs, *once in the core*, to undergo strong interactions with the IMBH–BH binary, which we call the 'strong interaction time-scale'. This should depend on the IMBH–BH semi-major axis, which is not accounted for by the central relaxation time. To calculate this time-scale, we follow the prescription of Sesana et al. (2012), with a few minor adjustments. In particular, based on equation (13) in Sesana et al. (2012), we write the strong interaction time-scale as:

$$\tau_{\rm si} = 4.7 \times 10^7 \left(\frac{r_{\rm h}}{1\,{\rm pc}}\right)^3 \left(\frac{v_{\rm m}}{10\,{\rm km\,s^{-1}}}\right)^3 \left(\frac{\bar{m}}{1\,{\rm M}_\odot}\right)^2 \times \left(\frac{10\,{\rm M}_\odot}{m_{\rm BH}}\right)^3 \left(\frac{10^3\,{\rm M}_\odot}{m_{\rm IMBH}}\right) {\rm yr}, \qquad (6)$$

where $r_{\rm h}$ is the half-mass radius in parsec, $v_{\rm m}$ is the cluster root-mean-square velocity, $m_{\rm IMBH}$ denotes the mass of the central IMBH and $m_{\rm BH}$ is the mass of its binary companion (both in $M_\odot$). We have confirmed that the time-scale given by equation (6) agrees quite well with the results of our *N*-body simulations. For example, equation (6) predicts $\tau_{\rm si} + \tau_{\rm rh} \sim$ a few Gyr for our *N*-body model with 128 k stars initially, which agrees quite well with the simulated ejection time of ∼4 Gy shown in Fig. 1.

To obtain equation (6), we set:

$$\tau_{\rm si} = \frac{V_{\rm BH}}{\sqrt{3}\sigma_{\rm BH}\Sigma}, \qquad (7)$$

where $V_{\rm BH} = (4/3)\pi(m/m_{\rm BH})^{3/2} r_{\rm h}^3$ is the volume within which all stellar-mass BHs in the cluster are confined after mass segregation (Heggie & Hut 2003) (which is approximately equal to one core radius in a typical GC; see below). We express the BH velocity dispersion as $\sigma_{\rm BH} = \sigma_0\sqrt{m/m_{\rm BH}}$, where we have assumed energy equipartition and denote the central velocity dispersion as σ_0.[9] The gravitationally focused cross-section is $\Sigma = \pi b^2$, with the impact parameter b set to:

$$b^2 = \frac{2GMa_{\rm h}}{3\sigma_{\rm BH}^2}, \qquad (8)$$

where M is the total mass of the IMBH–BH binary and $a_{\rm h}$ is the hard binary semi-major axis given in equation (1). Here, we assume that the collisional cross-section is dominated by gravitational focusing. Effectively, the strong interaction time-scale given in equation (6) gives the time for the relevant volume within the cluster, namely the (approximate) volume of the core, to flow through the loss-cone.

In Fig. 6 we show for several Milky Way GCs each of the half-mass relaxation time-scale (filled circles), the single-binary encounter time-scale (open squares) and the tidal capture time-scale (crosses) as a function of the strong interaction time-scale. To calculate the strong interaction time-scales, we assume an IMBH mass equal to 10^3 $M_\odot$ in every cluster. The GC sample and data are the same as used in Leigh et al. (2013c) (see Section 2 for a description of our sample), with the addition of M22 (NGC 6656). All of these clusters are non-core-collapsed, and can be accurately fit by King models. A summary of the clusters used in our sample along with the basic cluster parameters taken from Harris (1996) and used to produce Fig. 6 can be found in Table 1. We also show in the last column of Table 1 the upper limit for the IMBH mass in each cluster, found by setting the sum of equations (6) and (5) equal to 12 Gyr and solving for the IMBH mass. Note that if the obtained upper limit on the IMBH mass is less than the mass of a typical stellar-mass BH (i.e. a few tens of $M_\odot$), then our results suggest that the detection of even a few BH binaries suggests that an IMBH does not exist in that cluster.

[9] Note that the assumption of approximate energy equipartition here is only valid before the BHs decouple dynamically and undergo strong interactions with each other if the *Spitzer* instability is present. This assumption is also valid whenever the number of BHs is very small, since in this limit the *Spitzer* instability does not occur.

Table 1. Properties of the GCs used in our sample, taken from Harris (1996) and identified by their NGC number in Column 1. Columns 2, 3 and 4 list, respectively, the cluster absolute *V*-band magnitudes, distance moduli and extinctions. Columns 5 and 6 give the core and half-mass radius in arcmin, respectively. Column 7 gives the logarithm of the central luminosity density in $L_\odot$ pc^{-3}. Column 8 gives the core binary fraction taken from Milone et al. (2012), or supplemented with the values calculated in Leigh et al. (2013c). Columns 9 and 10 give the logarithms of the central and half-mass relaxation times, respectively, in years, taken directly from Harris (1996). In Columns 11, 12 and 13 we provide the (logarithm of) strong interaction time-scales, tidal capture time-scales and single-binary exchange time-scales in years, calculated using equation (6), equation (4) and equation (3), respectively. Finally, in Column 14, we provide the (logarithm of) upper limit for the IMBH mass (in $M_\odot$), calculated by setting the sum of equations (6) and (5) equal to 12 Gyr and solving for the IMBH mass.

