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Max Planck Institute for Human Development and Education

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Two Contributions to the Analysis of Developmental Sequences

Nr. 15/ES November 1986

Beiträge aus dem Forschungsbereich Entwicklung und Sozialisation

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A Model of the Development of Concrete Operations:
Synchrony or Dĉcalage?
(Ein Modell der Entwicklung konkreter Operationen:
Synchronie oder Dĉcalage?) - page 1

Abstract

In a longitudinal study conducted in Reykjavik, Iceland, subjects (121 children, 60 male, 61 female) were tested at seven, eight, and nine years of age in order to investigate their comprehension of three concrete-operational concepts (conservation, class inclusion and logical multiplication). The present study focuses on a logical reconstruction of the developmental sequence of the three concrete-operational concepts. Based on a structural task analysis, a developmental model was postulated that defines the emergence and consolidation of concrete operations as a successive (cumulative) process. Empirically, all children followed an invariable developmental sequence corresponding to a unidirectional model. Methodological implications of the longitudinal design and the logical formulation of multilateral developmental models are discussed. An empirical evaluation and statistical test of the developmental model is provided.

Zusammenfassung

In einer Längsschnittstudie wurden drei konkret-operationale Konzepte (Konservierung, Klasseninklusion und Logische Multiplikation) bei 121 isländischen Kindern (60 Jungen und 61 Mädchen) im Alter von sieben, acht und neun Jahren getestet. Die vorliegende Untersuchung geht von einer Verschiebungshypothese der Entwicklung (décalage horizontal) aus. Diese Hypothese beruht auf der Rekonstruktion der Entwicklungssequenz der untersuchten konkreten Operationen und wird mittels einer aufgabenstrukturellen Analyse validiert. Auf der Grundlage dieser konzeptuellen Analyse wird ein Entwicklungsmodell postuliert, das die Emergenz und Konsolidierung konkreter Operationen als einen sukzessiven und kumulativen Vorgang ausweist. Die Daten zeigen, daß die kognitive Entwicklung der untersuchten Kinder einer invariablen Entwicklungssequenz folgt. Das postulierte Entwicklungsmodell entspricht einem unidirektionalen Modell des Erwerbs konkreter Operationen. Methodologische Implikationen des längsschnittlichen Erhebungsplanes und die aussagenlogische Formulierung des multilateralen Entwicklungsmodells werden diskutiert. Desweiteren wird eine empirische Evaluation und ein statistischer Test des Modells vorgelegt.

Analysis of Developmental Sequences within the Structural Approach: Conceptual, Empirical, and Methodological Considerations (Die Analyse von Entwicklungssequenzen im Rahmen struktureller Entwicklungstheorien: Ein konzeptueller, empirischer und methodologischer Beitrag) - page 17

Abstract

Analysis of developmental sequences is central to a theory of cognitive development according to Piaget. The concept of developmental sequences characterizes such ordered successions of developmental steps that are theoretically defined by the regularity of their occurence. Analysis of developmental sequences focuses on the internal dynamics of development and the intraindividual changes in the domains investigated. In this paper conceptual and methodological issues in the analysis of developmental sequences are discussed. Conceptually, the reconstruction of the logic of acquisition calls for the use of task or structure analysis. Methodologically, it calls for an individual-oriented approach, the use of statement calculus for formulation of the postulated developmental relationships, and confirmatory fitting of the developmental model. This approach is illustrated by longitudinal data of operatory intelligence collected in an Icelandic sample (N = 121) of children aged 7, 9 and 12. Developmental relationships within (and between) concrete-operational and formal-operational abilities are specified with respect to synchronous and diachronous development. The postulated relations are validated by structure analysis and integrated into a consistent model of operatory development. This model represents intraindividual trajectories, or profiles, of cognitive development, and on an aggregate level the formation processes of emerging operativity.

Zusammenfassung

Die Analyse von Entwicklungssequenzen ist von zentralem Interesse in der Theorie der kognitiven Entwicklung von Piaget. Das Konzept der Entwicklungssequenz bezeichnet jene geordneten Abfolgen von Entwicklungsschritten, die sich theoretisch durch die Regelmäßigkeit ihres Auftretens bestimmen lassen. Die Analyse von Entwicklungssequenzen hat die interne Dynamik des Entwicklungsgeschehens und darüber hinaus die intraindividuellen Veränderungen der Entwicklung zum Gegenstand. Die vorliegende Arbeit diskutiert sowohl die konzeptuellen als auch die methodologischen Implikationen bei der Analyse von Entwicklungssequenzen. Auf der konzeptuellen Ebene wird eine aufgabenstrukturelle Analyse für die Rekonstruktion der Entwicklungsabfolgen gefordert. Auf der methodologischen Ebene werden ein individuum-orientierter Auswertungsansatz, die aussagenlogische Formulierung der postulierten Entwicklungsrelationen und eine konfirmatorische Vorgehensweise bei der statistischen Evaluation der Entwicklungsmodelle

vorgeschlagen. Dieser Auswertungsansatz wird an Längsschnitt-Daten der operationalen Intelligenz veranschaulicht, die bei 121 isländischen Kindern im Alter von sieben, neun und zwölf Jahren erhoben wurden. Die Entwicklungsrelationen innerhalb der (und zwischen den) konkret-operationalen und formal-operationalen Aufgaben werden in Hinblick auf die synchrone und diachrone Entwicklung bestimmt. Die postulierten Einzelrelationen werden durch eine aufgabenstrukturelle Analyse validiert und in ein Gesamtmodell der operationalen Entwicklung integriert. In diesem Modell lassen sich die intraindividuellen Entwicklungsverläufe der Kognition rekonstruieren. Aggregiert man diese individuellen Entwicklungsverläufe, so repräsentiert das Modell den Formationsprozeß der operationalen Entwicklung.

A Model of the Development of Concrete Operations: Synchrony or Décalage?

Theoretical Background and Hypothesis

The present research was aimed at analyzing individual developmental paths during the acquisition of concrete operations. The analysis purports to trace intraindividual change in the domain of operational intelligence back to general formation processes. These ordered formation principles are described and explained within the theoretical framework of genetic epistemology in terms of progressive consolidation, structuring and integration (Pinard & Laurendeau, 1969). Thus, the regularities in the genesis of concrete operations can be defined as developmental sequences (Flavell & Wohlwill, 1969; Hoppe, Schmid-Schönbein, & Seiler, 1977; Wohlwill, 1973). These developmental sequences (defined as relations between variables) can be represented on a conceptual level by logical propositions (v. Eye & Brandtstädter, in press; Hoppe-Graff, 1982). For instance, if the acquisition of a concept A is a prerequisite structure for the emergence of concept B, the relation of A and B can be conceptualized as necessary condition or as logical implication.

Three concrete-operational subtasks were investigated: Conservation, class inclusion, and logical multiplication. Contrary to the assumption that qualitatively different substructures emerge synchronously (synchrony-hypothesis; Achenbach & Weisz, 1975; Little, 1972; Piaget, 1941; Inhelder &

Piaget, 1967; Piaget & Szeminska, 1952), it was postulated that the three subtasks examined develop successively and cumulatively (décalage hypothesis; Brainerd, 1973; Kingma, 1983; Winer & Kronberg, 1974). The predicted horizontal décalage is based on a task analysis of the substructures. According to this analysis which corresponds to an analysis presented by Kofsky (1966), conservation is a necessary condition for the acquisition of class inclusion and class inclusion a necessary condition for logical multiplication. First: The operation of addition and inclusion of classes implies the concept of conservation since through the addition of both subclasses A1 + A2 = B the different classificatory attributes have to be conserved. Second: When comparing addition or inclusion of classes and multiplication of classes it was assumed that the second operation emerges at a later point of time because of simultaneous combination of multiple attributes. If this is true, there exist implicative relations between class inclusion and conservation, and, furthermore, between logical multiplication and class inclusion. Regarding the development of operativity, this ordered series of tasks or substructures corresponds to an invariable developmental sequence. It follows that conservation should emerge before class inclusion and class inclusion should emerge before logical multiplication.

