*34th EPS Conference on Plasma Phys. Warsaw, 2 - 6 July 2007 ECA Vol.**31F**, P-5.051 (2007)*

Beam Tracing description of LH waves in tokamaks

N. Bertelli[1], G. V. Pereverzev[2], E. Poli[2]

[1] *Physics Department "A. Volta", University of Pavia, 27100 Pavia, Italy*

[2] *Max-Planck-Institut für Plasmaphysik, 85748 Garching, Germany*

Introduction

Despite the propagation of lower hybrid (LH) waves in a tokamak plasma has been intensively studied there are still some debated issues. The most challenging one is known as the spectral gap problem. Although many explanations have been proposed in the meantime, no one is yet fully accepted. One of the candidates to explain the observed wave spectrum broadening and the spectral gap filling is the diffraction phenomenon [1] that has not been taken into account in the majority of former studies. The reason of the disregard is that the propagation of LH waves in plasmas is usually investigated on the basis of ray tracing [2]. This technique describes correctly the refractive effects but does not take into account the diffractive phenomena. In particular, in most cases of practical interest for LH waves, the sufficient condition of the applicability of the ray tracing (i.e., $W \gg \sqrt{\lambda L}$, where W is the beam width, λ the wavelength and L the plasma inhomogeneity scale) is violated [3,4]. In other words, the diffraction effects become significant and can strongly affect both wave propagation and absorption.

For these reasons, the beam tracing method [5] is employed in this paper. This approach reduces the full wave equation to a set of ordinary differential equations, including the ray tracing as a particular case, and also describes the diffraction effects of the wave. In order to evaluate the significance of the diffraction for LH wave propagation, a new code, called `LHBEAM`, is presented which solves the beam tracing equations in a tokamak geometry for arbitrary launching conditions and for analytic magnetic equilibria. The importance of the diffraction effects for the space broadening of the LH wave beams is shown by comparing beam tracing and ray tracing results for typical tokamak parameters.

Outline of the beam tracing technique and brief description of the `LHBEAM` code

The beam tracing method provides a solution of Maxwell's equation $\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\boldsymbol{\varepsilon} \cdot \mathbf{E} = 0$ (where $\boldsymbol{\varepsilon}$ is the cold plasma dielectric tensor) in the form

$$\mathbf{E}(\mathbf{r}) = A(\mathbf{r})\mathbf{e}(\mathbf{r})e^{i\kappa[s(\mathbf{r})+i\phi(\mathbf{r})]} \tag{1}$$

where $\kappa = 2\pi L/\lambda$ is a large dimensionless parameter , A and $\mathbf{e}$ are, respectively, the amplitude and the unit polarization vector. The two functions $s(\mathbf{r})$ and $\phi(\mathbf{r})$ are given by (summation over repeated indices is adopted)

$$s(\mathbf{r}) = s_0(\mathbf{r}) + K_\alpha(\tau)[x_\alpha - q_\alpha(\tau)] + \frac{1}{2}s_{\alpha\beta}(\tau)[x_\alpha - q_\alpha(\tau)][x_\beta - q_\beta(\tau)] \quad (2)$$

$$\phi(\mathbf{r}) = \frac{1}{2}\phi_{\alpha\beta}(\tau)[x_\alpha - q_\alpha(\tau)][x_\beta - q_\beta(\tau)] \quad (3)$$

where $q_\alpha(\tau)$ and $K_\alpha(\tau)$ are, respectively, the components of the position vector $\{x_\alpha\} \equiv \mathbf{r}$ and the wave vector $\{k_\alpha\} \equiv \mathbf{k}$ that satisfy the set of Hamiltonian differential equation of the ray tracing

$$\frac{dq_\alpha}{d\tau} = \frac{\partial H}{\partial k_\alpha}, \quad \frac{dK_\alpha}{d\tau} = -\frac{\partial H}{\partial x_\alpha}, \quad (4)$$

where H is the (real) determinant of the dispersion tensor $\mathbf{\Lambda} = (c^2/\omega^2)(\mathbf{kk} - k^2\mathbf{I}) + \boldsymbol{\varepsilon}$.