ID (NGC) (1)	M_V (2)	$(m-M)_V$ (3)	$E(B-V)$ (4)	r_c (arcmin) (5)	r_h (arcmin) (6)	$\log \rho_0$ ($L_\odot$ pc^{-3}) (7)	f_b (8)	$\log \tau_{rc}$ (yr) (9)	$\log \tau_{rh}$ (yr) (10)	$\log \tau_{si}$ (yr) (11)	$\log \tau_{tc}$ (yr) (12)	$\log \tau_{sb}$ (yr) (13)	$\log M_{IMBH}$ ($M_\odot$) (14)
104	−9.42	13.37	0.04	0.36	3.17	4.88	0.055	7.84	9.55	10.79	8.82	10.21	3.717
1261	−7.80	16.09	0.01	0.35	0.68	2.99	0.046	8.59	9.12	9.288	10.31	11.78	2.211
2298	−6.31	15.60	0.14	0.31	0.98	2.90	0.154	7.91	8.84	8.399	10.18	11.07	1.321
3201	−7.45	14.20	0.24	1.30	3.10	2.71	0.128	8.61	9.27	9.43	10.54	11.53	2.354
4147	−6.17	16.49	0.02	0.09	0.48	3.63	0.262	7.41	8.74	8.467	9.587	10.19	1.389
4590	−7.37	15.21	0.05	0.58	1.51	2.57	0.114	8.45	9.27	9.15	10.57	11.62	2.075
5024	−8.71	16.32	0.02	0.35	1.31	3.07	0.087	8.73	9.76	10.5	10.33	11.5	3.432
5272	−8.88	15.07	0.01	0.37	2.31	3.57	0.054	8.31	9.79	10.6	9.845	11.24	3.532
5466	−6.98	16.02	0.00	1.43	2.30	0.84	0.142	9.35	9.76	9.437	12.04	12.97	2.369
5904	−8.81	14.46	0.03	0.44	1.77	3.88	0.06	8.28	9.41	10.14	9.634	10.98	3.062
5927	−7.81	15.82	0.45	0.42	1.10	4.09	0.104	8.39	8.94	9.841	9.541	10.63	2.763
5986	−8.44	15.96	0.28	0.47	0.98	3.41	0.048	8.58	9.18	9.618	10.04	11.49	2.541
6101	−6.94	16.10	0.05	0.97	1.05	1.65	0.1	9.21	9.22	9.035	11.43	12.53	1.959
6121	−7.19	12.82	0.35	1.16	4.33	3.64	0.148	7.90	8.93	9.047	9.691	10.6	1.969
6171	−7.12	15.05	0.33	0.56	1.73	3.08	0.186	8.06	9.00	8.813	10.13	10.93	1.736
6205	−8.55	14.33	0.02	0.62	1.69	3.55	0.01	8.51	9.30	9.91	9.906	12.05	2.834
6218	−7.31	14.01	0.19	0.79	1.77	3.23	0.114	8.19	8.87	8.789	10.05	11.09	1.711
6254	−7.48	14.08	0.28	0.77	1.95	3.54	0.078	8.21	8.90	9.096	9.824	11.05	2.018
6341	−8.21	14.65	0.02	0.26	1.02	4.30	0.057	7.96	9.02	9.628	9.238	10.61	2.551
6362	−6.95	14.68	0.09	1.13	2.05	2.29	0.12	8.80	9.20	9.213	10.87	11.89	2.137
6535	−4.75	15.22	0.34	0.36	0.85	2.34	0.092	7.28	8.20	6.369	10.29	11.44	−0.710
6584	−7.69	15.96	0.10	0.26	0.73	3.33	0.09	8.13	9.02	9.012	9.951	11.11	1.935
6637	−7.64	15.28	0.18	0.33	0.84	3.84	0.124	8.15	8.82	9.157	9.631	10.63	2.079
6652	−6.66	15.28	0.09	0.10	0.48	4.48	0.344	7.05	8.39	8.167	8.974	9.407	1.088
6656	−8.50	13.60	0.34	1.33	3.36	3.63	0.046	8.53	9.23	9.853	9.869	11.34	2.777
6723	−7.83	14.84	0.05	0.83	1.53	2.79	0.062	8.79	9.24	9.52	10.52	11.85	2.444
6779	−7.41	15.68	0.26	0.44	1.10	3.28	0.1	8.33	9.01	9.225	10.05	11.16	2.148
6838	−5.61	13.80	0.25	0.63	1.67	2.83	0.304	7.54	8.43	7.343	10.18	10.69	0.265
6934	−7.45	16.28	0.10	0.22	0.69	3.44	0.092	8.20	9.04	9.27	9.889	11.04	2.193
6981	−7.04	16.31	0.05	0.46	0.93	2.38	0.098	8.72	9.23	9.252	10.78	11.9	2.176
7089	−9.03	15.50	0.06	0.32	1.06	4.00	0.094	8.48	9.40	10.37	9.641	10.78	3.296