Subjects and Procedure

The research is part of a longitudinal study conducted in Reykjavik, Iceland, since 1977 (Edelstein, Schröder, Kliegl et al., 1984). Subjects were 121 children, 60 male and 61 female, from different socio-economic backgrounds. For the examination of the development of concrete operations, performance on three tasks (conservation, class inclusion, and logical multiplication) was assessed at ages seven, eight, and nine. For the present purpose only the data from the 7 and 9 year old subjects will be used. Moreover, conservation was dropped from the study at age 9, as ceiling effects appeared. The tasks were presented in accordance with Piaget's clinical method. For the measurement of conservation, subjects were administered the "Concept Assessment Kit" designed by Goldschmid & Bentler (1968); for the examination of class inclusion and logical multiplication an adaptation of Smedslund's (1964) tasks was used.

Conceptual and Methodological Implications

Starting from the sequence hypothesis a developmental model will be presented and discussed in this section. The developmental relations within this acquisition model are formulated as logical propositions. Within the model only those progress patterns were specified that match the conditions of the décalage hypothesis.

Analyzing intraindividual changes, the longitudinal design permits the distinction of two analytical perspectives. First: The synchronous perspective represents the analysis of developmental sequences within one measurement point, that is, configurations of variables within the subject. This sequence is not observed but inferred. Second: The diachronous perspective represents the examination of the observed developmental changes (constellations) across measurement occasions. The second analytical perspective necessitates longitudinal designs, because it focuses on the analysis of change across different measurement occasions. These differentiations correspond to the distinctions within the methodological framework proposed by Buss in his 1979 paper.

Insert Figure 1 about here

On the basis of the distinction between synchronous and diachronous perspectives the postulated developmental relations can be reformulated as logical propositions. It will be remembered that three developmental relations had emerged from the task analysis (conservation is a necessary condition for class inclusion and logical multiplication, and class inclusion is a necessary condition for logical multiplication). These three developmental relations are adequately represented by the logical proposition type of implication (see Fig. 2). Logical implication does not admit the conjunction of two successively emerging variables of the following kind: "Absence of prerequisite structure A" and "Presence of later substructure B". For example, the following developmental paths would be inadmissible:

Classification operations are available to the subject while conservation has not been acquired.

Insert Figure 2 about here

Logical implication, however, only matches the ordered series of variables on the synchronous level; it adequately describes the ordered set of relationships between variables at one time of measurement (configurations). In contrast, the diachronous level of analysis focuses on the emergence of concepts across times of measurement. In this case an additional assumption has to be examined and included in the developmental model. It concerns the retention of an acquired ability. According to the notion of progressive consolidation and integration of the concrete-operational structure of the whole, it was postulated that the different substructures emerge cumulatively during development (cumulativity); that is, once an ability is acquired, it must be retained across development. This relation can be formulated as a sufficient condition which corresponds to the logical proposition type of replication (see Fig. 3). Considering the case of two measurement occasions (t1 and t2), replication implies that an ability that exists at the first time of measurement (A1) must be present on the follow-up measurement occasion (A2). For instance, the occurrence of ability A on the first measurement occasion represents a sufficient condition for the occurrence of the same ability at the following time of measurement. Note that this is not the case in most instances of psychological assessment of a trait.

Insert Figure 3 about here

The two hypotheses (transitivity and cumulativity) and the corresponding developmental relations can be summarized in a more complex model of development (see Fig. 4). This model integrates both the synchronous and diachronous perspectives.

Results

?r the statistical evaluation of the order-theoretical analyses conducted on the data, a procedure known as "scalability models" was used (Henning & Rudinger, in press). In particular, Dayton & McReady's probabilistic validation procedure (1976) was applied. This procedure permits the reconstruction of a postulated developmental model from observed developmental data. The acquisition model specifies the admissible paths of development which, in turn, are depicted as the configuration or pattern of the variables. In our case, the patterns represent the developmental paths of the children in the domain of concrete-operational intelligence (the three tasks invariance, classification, and logical multiplication at ages seven and nine).

The procedure permitted a statistically significant reconstruction of the postulated model with the exception of one effect of vertical décalage (Schröder, in press, see Tab. 1 and 2). Further details will not be discussed in this paper. It should be emphasized, however, that the effect of the vertical décalage did not contradict the postulated developmental model and the implicit hypothesis of horizontal décalage.

Insert Tables 1 and 2 about here

Conclusions and Discussion

The model of development shown in Fig. 4, which represents the formation process of the structure of the whole of concrete operations, is based on the hypothesis of horizontal décalage. Several methodological and analytical reasons necessitated the transformation of the décalage hypothesis into two separate hypotheses serving both the synchronous and diachronous perspective. The developmental relations were formulated as logical propositions since the propositional calculus was sufficient to describe the assumed developmental processes as theoretically defined.

It was demonstrated that all children of the sample followed an invariable sequence. This lends credence to the assertion that development of concrete operations and the consolidation of the structure of the whole is a unitary and unidirectional process.

On the one hand, the horizontal dêcalage hypothesis contradicts the assumption of a synchronous acquisition of qualtitatively different concrete-operational substructures. On the other hand, it is congruent with various empirical findings that support an ordered sequence in the acquisition of concrete operations. The consolidation of concrete operations proceeds in the form of an invariable developmental sequence. Therefore, the results obtained through the microanalytic approach presented here may be viewed as additional theoretical and statistical evidence for the dêcalage hypothesis of the development of concrete operations.

References

Achenbach, T. M., & Weisz, J.R. (1975). A longitudinal study of developmental synchrony between conceptual identity, seriation and transitivity. Child Development, 43, 1024-1034.

Brainerd, C. J. (1973). Order of acquisition of transitivity, conservation, and class inclusion of length and weight. Developmental Psychology, 8, 105-116.

Buss, A. R. (1979). Toward a unified framework for psychometric concepts in multivariate developmental situations. In J.R. Nesselroade & P.B. Baltes (Eds.), Longitudinal research in the study of behavior and development: Design and analysis (pp. 41-59). New York: Academic Press.

Dayton, G. M., & MacReady, G.B. (1976). A probabilistic model for validation of behavioral hierarchies. Psychometrika, 44, 189-204.

Edelstein, W., Schröder, E., Kliegl, R. et al. (1984). Das Projekt Kindliche Entwicklung und Soziale Struktur. In K. E. Grossmann & P. Lütkenhaus (Eds.), Bericht über die 6. Tagung Entwicklungspsychologie in Regensburg (Bd. II, pp. 274-296). Regensburg: Universität Regensburg.

v. Eye, A., & Brandstädter, J. (in press). Evaluating developmental hypotheses using statement calculus and non-parametric statistics. In P.B. Baltes, D.L. Featherman, & R.M. Lerner (Eds.), Life-span development and behavior (Vol. 8). Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. H., & Wohlwill, J.F. (1969). Formal and functional aspects of cognitive development. In D. Elkind & J. H. Flavell (Eds.), Studies in cognitive development: Essays in honor of Jean Piaget (pp. 67-120). New York: Oxford University Press.