The remaining functions $s_{\alpha\beta}(\tau)$ and $\phi_{\alpha\beta}(\tau)$ which are connected, respectively, with the curvature of the wave front and the width of the wave packet, obey the equations

$$\frac{ds_{\alpha\beta}}{d\tau} = -\frac{\partial^2 H}{\partial x_\alpha \partial x_\beta} - \frac{\partial^2 H}{\partial x_\beta \partial k_\gamma}s_{\alpha\gamma} - \frac{\partial^2 H}{\partial x_\alpha \partial k_\gamma}s_{\beta\gamma} - \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta}s_{\alpha\gamma}s_{\beta\delta} + \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta}\phi_{\alpha\gamma}\phi_{\beta\delta}, \quad (5)$$

$$\frac{d\phi_{\alpha\beta}}{d\tau} = -\left(\frac{\partial^2 H}{\partial x_\alpha \partial k_\gamma} + \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta}s_{\alpha\delta}\right)\phi_{\beta\gamma} - \left(\frac{\partial^2 H}{\partial x_\beta \partial k_\gamma} + \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta}s_{\beta\delta}\right)\phi_{\alpha\gamma}. \quad (6)$$

All the derivatives in the Eqs. (4-6) are calculated at $x_\alpha = q_\alpha(\tau)$ and $s_\alpha = K_\alpha(\tau)$ and, moreover, the matrices $s_{\alpha\beta}$ and $\phi_{\alpha\beta}$ are symmetric. There are two other relations connected with this two matrices, namely, $s_{\alpha\beta}\partial H/\partial k_\beta + \partial H/\partial x_\alpha = 0$ and $\phi_{\alpha\beta}\partial H/\partial k_\beta = 0$ which can be used as constraints to control of the solution accuracy.

In order to investigate LH propagation with allowance for the diffraction effects, a new code, called `LHBEAM` has been developed, which solves numerically the Eqs. (4-6). Part of the code's framework is based on `TORBEAM` code [6] and the main features are

(i) the plasma dielectric tensor is computed in the cold plasma limit and in the range of LH frequency approximation (i.e., $\omega_{ci}^2 \ll \omega^2 \ll \omega_{ce}^2$). In particular, the elements of the (cold) dielectric tensor are [7]

$$S = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2}, \quad D = \frac{\omega_{pe}^2}{\omega\omega_{ce}}, \quad P = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2}, \quad (7)$$

where ω_{ce} (ω_{ci}) is the electron (ion) cyclotron frequency and ω_{pe} (ω_{pi}) the electron (ion) plasma frequency;

(ii) the dispersion function H can be chosen to be the full electromagnetic dispersion function H_{ELM} or the electrostatic dispersion function H_{ELS} and which read [7], respectively,

$$H_{ELM} = SN_\perp^4 - \left[\left(S - N_\parallel^2\right)(P + S) - D^2\right]N_\perp^2 + P\left[\left(S - N_\parallel^2\right)^2 - D^2\right] \quad \text{and} \quad (8)$$

$$H_{ELS} = SN_\perp^2 + PN_\parallel^2 \quad (9)$$

where $N_\perp$ ($N_\|$) is the perpendicular (parallel) component of the refractive index with respect to magnetic field;

(iii) arbitrary initial conditions for the wave beam can be assigned;

(iv) the plasma equilibrium is prescribed analitically.

Numerical results

By means of the `LHBEAM` code, one can show the importance of diffraction effects during the propagation of LH beams in a tokamak. In particular, in the example shown here, JET parameters are employed. The major radius is $R_0 = 296$ cm, the minus radius $a = 125$ cm, the magnetic field $B(R_0) = 3.45$ T, the frequency $\omega/2\pi = 3.7$ GHz. The safety factor profile is $q = 1+3\rho^2$, where ρ is the normalized minor radius. The central electron density is $n_{e,0} = 3\times 10^{13}$ cm^{-3}. The initial wave front is flat and has a circular symmetry in a cross-section orthogonal to the group velocity, the beam width being $W = 4$ cm. The initial value of parallel refractive index is $N_{\|,0} = 1.8$.

In Fig.1(a) the 3D propagation of the LH beam launched in equatorial plane is plotted, comparing the ray tracing (blue line) with the beam tracing (red line). The difference betweeen the two approaches is evident, in particular, the spatial wave beam broadening is very significant as is shown both in the toroidal projection (cf. Fig. 1(b)) and in the poloidal projection (cf. Fig. 2(a)). Moreover, in Fig. 2(a), a comparison between a electrostatic and electromagnetic