To produce Fig. 6, we multiplied the half-mass relaxation time-scales taken from Harris (1996) by the ratio of the average stellar mass in the core (assumed to be 0.5 $M_\odot$ for all clusters) to the mass of the BH (Vishniac 1978) so that our estimates for the relaxation times are suitable to a 10 $M_\odot$ object (instead of an object with a mass equal to the average single star mass in the cluster core). Similarly, both the single-binary encounter and tidal capture time-scales are calculated for a single stellar-mass BH of mass 10 $M_\odot$. For the encounter time, we take the average semi-major axis to be equal to 0.1 au for all clusters. This is because, we are primarily concerned with BH binaries that are actually observable, and the separation must be small in order for BH–MS binaries to emit significantly in the X-ray, independent of the host cluster properties (assuming all clusters are comparably old, as is the case for Mily Way GCs). Most confirmed stellar-mass BH binaries have separations slightly less than 0.1 au (e.g. McClintock & Remillard 2006; Kreidberg et al. 2012); however we do not account for any orbital decay which is likely for an initially eccentric orbit due to tidal dissipation at periastron.

As is clear from Fig. 6, we confirm that the half-mass relaxation time is shorter than the strong interaction time-scale in nearly all clusters. More importantly, the sum of these time-scales is typically much shorter than the cluster age, as is the case for ∼5/6 of the GCs in our sample. It follows that, in these clusters, a sub-population of stellar-mass BHs should have sufficient time to not only mass segregate into the core, but also diffuse through the loss-cone of the IMBH–BH binary, assuming a central massive IMBH is present. Therefore, *in ∼83 per cent (i.e. 5/6) of the clusters considered here, the detection of even a single BH binary provides strong evidence against the presence of an IMBH more massive than* 10^3 $M_\odot$. Our results suggest that this process should typically take on the order of a few Gyr, but it can approach and even exceed a Hubble time in the most massive MW GCs (in our sample these clusters are NGC 104, NGC 5024, NGC 5272, NGC 5904 and NGC 7089). This is in rough agreement with the results of our *N*-body simulations, which suggest that an $\sim 10^5$ $M_\odot$ GC will have lost all its stellar-mass BHs within ∼4 Gyr. In fact, nearly all clusters in Fig. 6 with a total mass $\sim 10^5$ $M_\odot$ have a strong encounter time-scale on the order of a few Gyr, in rough agreement with the results of our *N*-body models.

Interestingly, the time-scale for a stellar-mass BH to encounter a binary is longer than the tidal capture time-scale in most GCs in our sample. This implies that tidal captures should typically dominate the rate of BH binary formation.[10] However, provided the fraction of single-binary encounters that result in an exchange event is a significant fraction of the total number of such encounters, our results suggest that the difference is small, and hence the two mechanisms are of comparable significance.

If, at a given time, the time-scale for the formation of BH binaries is shorter than the strong interaction time-scale, then a substantial population of BH binaries could form in the core before all BHs interact with the central IMBH. If, on the other hand, the strong interaction time-scale is shorter than the time-scale for BH binary formation, then BHs should typically interact with the IMBH and be kicked out of the core before they acquire binary companions. Whether or not a BH binary can co-exist in the cluster core with an IMBH and, if so, which formation mechanism is expected to dominate depends on the cluster in question, in particular its structural parameters, binary fraction and BH fraction. The latter quantity is defined as the number of remaining BHs divided by the total number of objects in the cluster.

In Fig. 7, we show *for a given time* the regions of parameter space where either exchange interactions and/or tidal capture events (and hence the formation of a BH binary) should occur before all BHs in the core are ejected from the cluster, and vice versa. This illustrates the minimum BH fraction required in the core *at a given time* for even one BH binary to form before the entire BH sub-population passes through the IMBH–BH binary loss-cone and are ejected from the cluster, in addition to the most likely BH binary formation mechanism. If BHs segregate into the core at a rate that keeps the BH fraction in the core below this minimum value, then very few if any BH binaries should be present in the cluster at any given time. If observational constraints are available both for the BH and for binary fractions in a given cluster core, a point can be placed in Fig. 7 for that cluster. The dashed vertical line separates the tidal capture-dominated regime from the exchange-dominated regime – if the point falls to the right (left) of this line, then exchanges (tidal captures) dominate.