Goldschmid, M. L., & Bentler, P.A. (1968). Manual: Concept assessment kit - conservation. San Diego, CA: Educational and Industrial Testing Service.

Henning, H. J., & Rudinger, G. (in press). Analysis of qualitative data in developmental psychology. In J. R. Nesselroade & A. v. Eye (Eds.), Individual development and social change: Explanatory analysis. New York: Academic Press.

Hoppe, S., Schmid-Schönbein, C., & Seiler, T. B. (1977). Entwicklungssequenzen. Bern: Huber.

Hoppe-Graff, S. (1982). Bedingungsanalysen zur Genese der Klasseninklusion. Unpublished dissertation, Technische Hochschule Darmstadt.

Inhelder, B. & Piaget, J. (1967). La genèse des structures logiques élémentaires. Classifications et sériations. Neuchâtel: Delachaux & Niestlé.

Kingma, J. (1983). The development of seriation, conservation, and multiple classification: A longitudinal study. Genetic Psychology Monographs, 108, 43-67.

Kofsky, E. (1966). A scalogram study of classificatory development. Child Development, 37, 191-204.

Little, A. (1972). A longitudinal study of cognitive development in young children. _Child Development_, _43_, 1024-1034.

Piaget, J. (1941). Le mécanisme du developpement mental. _Archives de Psychologie_, _17_(112), 215-285.

Piaget, J., & Szeminska, A. (1952). _The child's conception of number_. London: Routledge & Kegan Paul.

Pinard, A., & Laurendeau, M. (1969). "Stage" in Piaget's cognitive developmental theory: Exegesis of a concept. In D. Elkind & J. H. Flavell (Eds.), _Studies in cognitive development: Essays in honor of Jean Piaget_ (pp. 121-170). New York: Oxford University Press.

Smedslund, J. (1964). Concrete reasoning: A study in intellectual development. _Monographs of the Society for Research in Child Development_, _19_(2), 93.

Schröder, E. (in press). _Sequenzanalyse konkreter Operationen_. Berlin: Max-Planck-Institut for Human Development and Education.

Winer, G. A., & Kronberg, D. D. (1974). Children's responses to verbally and pictorially presented class-inclusion items and to a test of number conservation. _Journal of Genetic Psychology_, _125_, 141-152.

Wohlwill, J. (1973). _The study of behavioral development_. New York: Academic Press.

Figure 1: Sequence Analysis
Synchronous and Diachronous Perspectives

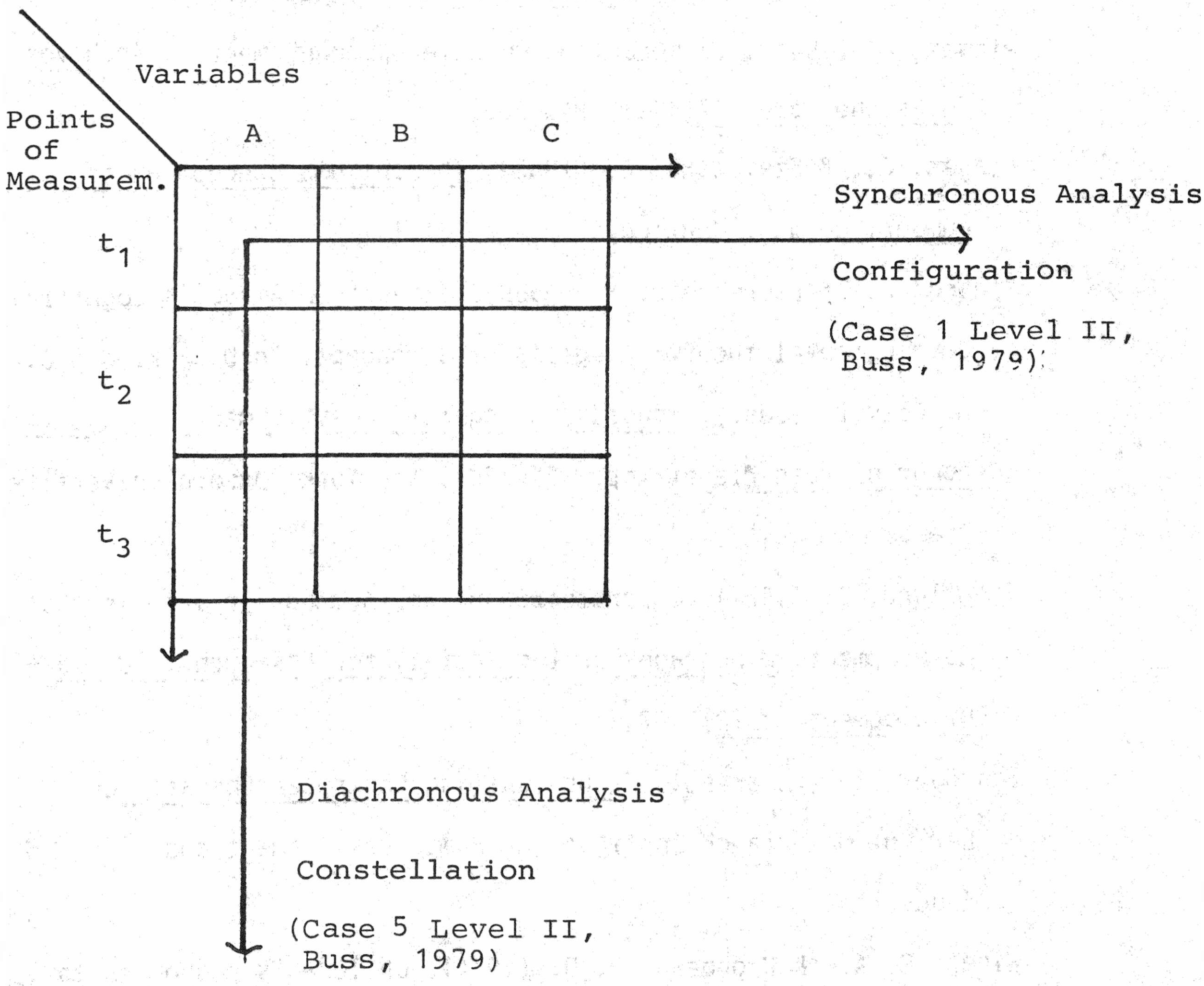
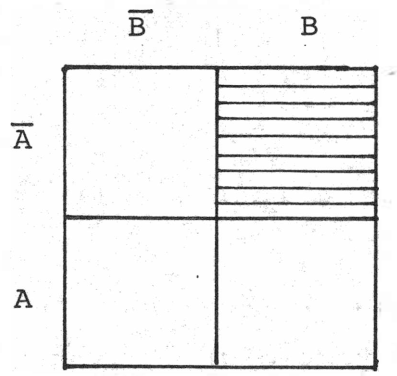


Figure 2 and 3: Types of Logical Relations and Patterns of Error in Contingency Tables