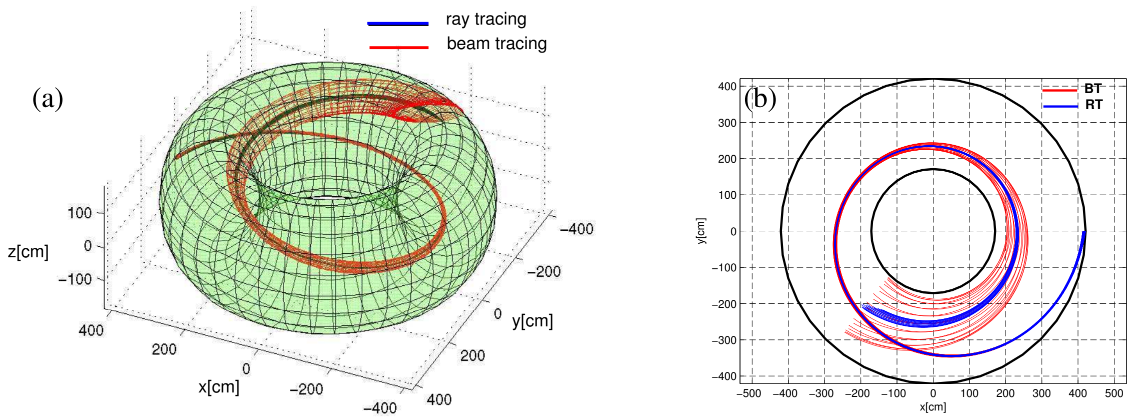

Figure 1: *(a) Evolution of the LH beam in 3D ;(b) toroidal wave beam propagation: comparison of RT (blue lines) and BT (red lines).*

case is shown. In addition, for both the cases, the ray tracing result is plotted. The significant difference between elctromagnetic and electrostatic case is due to the small initial value of $N_\|$ ($N_{\|,0} = 1.8$). In fact, it can be shown that for large value of $N_\|$ the two cases are almost the same,

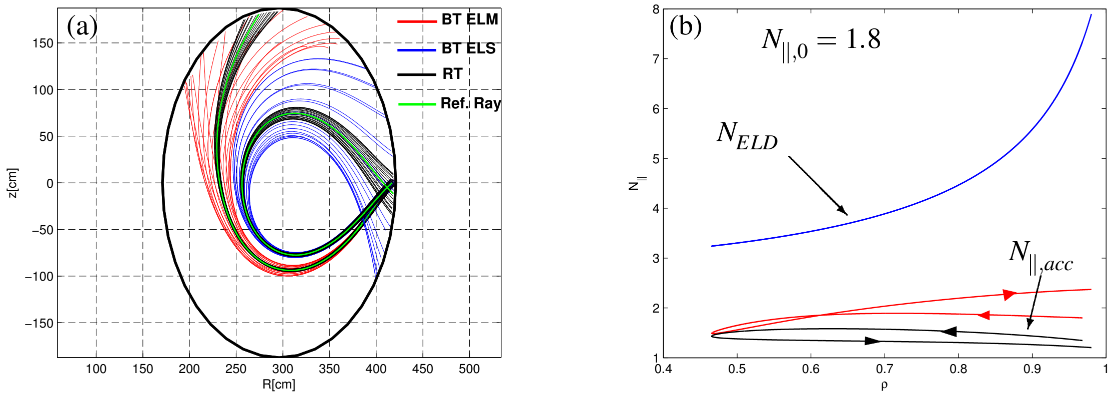

Figure 2: *(a) Poloidal wave beam propagation: comparison beetween electromagnetic (ELM) and electrostatic (ELS) case. The reference ray (beam axis) and the ray tracing results, for both case, are shown; (b) $N_{\|}$ as a function of normalized minor radius ρ along with the accessibility condition $N_{\|,acc}$ and the electron Landau damping (ELD) criterion N_{ELD}.*

as it is expected, because for $N_{\|} \gg 1$ the electromagnetic dipersion function tends to electrostatic dispersion function $(H_{ELM} \longrightarrow H_{ELS})$. In Fig. 2(b) one can note the $N_{\|}$-upshift (red line), along with the accessibility condition (black line) [7], $N_{\|,acc} = \sqrt{S} + \frac{\omega_{pe}}{\omega_{ce}}$ and the condition of linear electron Landau damping (ELD), $N_{ELD} = \frac{6.5}{\sqrt{T_e(\mathrm{keV})}}$ (see Ref.[8]), where it is assumed a parabolic profile of electron temperature with $T_{e,0} = 5$ keV.

Acknowledgments
The first author is grateful to M. Bornatici for many useful suggestions. For the development of the `LHBEAM` code, some help of A. Bottino and A. Manini is gratefully acknowledged.

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