To construct Fig. 7 (including the parameter space where each mechanism dominates), we adopt the observed parameters for the Galactic GC M22 (NGC 6656), including a central luminosity density log $\rho_0 = 3.63$ (Harris 1996), a core radius 1.7 pc (Harris 1996) and a core binary fraction $f_b = 0.046 \pm 0.006$ (Milone et al. 2012). Following Strader et al. (2012b), we assume a population size of 5–100 stellar-mass BHs in M22, all assumed to reside in the core, and an average BH mass $m_{BH} = 10\ M_\odot$ (e.g. Kalogera et al. 2004). The central velocity dispersion is calculated from the virial theorem assuming a central mass-to-light ratio of 2, and an average stellar mass 0.5 $M_\odot$. The average binary semi-major axis is chosen to be equal to the semi-major axis corresponding to the hard-soft boundary (e.g. Heggie & Hut 2003), since these binaries have the largest cross-sections for collision and are hence the most likely to undergo interactions with single BHs on the shortest time-scales. We note that this is likely an overestimate for the formation rate of BH binaries via binary exchange encounters, since not all single-binary interactions will result in an exchange, nor will they necessarily produce a BH binary with a sufficiently short period to be observable as an X-ray binary. Adopting a smaller average binary separation for BH binary formation via exchanges would increase the lower limit for the BH fraction needed for exchanges to dominate over BH depletion via strong interactions with the central IMBH–BH binary, and increase the binary fraction needed for exchanges to dominate over tidal capture.

Fig. 7 illustrates that, if 5–100 stellar-mass BHs are present in the core of M22 at the present-day cluster age (Strader et al. 2012b), a handful of BHs should have had sufficient time to capture binary companions before being ejected from the cluster by the IMBH–BH binary, and could therefore be observable. But do we expect so many BHs to remain at an age of 12 Gyr if an IMBH is also present? Assuming an IMBH mass of 10^3 $M_\odot$, the answer is no, since the strong interaction time-scale is less than a Hubble time, as shown by the red point in Fig. 6. In fact, the sum of the strong interaction and half-mass relaxation times is very nearly a Hubble time, yielding an upper limit on the mass of a possible IMBH of log $(M_{IMBH}/M_\odot) \lesssim 2.8$ (see Table 1). Thus, given that Strader et al. (2012b) estimate a population size of 5–100 stellar-mass BHs with two confirmed detections, these results argue against the presence of an IMBH

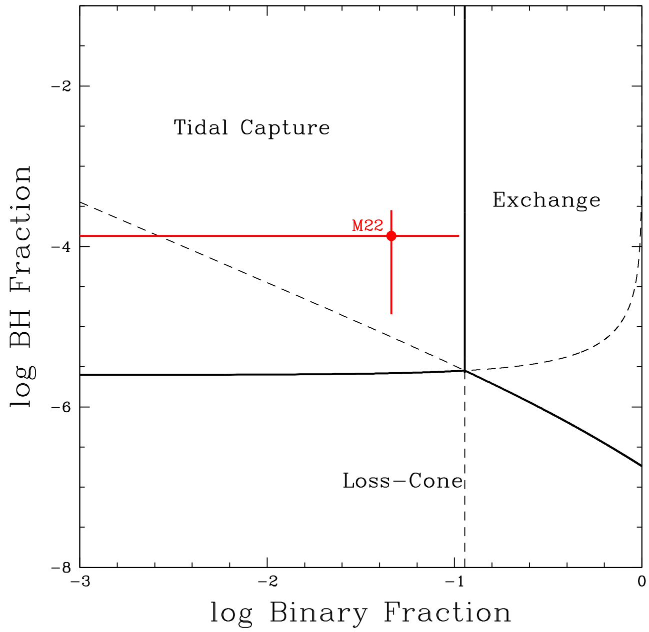

Figure 7. The binary fraction in the cluster core is plotted against the fraction of objects that are BHs. This illustrates the parameter space for which the rate of BH depletion due to strong interactions with the IMBH–BH binary dominates over the rate of BH binary formation via either tidal capture of exchange interactions. This serves as a rough guide for the size of the BH population required to ensure that at least some BH binaries will form before undergoing a strong interaction with the IMBH–BH binary. The BH and binary fractions are defined as the numbers of BHs and binaries, respectively, divided by the total number of cluster objects. The dashed lines are obtained by equating the relevant time-scales, namely those for strong interactions, tidal captures and binary exchanges. The solid lines sector off the parameter space where each of these three processes dominates. If the BH fraction is sufficiently low and strong interactions dominate, then the rate at which all BHs pass through the loss-cone exceeds the rate at which they acquire binary companions, and it is unlikely that any BH binaries should form before being ejected. The red point illustrates the observed estimates for the Galactic GC M22 (NGC 6656), taken from Strader et al. (2012b) and Milone et al. (2012).