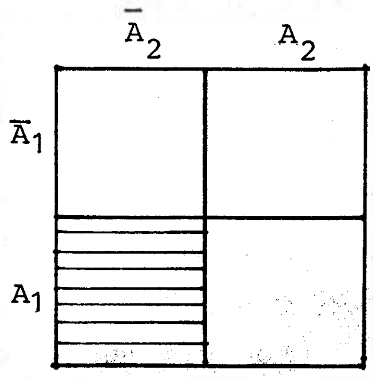


Error Pattern I

Necessary Condition

Logical Implication

Error Celles Barred

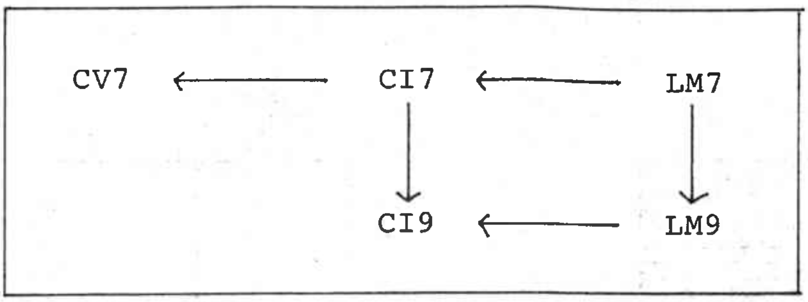


Error Pattern II

Sufficient Condition

Logical Replication

Figure 4: Developmental Model of Concrete Operations



CV7 : Conservation (7 year -old)

CI7,9: Class Inclusion (7,9 year-old)

LM7,9: Logical Multiplication (7,9 year-old)

←——: Necessary Condition (Tansitivity): Synchronous Level

↓ : Sufficient Condition (Cumulativity): Diachronous Level

Table 1: Admissible Patterns
in Accordance with Test Model

7-Year			9-Year		
CV7	CI7	LM7	CI9	LM9	
0	0	0	0	0	Invariant
1	0	0	0	0	I
1	1	0	1	0	I
1	1	1	1	1	I
0	0	0	1	0	Progression
0	0	0	1	1	P
1	0	0	1	0	P
1	0	0	1	1	P
1	1	0	1	1	P
1	0	1	0	1	Vertical Décalage
1	0	1	1	1	V
1	1	1	1	0	V

12 Admissible Patterns out of
32 Possible Patterns

Table 2: Statistical Parameters of Testing Model
(Procedure by Dayton & MacReady, 1976)

Pattern	Theta	Observed N	Predicted N	Chi-Square
00000	0.025272	4	3.8725	0.0042
10000	0.072291	9	8.7862	0.0052
01000		0	0.0255	0.0255
11000		0	0.3376	0.3376
00100		0	0.0595	0.0595
10100		0	0.7881	0.7881
01100		0	0.0251	0.0251
11100		0	0.3320	0.3320
00010	0.043526	6	5.8701	0.0029
10010	0.136858	16	15.7725	0.0033
01010		0	0.3376	0.3376
11010	0.039726	5	4.4716	0.0625
00110		2	0.7881	1.8636
10110	0.103947	10	10.4375	0.0183
01110		0	0.3320	0.3320
11110	0.041900	5	4.3975	0.0826
00001		1	0.2116	2.9367
10001		1	1.3973	0.1130
01001		0	0.0450	0.0450
11001		1	0.5960	0.2739
00101		0	0.0550	0.0550
10101		0	0.7286	0.7286
01101		0	0.0648	0.0648
11101		2	0.8588	1.5163
00011	0.014388	2	2.8029	0.2300
10011	0.189870	19	18.5060	0.0132
01011		0	0.5960	0.5960
11011	0.083288	8	7.8930	0.0015
00111		2	0.7286	2.2186
10111	0.104087	9	9.6496	0.0437
01111		0	0.8588	0.8588
11111	0.144848	11	11.3744	0.0123
Σ		113		χ^2 = 13.9875

obs. χ^2 = 13.9875 (18 df)
crit. χ^2 = 21.6049 (18 df and α = 75%)
alpha (α_i) ≤ 0.001; beta (β_i) = 0.07205

Pattern:	Configuration (CV7-CI7-LM7-CI9-LM9)
Theta:	Probability of Pattern j
Observed N:	Observed Frequency
Predicted N:	Predicted Frequency
alpha:	"Positive Error" (Guessing Error)
beta:	"Negative Error" (Forgetting Error)

Analysis of Developmental Sequences Within the Structural Approach: Conceptual, Empirical, and Methodological Considerations

Conceptual and methodological considerations

The constructs of developmental sequence and stage are central to the theory of cognitive development since they document intraindividual changes in the acquisition of cognitive operations. Within the framework of genetic epistemology, the concept of sequence means the ordered acquisition of distinct developmental steps or stages. Two types of developmental sequences, precursor sequences and prerequisite structures are postulated (Campbell & Richie, 1983). Precursor sequences merely reflect the empirical order in the acquisition of tasks. This implies that some tasks are easier to solve than others, since specific performance conditions (difficulty of tasks or presentation modes) result in differences when solving the tasks. The postulation of a prerequisite structure, however, requires structural or functional analysis of the relations between operations. Conceptual arguments are needed to validate the interrelationship examined. Therefore, the concept of developmental sequence commands an explanatory status only if theoretical and conceptual arguments support the postulated order of acquisition stages. Such arguments are, therefore, prerequisite conditions for meaningful empirical analyses of developmental sequences.

The construct of developmental sequence refers to the functional or formal (structural) aspects of development. It not only represents inclusive or implicative relationships between developmental variables, but it also represents synchronous acquisition sequences of different abilities in form of point- or interval-synchronies (Fischer & Bullock, 1981).

A review of empirical research shows that assumptions about sequentiality in the emergence of cognitive operations are mostly based on the analysis of group sequentiality in one-group designs or across-group-sequentiality in cross-sectional-designs. Few studies examine intraindividual change, although sequence hypotheses focus explicitly on this perspective of individual development. To emphasize this argument, hypotheses dealing with intraindividual changes will be represented within the methodological framework of Buss (1979). There are three analytical dimensions which are relevant in the analysis of individual development, namely, the person (individual), the dimension (developmental variables), and the measurement occasions. According to Buss (1979), intraindividual changes and differences in development can only be analyzed with respect to the following types of data-aggregations:

case 1: Interindividual differences in intraindividual differences, in which individuals are compared in terms of sampling across variables at one occasion.

case 5: Interindividual differences in intraindividual changes, in which individuals are compared in terms of sampling across occasions for one variable.

Figure 1 shows both analytical perspectives within a three-dimensional data-matrix whereby persons by variables by occasions are aggregated (see also Schröder, 1986). Synchronous and diachronous development are specified and distinguished.

Insert Figure 1 about here

In the following discussion, hypotheses of the first type will be called Configuration hypotheses or synchronous profiles and assumptions of the second type Constellation hypotheses or diachronous profiles.

Three different procedures have been used for statistical evaluation of sequence hypotheses (see Spiro, 1984). These strategies are:

1) Difference-testing procedures: Differences in one or more developmental variables in different age-groups are taken as evidence of intraindividual changes in development. The inference from the developmental function of a population or sample to intraindividual changes within the subjects is valid only if the developmental function of the group is congruent to those of the individuals. This implicit assumption is unlikely, since interindividual differences exist between the individual

developmental courses that are not revealed in the global developmental function (Bakan, 1967). Therefore, developmental paths of the individual cannot be predicted or approximated by the developmental function of the group.

2) Correlational procedures: According to Winer (1980) correlational testing procedures are appropriate to the analysis of developmental sequences since they focus on the functional or formal (structural) relations between developmental variables. Correlations are taken to represent the degree of consistency between variables or stability over time. However, conclusions of sequential relationships in data-matrices are not justifiable by correlational analysis, since hierarchical relations between variables could exist without statistically significant covariations between those variables (Edelstein, Keller & Wahlen, 1984; Henning, 1981; Hudson, 1978; Rudinger, 1978).

3) Unidimensional Guttmann Scaling: This procedure is adequate for analysis of developmental sequences since it takes into account the individual patterns or configurations. Each individual can be definitively classified on the developmental scale. Unfortunately, Guttmann-scaling is restricted to transitive (linear) relationships and does not allow investigation of cumulative or synchrony relationships. Further, it is not readily applicable to the longitudinal analysis of developmental sequences.

Since these procedures do not analyze intraindividual change, new empirical and methodological strategies have been developed which are more appropriate to the analysis of developmental sequences (Bart & Airasian, 1974; Bart & Krus, 1973; Dayton & McReady, 1976; Hildebrand, Laing & Rosenthal, 1977; Rudinger, Chaselong, Zimmermann & Henning, 1985; von Eye & Brandtstädter, 1985). These procedures, deriving partly from the fields of biostatistics, latent attribute scaling or order theory, are appropriate to categorial data. In comparison with the often used parametric statistics these procedures can be characterized systematically by the distinction between variable- and person-(individual-) oriented approaches (Bergman & Magnusson, 1983; Magnusson, 1985). Within the individual-oriented approach, assumptions of individual development can be formulated and expressed in terms of configuration or constellation-hypotheses based on intraindividual developmental change according to Buss' (1979) methodological framework. In this case the variables will be aggregated to form specific configurations of attributes or constellations of time occasions which represent appropriate individual courses of development with respect to a synchronous or a diachronous perspective of development.

Within the individual-oriented approach the formal or functional relations between variables or time occasions, as postulated and validated by structure or task analysis, can be formulated in terms of a statement calculus. The formulation by logical propositions allows the specification of occurring patterns as admissible or inadmissible according to the sequence hypothesis. For example, if attribute A is the precursor of B (A and B coded in dichotomous form) the combination of non-A and B is inadmissible due to the implicative relationship of A and B. The other three combinations (A B, A non-B and non A non-B) are admissible, since they do not contradict the sequence hypothesis. In Figure 2 the first contingency table represents an implicative relation between two developmental variables A and B. The second contingency table in Figure 2 represents the replicative relation (cumulativity) between two measurement occasions A1 and A2. The error cells are barred.

Insert Figure 2 about here

These examples of bivariate relations can easily be extended to multivariate relationships (see also Schröder, 1986). Additionally, different types of relations could be specified.

In the following section the approach of analyzing developmental sequences as described above will be illustrated by an empirical example taken from the study of operatory development. It should evidence how structural relationships between developmental variables are specified with respect to synchronous and diachronous perspectives of individual development and how the resulting model of operatory development is fitted statistically to the observed profiles of acquisition by the probabalistic validation procedure of Dayton and MacReady (1976).

Example

Subjects and instrumentation

In a longitudinal study conducted in Reykjavik, Iceland, a number of Piagetian tasks was presented to 60 girls and 61 boys at ages 7, 9 and 12 (Edelstein, Schröder, Kliegl, Spellbrink, Zebergs & Baker, 1984). The following tasks were administered: Conservation (Goldschmid & Bentler, 1968) at age of 7, Class Inclusion (Smedslund, 1964) at ages 7 and 9, Multiplicative Compensation (Invariance of Volume: Piaget & Inhelder, 1975) at ages 9 and 12 and the Pendulum Task (Piaget & Inhelder, 1977; Somerville, 1974) at age 12. All tasks were administered individually. In accordance with Piagetian theory, dichotomous competence scores were given for subjects' explanations of their judgments.

The longitudinal measurement design of the study is shown in Figure 3. Depending on the presumed general developmental status of the age groups different concrete- or formal-operational tests were presented at ages 7, 9 or 12. They cover cognitive abilities appropriate to the age groups. The four operational concepts investigated in the analysis of developmental sequence are described as follows:

Insert Figure 3 about here

Conservation: early or emerging concrete-operational ability; quantity has to be conserved (without a second reference system) by compensation or reversibility (identity).

Class Inclusion: mature concrete-operational ability; addition of classes and classification of hierarchically ordered attributes.

Multiple Compensation: early or emerging formal-operational ability; volume has to be conserved within a second reference system (decompensation of water) by multiple compensation; the conservation judgment must be transferred to a second frame of reference.

Pendulum Task: mature formal-operational ability; identification of operative variables within a multivariate system by controlling operative variables and exclusion of inoperative variables by applying propositional logic (hypothetical and deductive thinking using verification and falsification experimentally).

Developmental hypotheses and the formulation of a model of operatory development

For the specification of synchronous and diachronous structural relationships between these four developmental variables the following assumptions were made:

Synchronous development (interrelations of variables at one measurement occasion)

1) Concerning concrete operational tasks: The operation of addition and inclusion of classes implies the operation of conservation since through the addition of both subclasses A1 + A2 = B the different, but hierarchically ordered classificatory attributes must be conserved. According to this dêcalage hypothesis, Conservation is a necessary condition (or prerequisite structure) for the emergence of Class Inclusion and has to be acquired earlier than Conservation. With respect to the binary-matrix shown above, the following combination is inadmissible to the developmental hypothesis: Class Inclusion is acquired, but subject is unable to give an explanatory statement about Conservation (see also Fig. 2; Pattern I).

2) Concerning the formal-operational tasks: It was assumed that Multiple Compensation is easier to acquire and thus acquired earlier than identification of operative variables. Conservation of volume requires decompensation in a second reference system, whereas the Pendulum Task calls for the exclusion of variables and the deduction of operative variables within a multivariate reference system. Since in the Pendulum Task subjects must process and coordinate much more information than in the compensatory operation, we postulate that Multiple Compensation is a precursor (but not prerequisite) structure for the emergence of the multivariate exclusion task (see also Fig. 2; Pattern I).

3) Concerning the structural relation between concrete and formal operations (Class Inclusion and Multiple Compensation): The consolidation of the concrete-operational structure is a prerequisite condition for the emergence of transitional formal operations. For example, unidimensional conservation (concrete-operational) is a necessary condition for multiplicative conservation since it implies a transformation (INRC-group) to a second reference frame (see also Fig. 2; Pattern I).

Diachronous development (interrelation of measurement occasions for one variable)

1) Concerning the developmental relation between measurement occasion 1 and 2 (ages 7 and 9) for Class Inclusion: In accordance with the décalage hypothesis it is assumed that concrete operations develop cumulatively. Whenever an ability has been acquired at the first time of measurement, it is retained at the second time of measurement. Regarding inadmissible patterns of occurence of one variable over time, those combinations are inadmissible which represent the acquisition of an ability at the first time of measurement and the lack of that ability at the second time of measurement (see also Fig. 2; Pattern II).

2) Concerning the relation between measurement occasion 2 and 3 (ages 9 and 12) for Multiple Compensation: As argued above for the case of development of Class Inclusion, it was also assumed that Multiplicative Compensation develops cumulatively over time (see also Fig. 2; Pattern II).

The transformation of these hypothesis into a consistent model of operatory development is shown in Figure 4. The developmental relations are formulated in terms of statement calculus, since they adequately represent the assumed processes of development: Synchronous relations are formulated as logical implications (transitivity) and the diachronous relations as logical replication (cumulativity) (see also Fig. 2).

Insert Figure 4 about here

Admissible patterns or configurations according to the model of operatory development are shown in Table 1. Only those patterns are tabulated that are admissible with regard to the multiple relationships postulated above. The 21 admissible patterns are the results of a transformation of the constraints or conditions of the assumed model of development into binary-matrix language. For example, the pattern 11 10 00 means that only Conservation and Class Inclusion were acquired at ages 7 and 9 (occasions 1 and 2), but formal operational abilities did not emerge at ages 9 or 12 (occasions 2 and 3).