[10] We note that this conclusion is based on the *present-day* binary fractions. If the binary fractions were higher in the past, this could increase the significance of binary exchanges relative to tidal captures.

in M22. We emphasize that the detection of any additional BH binaries in M22 should further constrain the presence of an IMBH by lowering the upper limit on its mass. Thus, the presence of even a handful of BH binaries in M22 argues against the presence of an IMBH (with a non-negligible mass). Independent of the presence of an IMBH, our results suggest that most BH binaries likely formed via tidal capture in M22.

To better illustrate the points raised above, we construct a toy Monte Carlo model to investigate the probability that a stellar-mass BH binary will remain in a GC that contains an IMBH. In our model, 10 $M_{\odot}$ BHs form in a static GC potential described by a Plummer model. Initial cluster-centric radii for the BHs are drawn from a Plummer density profile, and the BHs are then assumed to migrate inwards on a dynamical friction time-scale (Binney & Tremaine 1987).[11] We run two models; one where all BHs are born single, and a second where all BHs are born in binaries.

We use the same GC parameters for both models, chosen to be representative of a typical Milky Way GC. Specifically, we choose the cluster to have 10^6 total stars with a mean mass of 0.5 $M_{\odot}$, a binary fraction of 10 per cent, a half-mass radius of 3 pc, and a present-day age of 10 Gyr. The resulting central density is about 2×10^4 pc^{-3}. We insert a 10^3 $M_{\odot}$ IMBH with a 0.04 pc radius of influence in the centre of the cluster. Then, we migrate the BHs inward towards the centre of the cluster over 10 Gyr at a rate defined by the local dynamical friction time-scale (updated at 1 Myr time-steps). If a stellar-mass BH reaches the radius of influence of the IMBH before 10 Gyr, we assume that it is promptly ejected from the cluster or captured by the IMBH (and in either case would not be observed as an X-ray or radio source in the cluster). Note that our results are insensitive to the precise distance used, and our answer is more or less unaffected if we adopt the hard binary separation given in equation (1) instead of the radius of influence.

For the model where all BHs are born single, we check if a given BH will obtain a companion along its inward migration either through a binary–single exchange encounter or a tidal capture event, following the time-scales given in Kalogera et al. (2004), and accounting for the binary fraction in the tidal capture time-scale as mentioned above. We set the exchange encounter radius to 1 au, and the tidal capture radius to 5 $R_{\odot}$. We take a running average of these respective time-scales for a given BH as it migrates towards the cluster core. To account for the stochastic nature of stellar encounters in star clusters, at each time-step we draw a random number between 0 and 1 and if the encounter time-scale multiplied by this random number is less than the current model time we assume the BH gained a binary companion (following Ivanova et al. 2005b). Certainly, a more detailed model is desirable (for instance as in Morscher et al. 2013), but is beyond the scope of this paper.

Nonetheless, the results from our toy model are instructive, and are shown in Fig. 8. First, we note that the vast majority of the single BHs that obtain companions do so within one core radius (r_c) from the cluster centre, where the density is high and therefore the encounter time-scales are low. Given our assumptions about the cluster structure and binary frequency, exchange encounters are the most likely mechanism for initially single BHs to obtain companions. Interestingly, of those BH binaries expected to remain in the cluster, our model predicts that they will most likely be found at radii of $\sim$10 r_c or beyond. These BHs were born significantly further out from the centre, at radii of around 20 r_c, where the dynamical friction time-scale is long, and therefore these BHs do not have time to migrate fully into the core.

Given a typical GC hosting an IMBH, our model also predicts that there is only about a 0.5 per cent (2 per cent) conditional probability that a stellar-mass BH that forms single (in a binary) will remain in a binary within a typical GC at the present day (which we call P(A)), given that this GC has an IMBH (which we call P(B)). If we assume that BHs are equally likely to form single as they are to form in binaries, then this conditional probability, P(A|B), equals

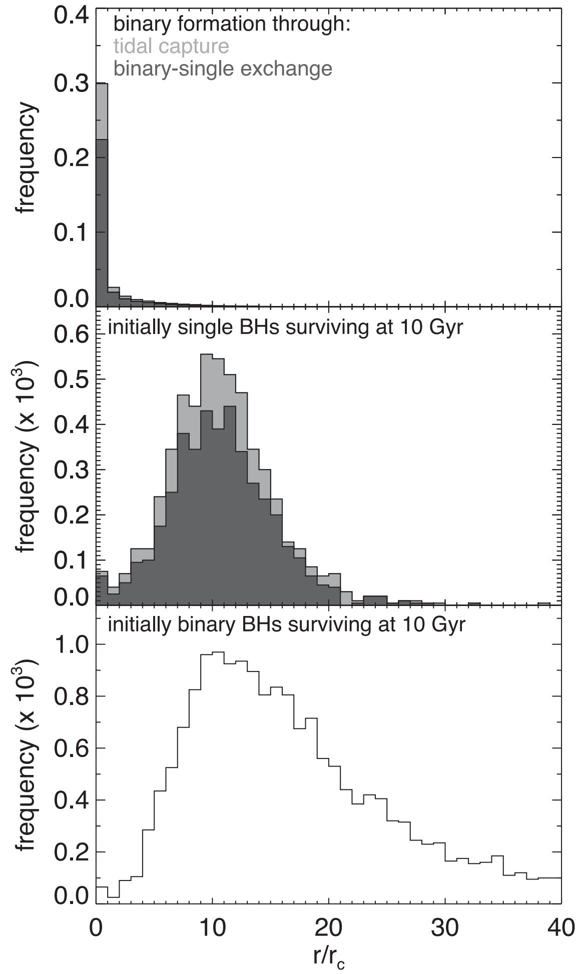

Figure 8. Results from our toy Monte Carlo model in which 10 $M_{\odot}$ BHs are born within a Plummer density distribution and allowed to migrate inwards towards the cluster core on a dynamical friction time-scale, over a cluster lifetime of 10 Gyr. The cluster hosts a 10^3 $M_{\odot}$ IMBH at its centre. In both insets, we plot stacked histograms and designate between BHs that obtain companions through tidal capture and exchange events via the light- and dark-grey bins, respectively. The top inset shows the clustercentric radii at which isolated BHs obtain companions at any time during the inward migration, the vast majority of which obtain companions within 1 core radius from the cluster centre. The middle and bottom insets show the frequencies of BH binaries that remain in the cluster at 10 Gyr, assumed either to be born single (middle inset) or in a binary (bottom inset) at the indicated clustercentric radius.

[11] We note that a Plummer density profile may not be realistic throughout the entire extent of the cluster, particularly near the cluster centre where the influence of the IMBH could steepen the density profile to better resemble a Bahcall–Wolf cusp. However, this should not significantly affect our calculations for the dynamical friction time-scale, which is dominated by the migration time at large clustercentric radii. Regardless, adopting a cuspier density profile will only contribute to (slightly) decreasing the derived dynamical friction time-scales. Thus, the results of our toy Monte Carlo model can be regarded as upper limits.

1.25 per cent. To obtain these probabilities, we ran 2×10^5 BHs through our Monte Carlo model for each case (i.e. all BHs are born single and all BHs are born in binaries), and counted the number of BHs that exist in a BH in a binary within the cluster at 10 Gyr. The results are plotted in Fig. 8 in the form of histograms, which are then integrated to obtain the total probabilities listed above.

Thus, to first-order, these results suggest that, of all stellar-mass BHs born in the cluster, on the order of ∼1 per cent will still remain in the cluster by avoiding a direct encounter with the central IMBH, and also have a binary companion at a cluster age of 10 Gyr. This estimate could increase, depending on how many BHs remain bound to the cluster and also retain their binary companion after undergoing one or more interactions with the IMBH. Additionally, note that if P(A) and P(B) are known (e.g. through future observational efforts), one can use Bayes' Theorem with such a model to calculate P(B|A) = P(A|B)P(B)/P(A), the probability that a GC has an IMBH given the detection of a stellar-mass BH in a binary.

We caution that the results presented in this section are sensitive to the initial cluster conditions, which are poorly known. This includes the initial stellar-mass function at the high-mass end, the initial–final mass relation for BHs, the magnitude and frequency of natal kicks, the initial cluster mass and structural properties, etc. An interesting example that relates to the issue of BH retention is the influence of gas damping during the gas-embedded phase of cluster evolution. In particular, gas damping can significantly accelerate the rate of dynamical decoupling of a massive sub-population such as BHs, and even prevents any stable energy equilibrium from occurring between different mass species, even if *Spitzer*'s criterion is satisfied. Thus, gas damping can cause BHs to be ejected on shorter time-scales, or even to be ejected when they otherwise would not be (Leigh et al. 2013a).

4.5 Will a BH binary survive a direct encounter with an IMBH–BH binary?

In this section, we obtain a rough estimate for an upper limit for the BH binary survival probability during a strong interaction with an IMBH–BH binary, and defer more detailed numerical scattering experiments (e.g. Colpi, Mapelli & Possenti 2003) to a future paper.