Insert Table 1 about here

Results

For the statistical evaluation of the developmental model a procedure according to Dayton & MacReady (1977) was used. This statistical program tests the fit of an empirical distribution to a postulated structure of the data. Within the procedure only those patterns are treated as true scores or "non error", which are admissible with respect to the model of operatory development. Specific statistical parameters (positive and negative error and Chi-square statistics) and the general fit of the model are shown in Table 2. In the first column the 64 possible patterns are listed. In the second column only those patterns are numbered (1 through 21) which are admissible with respect to the model. The third and fourth column stand for the observed and predicted frequencies according to the model respectively. The last column shows parameters of the Chi-Square tests. At the bottom, statistics for the general fit of the model are given.

Insert Table 2 about here

About 89% of the individual paths of development in the sample could be predicted appropriately by the operatory model of development. Only 11% showed inadmissible patterns of cognitive development. Roughly that percentage was classified as positive or negative errors according to the postulated model. In an overall test, the model fits satisfactorily, since the general Chi-square was less than or equal to the degrees of freedom.

In this paper we do not discuss further substantial implications of this model of operatory development, since the empirical analysis serves only to demonstrate the proposed methodological approach in the analysis of developmental sequences.

Discussion

The present paper represents a contribution to the theory and methodology of the analysis of developmental sequences. The order theoretical and latent attribute procedures used here represent an advance beyond traditional procedures of analysis of variance and correlational analyses focusing on intraindividual changes in development. These strategies are characterized by attention to conceptual implications of cognitive developmental theory (task or structure analysis, order of acquisition, precursor and prerequisite structures), a methodology which specifically focuses on intraindividual changes (individual-oriented approach, formulation of developmental relations in terms of statement calculus) which generates a mathematical model which is statistically testable. Various approaches to the analysis of acquisition sequences are suggested, evaluated, and finally integrated within a consistent sequence model.

Regarding the empirical example it was shown that the development of concrete and formal operations is a highly ordered and cumulative process showing both synchronous and diachronous development. With respect to the intraindividual changes in the emergence of operativity nearly all variations of the observed profiles of acquisition could be described and explained by the developmental model. Individuals differ in their initial cognitive status and their degree of progression or stagnation, but they do not differ with regard to the postulated developmental sequence. The results can also be interpreted as a partial replication of the investigated instruments over time. The concepts of Class Inclusion and Multiple Compensation are solved cumulatively over time (cumulativity hypothesis); in other words, there are no developmental regressions or unsystematic changes in acquisition over time. Therefore, the results represent a contribution to the analysis of micro-developmental processes.

The formulation in terms of statement calculus allows a highly complex modelling of development. It could be demonstrated that even temporal relationships (over time) can be specified precisely. Additionally, other developmental relationships (e.g. synchrony or substitution, Flavell, 1972) can be formulated and implemented in multiple models.

The transformation of the developmental relations into the language of binary matrices and multiplicative contingency tables is an appropriate and powerful means to study sequences of development within an individual-oriented framework.

References

Bakan, D. (1967). On method: Toward a reconstruction of psychological investigation. San Francisco: Jossey-Bass.

Bart, W. M., & Airasian, P. W. (1974). Determination of the ordering among seven Piagetian tasks by an ordering-theoretic method. Journal of Educational Psychology, 66(2), 277-284.

Bart, W. M., & Krus, D. J. (1973). An ordering-theoretic method to determine hierarchies among items. Educational and Psychological Measurement, 33, 291-300.

Bergman, L. R., & Magnusson, D. (1983). The development of patterns of maladjustment. Reports from the Department of Psychology, 50, Stockholm: University of Stockholm.

Buss, A. R. (1979). Toward a unified framework for psychometric concepts in multivariate developmental situations. In J. R. Nesselroade & P. B. Baltes (Eds.), Longitudinal research in the study of behavior and development: Design and analysis (pp. 41-59). New York: Academic Press.

Campbell, R. L., & Richie, D. M. (1983). Problems in the theory of developmental sequences. Human Development, 26, 156-172.

Dayton, C. M., & MacReady, G. B. (1976). A probabilistic model for validation of behavioral hierarchies. Psychometrika, 41, 189-204.

Edelstein, W, Schröder, E., Kliegl, R., Spellbrink, W., Zebergs, D. & Baker, D. (1984). Das Projekt Kindliche Entwicklung und Soziale Struktur. In K. E. Grossmann & P. Lütkenhaus (Hrsg.), Bericht über die 6. Tagung Entwicklungspsychologie in Regensburg (Vol. 2, pp. 274-296). Regensburg: Universität Regensburg.

Edelstein, W., Keller, M., & Wahlen, K. (1984). Structure and content in social cognition: Conceptual and empirical analyses. Child Development, 55, 1514-1526.

Fischer, K. W., & Bullock, D. (1981). Patterns of data: Sequence, synchrony, and constraint in cognitive development. New Directions in Child Development, 12, 1-20.

Flavell, J. H. (1972). An analysis of cognitive-developmental sequences. Genetic Psychology Monographs, 86, 279-350.

Goldschmid, M. L., & Bentler, P.A. (1968). Manual: Concept assessment kit - Conservation. San Diego, CA: Educational and Industrial Testing Service.

Henning, H. J. (1981). Suche und Validierung kognitiver Struktur, Entwicklungssequenzen und Lern-/Verhaltenshierarchien mit Hilfe probabilistischer Modelle. Zeitschrift für Psychologie, 189, 437-461.

Hildebrand, D., Laing, J., & Rosenthal, H. (1977). Prediction analysis of cross-classifications. New York: Wiley.

Hudson, L. M. (1978). On the coherence of role-taking abilities: An alternative to correlational analysis. Child Development, 129, 223-227.

Magnusson, D. (1985). Implications of an interactional paradigm for research on human development. International Journal of Behavioral Development, 9, 115-137.

Piaget, J., & Inhelder, B. (1975). Die Entwicklung des physikalischen Mengenbegriffes beim Kinde (Vol. 4, Studienausgabe). Stuttgart: Ernst Klett Verlag.

Piaget, J., & Inhelder, B. (1977). Von der Logik des Kindes zur Logik des Heranwachsenden. Olten: Walter Verlag.

Rudinger, G. (1978). Erfassung von Entwicklungsveränderungen im Lebenslauf. In H. Rauh (Ed.), Jahrbuch für Entwicklungspsychologie (pp. 157-214). Stuttgart: Klett-Cotta.

Rudinger, G., Chaselong, F., Zimmermann, E., & Henning, H. J. (1985). Qualitative Daten, Neue Wege sozialwissenschaftlicher Methodik. München: Urban & Schwarzenberg.

Schröder, E. (1986). Entwicklungssequenzen konkreter Operationen: Eine empirische Analyse individueller Entwicklungsverläufe. Berlin: Studien und Berichte, Nr. 43, Max-Planck-Institute for Human Development and Education.

Smedslund, J. (1964). Concrete reasoning: A study in intellectual development. Monographs of the Society for Research in Child Development, 19(2), 93.

Somerville, S. C. (1974). The Pendulum problem: Patterns of performance defining developmental stages. British Journal of Educational Psychology, 44, 266-281.

Spiro, A. (June, 1984). Methods for the analysis of categorial data on developmental sequences. Paper presented at the 14th Annual Symposium of the Jean Piaget Society, Philadelphia, USA.

von Eye, A., & Brandtstädter, J. (in press). Evaluating developmental hypotheses using statement calculus and non-parametric statistics. In P. B. Baltes, D. L. Featherman, & R. M. Lerner (Eds.), Life-span development and behavior (Vol. 8). Hillsdale, NJ: Lawrence Erlbaum.

Winer, G. A. (1980). Class-inclusion reasoning in children: A review of the empirical literature. Child Development, 51(2), 309-328.

Figure Captions

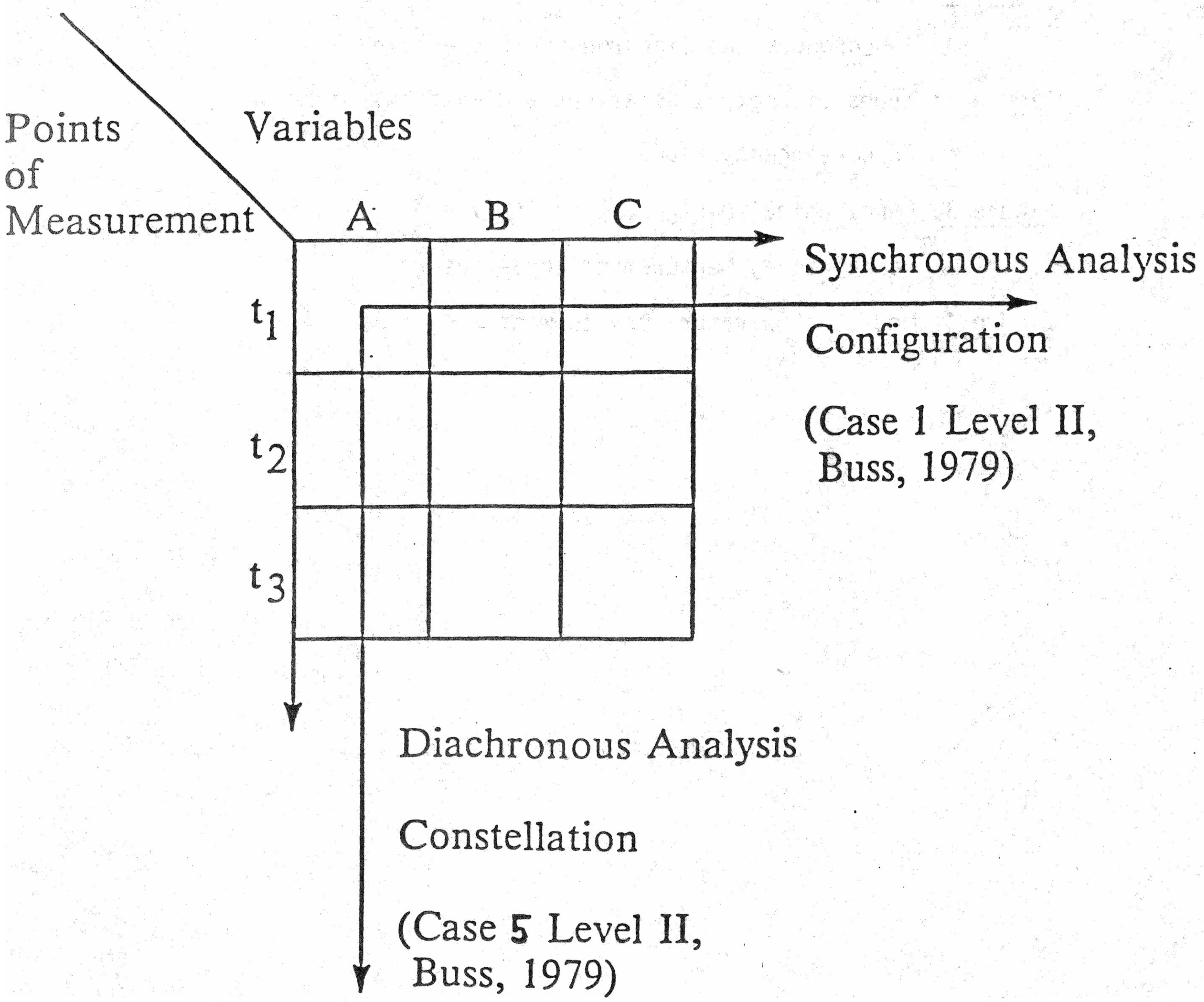
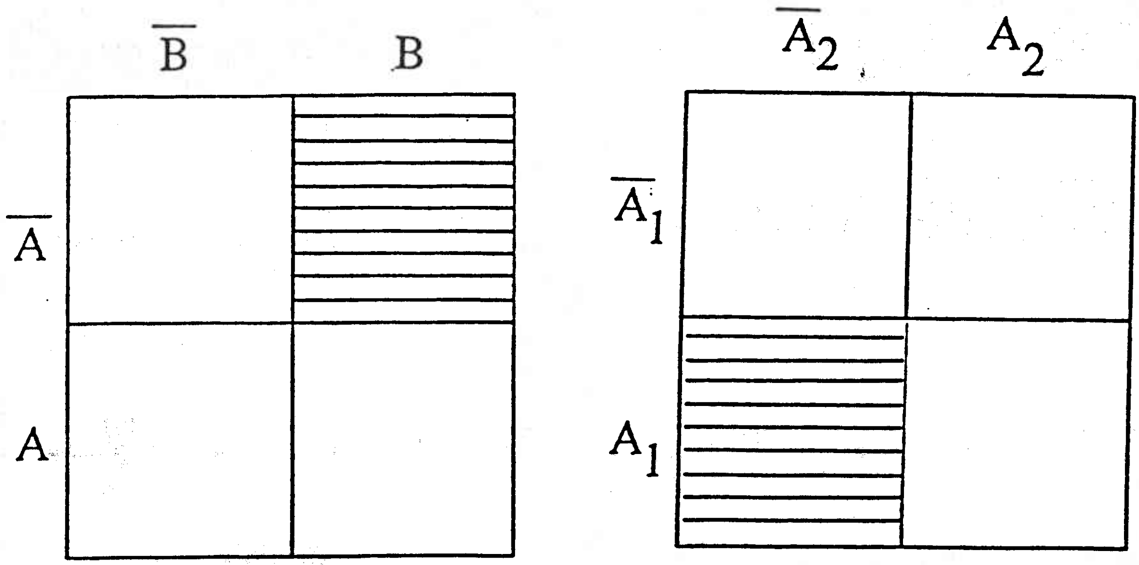

Points
of
Measurement
Variables
A
B
C
t_1
t_2
t_3
Synchronous Analysis
Configuration
(Case 1 Level II,
Buss, 1979)
Diachronous Analysis
Constellation
(Case 5 Level II,
Buss, 1979)