Consider two limiting cases. In the first scenario, the semi-major axis of the IMBH–BH binary is comparable to or smaller than that of the BH–MS binary. In this case, the disruption time for the BH–MS binary is given by equation (6), since the binding energy of the BH–MS binary is sufficiently low (in terms of its absolute value) compared to that of the IMBH–BH binary that a direct encounter will almost certainly disrupt the BH–MS binary. In the second scenario, the semi-major axis of the IMBH–BH binary is much larger than that of the BH–MS binary. In this case, the BH–MS binary could survive a direct encounter with the IMBH–BH binary, provided its distance of closest approach to the IMBH exceeds the tidal disruption radius for the BH–MS binary. The tidal disruption radius for the BH–MS binary is $r_{\rm t} \sim (M_{\rm BH-MS}/M_{\rm IMBH})^{1/3} a$, where $M_{\rm IMBH}$ is the IMBH mass, $M_{\rm BH-MS}$ is the mass of the BH–MS binary and a is its semi-major axis.

Based on the above, an order-of-magnitude estimate for the maximum fraction of strong interactions between a BH–MS binary and IMBH–BH binary (with a semi-major axis given in equation (1)) that leave the BH–MS binary intact can be obtained. Our estimate for the survival probability is an upper limit, and we come back to this below. It comes from the ratio of the gravitationally focused cross-sections, or:

$$f_{\rm surv} = 1 - \sqrt{\frac{(M_{\rm IMBH} + M_{\rm BH}) r_{\rm t}}{M_{\rm IMBH} a_{\rm h}}}. \qquad (9)$$

Here, we adopt for the IMBH–BH binary a semi-major axis given in equation (1), which corresponds to the point at which the main mechanism driving the orbital decay changes from dynamical friction to scattering interactions. For example, assuming $M_{\rm IMBH} = 10^3$ M$_\odot$, equation (9) yields a survival fraction of ∼65–95 per cent for the sample of Galactic GCs considered in the previous section.

As stated, our quoted estimate for the survival fraction of BH binaries after direct encounters with the IMBH–BH binary represents an upper limit. We do not consider the *rate* of orbital decay, and hence the time the IMBH–BH binary spends at each semi-major axis. The rate of orbital decay reaches a minimum just beyond $a_{\rm GW}$, since here GW emission is not yet effective and the rate of scattering interactions is low since the cross-section of the IMBH–BH binary is small. In general, as the orbital separation of the IMBH–BH binary falls below $a_{\rm h}$ due to scattering interactions, the survival fraction decreases. This continues until the semi-major axis of the IMBH–BH binary reaches that of the BH–MS binary (i.e. ∼0.1 au), at which point the survival probability drops to approximately zero. Additionally, our estimate ignores resonant interactions which increase the number of close approaches that can occur between the BH–MS binary and the IMBH, and hence the probability of disruption.

5 SUMMARY AND DISCUSSION

In this paper, we use a combination of *N*-body simulations and analytic methods to address the fate of a sub-population of stellar-mass BHs in clusters hosting a central IMBH. We further consider the possible presence of BHs with binary companions, weighed against the ejecting/disrupting influence of the IMBH. Our main results can be summarized as follows:

(i) A central IMBH will typically capture the most massive stellar-mass BH in a binary early on in the cluster lifetime. The IMBH will retain a BH companion until the BH spirals in, due to scattering interactions with neighbouring objects and gravitational wave emission, and merges.

(ii) For typical IMBH masses in GCs following from the M–σ relation (Kruijssen & Lützgendorf 2013), gravitational wave emission is likely to play an important role in deciding the evolution of the IMBH–BH binary orbital parameters, often while hardening interactions with the surrounding stellar population also drive orbital decay. Thus, GW emission should be accounted for in future related studies.

(iii) *In most Galactic GCs, the detection of one or more BH binaries can be used to place strong constraints on the possible presence of an IMBH.*[12]

(iv) If the (instantaneous) time-scale for BHs in the cluster core to flow through the loss-cone and undergo strong interactions with the central IMBH–BH binary is longer than the formation time for BH binaries, then at least some isolated BHs will have time to acquire binary companions *before* interacting with the IMBH, at which point they are typically ejected from the cluster.

[12] It is important to make the distinction, however, that the absence of detected BH binaries does not automatically mean that an IMBH is present.

Importantly, we assume throughout this paper that the IMBH mass is ∼1 per cent of the *present-day* cluster mass. This is larger than predicted by extending the M–σ relation observed for SMBHs in galactic nuclei (e.g. Ferrarese & Merritt 2000; Kruijssen & Lützgendorf 2013), but not unreasonable if GCs were once considerably more massive than their present-day masses suggest. For lower IMBH masses, even the IMBH can be ejected from the cluster due to dynamical interactions with other massive remnants. In this case, our results do not apply. If an IMBH is retained, our results for each initial total cluster mass can be extended to lower IMBH masses as follows. First, for a given number of BHs (and host cluster properties), lowering the central IMBH mass should increase the time-scale for all BHs to be ejected from the cluster. This is because individual interactions between the IMBH and BHs tend to be less energetic, and thus less likely to accelerate BHs to velocities that exceed the escape speed. Secondly, as shown in Fig. 4, for a given cluster mass and hence central velocity dispersion, lowering the central IMBH mass in turn increases the critical semi-major axis $a_{\rm h}$ at which hardening interactions begin to dominate the rate of orbital decay, and decreases the critical semi-major axis $a_{\rm GW}$ at which gravitational wave emission begins to dominate. Thus, for lower IMBH masses, the role of hardening interactions increases at small semi-major axes, where the IMBH–BH binary cross-section is at its smallest. This prolongs the lifetime of the IMBH–BH binary by increasing the time it spends with a semi-major axis $\gtrsim a_{\rm GW}$. The net result is an increase in the merger time of the IMBH–BH binary. The evolution of the IMBH–BH binary (along with the time for it to merge) can be accurately modelled with minimal computational expense for semi-major axes $a < a_{\rm h}$ using a combination of analytic methods and numerical scattering experiments. This can be done for any combination of IMBH–BH binary orbital parameters, as well as central cluster densities and velocity dispersions (Quinlan & Shapiro 1990; Quinlan 1996; Sesana, Haardt & Madau 2006).

We caution that the time-scales presented in this paper depend to some degree on the initial cluster conditions, which are in general quite uncertain. This includes the initial density and concentration, the initial cluster mass, the degree to which clusters are initially tidally over- or under-filling, the initial numbers and spatial positions of stellar-mass BHs and binaries, the maximum BH mass, etc. We also do not consider BHs formed through binary evolution due to the considerable uncertainties inherent to this formation pathway, although we expect this contribution to the total number of BHs to be small compared to the number of primordial BHs. All of these uncertainties affect our estimates for the initial numbers, ejection times and survival probabilities of BHs, as well as the characteristic semi-major axes of any central IMBH–BH binary (i.e. $a_{\rm h}$ and $a_{\rm GW}$) and thus its quantitative evolution. This paper illustrates that the numbers of BH binaries in GCs can potentially be used to constrain the possible presence of an IMBH (or at least an upper limit for its mass), and provides a useful benchmark for future studies focusing on individual clusters.

Interestingly, if an IMBH is present *and* it has a BH binary companion at the present-day cluster age, the distribution of ejected stars and/or compact binaries could be asymmetric. This is because the escapers are typically ejected in the plane of the IMBH–BH binary, but forming a wide jet if the binary is eccentric. This raises the interesting possibility of observing a weakly visible and wide jet of escaping stars and/or compact binaries. Indeed, such an asymmetry may already have been observed. For example, Grindlay (2006) reported a possible asymmetry in the spatial distribution of binaries (emitting X-rays) about the cluster centre in the core-collapsed GC NGC 6397. The issue of whether or not such a wide jet would actually be observable in some clusters will be the topic of a future paper. However, we note that, to the best of our knowledge, such an asymmetry in the distribution of ejected objects can only be produced via a massive central *binary*. This is because the distribution of ejection velocities should be isotropic if they originated from a strong interaction with an isolated IMBH, or even during a resonant encounter with a stellar-mass BH or even NS binary (e.g. Drukier & Bailyn 2003). Thus, an isotropic distribution of high-velocity escapers cannot be used to unambiguously identify the presence of an IMBH, since encounters with BH or NS binaries could also reproduce such a signature. Similarly, previous authors have argued that, although the kinematic signatures will be difficult to measure with present-day telescopes, an IMBH–IMBH binary could produce a population of a few hundred suprathermal stars moving at anomalously high velocities (Mapelli et al. 2005). The orbital angular momenta of these objects should be preferentially aligned with that of the IMBH–BH binary, creating an anisotropy in the angular momentum distribution of cluster stars. Thus far, no asymmetry is clearly present in either NGC 104 (McLaughlin et al. 2006) or ω Cen (Anderson & van der Marel 2010), although a signal may be present in NGC 2808 (Lützgendorf et al. 2012). Our results suggest that, in clusters with strong interaction times that are longer than the BH binary formation time, such an anisotropy may also be present (and easier to observe), and should be looked for, in the distribution of X-ray binaries in the cluster.

Our results can be extended to include as well neutron star X-ray binaries, although more work is needed in this direction. As shown in Fig. 1, it takes longer for a cluster's NS population to be depleted via interactions with a central IMBH than it does for its BH population. This is due to the smaller masses of NSs, and hence their longer relaxation times. This raises the interesting possibility of using the ratio of BH to NS X-ray binaries (or simply the absolute numbers of NS X-ray binaries in clusters with very few or no BHs) as a diagnostic for probing the possible presence of an IMBH, since it accelerates the rate of ejection of (preferentially massive) objects from the cluster. In massive clusters with very long relaxation times, it follows that the BH X-ray binary population should be more depleted due to strong interactions with the IMBH than the NS X-ray binaries. Moreover, NS X-ray binaries could potentially provide a more useful tracer of an anisotropy in the angular momentum distribution in clusters for which the time-scale for all BHs to be ejected is shorter than the cluster age.

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