Fig. 1

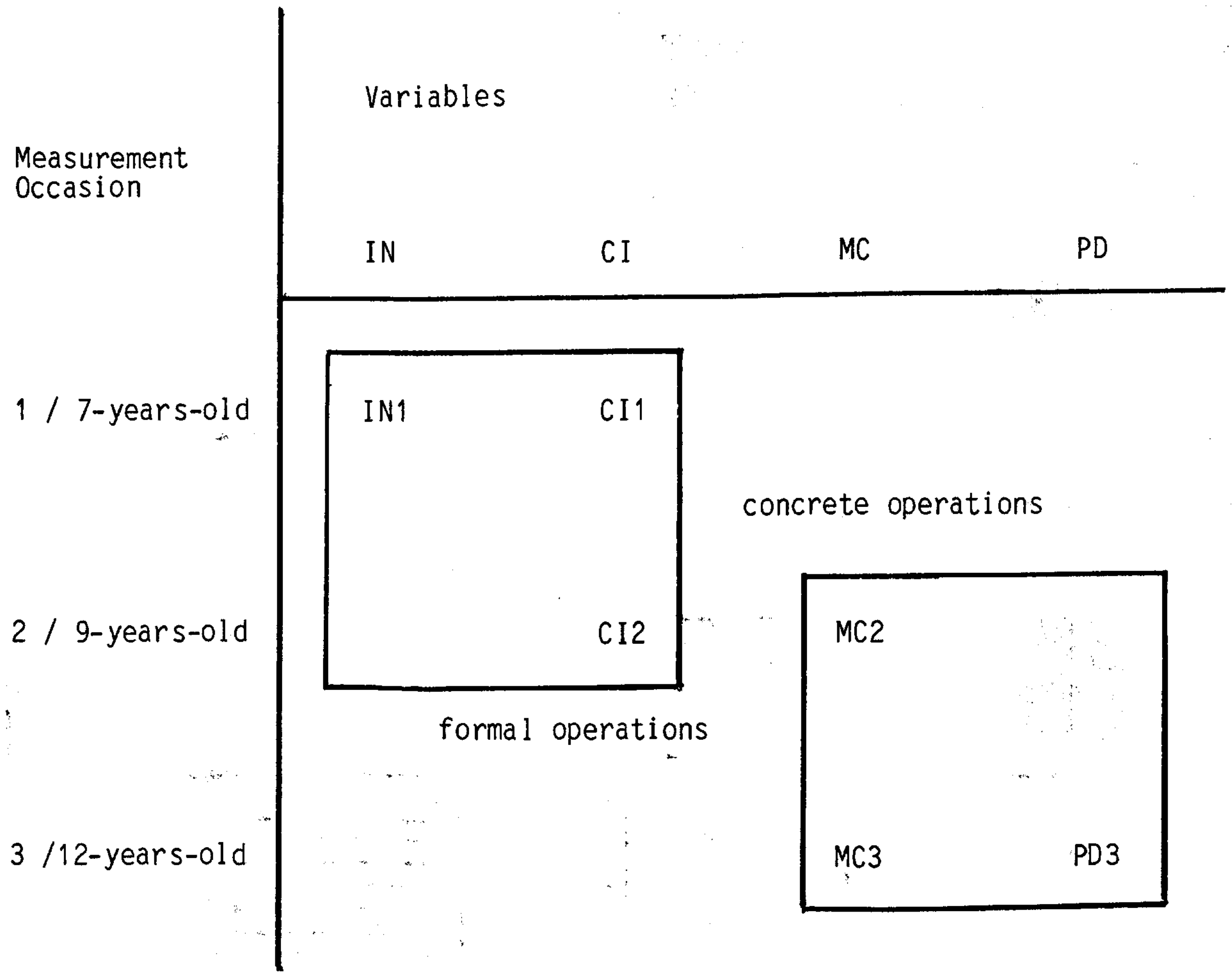


Error pattern I
Necessary condition
Logical implication

Error cells barred

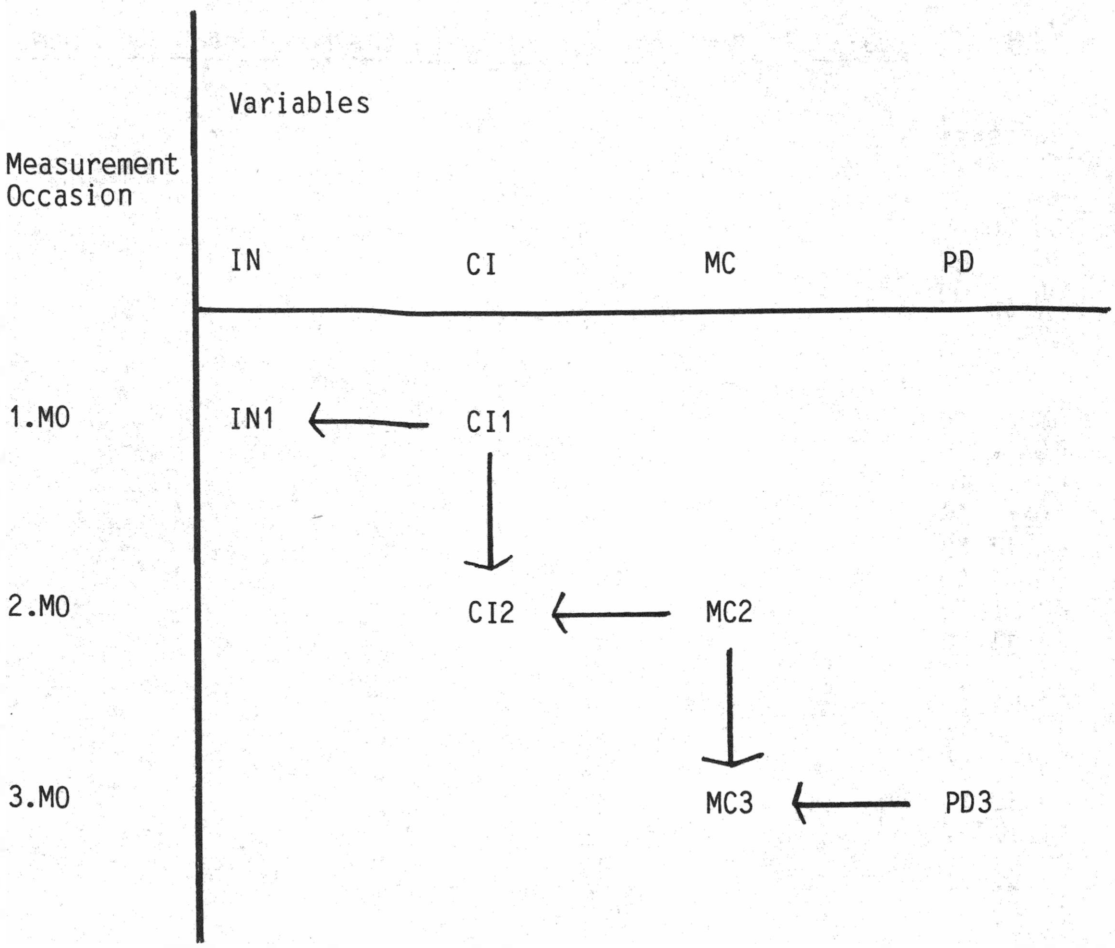
Error pattern II
Sufficient condition
Logical Replication

Fig. 2



IN1 : Invariance of Quantity at measurement occasion 1
CI1, CI2 : Class Inclusion at measurement occasions 1 and 2
MC2, MC3 : Multiple Compensation at measurement occasion 2 and 3
PD3 : Pendulum Task at measurement occasion 3

Fig. 3



IN1 : Invariance of Quantity at measurement occasion 1
CI1, CI2 : Class Inclusion at measurement occasions 1 and 2
MC2, MC3 : Multiple Compensation at measurement occasion 2 and 3
PD3 : Pendulum Task at measurement occasion 3

← : synchronous relationship (between variables)
logical implication - necessary condition (transitivity)

↓ : diachronous relationship (between occasions)
logical replication - sufficient condition (cumulativity)

Fig. 4

Table 1: Admissible Patterns according to the Model of Development

Patterns (IN1-KI1-KI2-MC2-MC3-PD3)	No
00 00 00	1
10 00 00	2
00 10 00	3
10 10 00	4
11 10 00	5
00 00 10	6
10 00 10	7
00 10 10	8
10 10 10	9
11 10 10	10
00 11 10	11
10 11 10	12
11 11 10	13
00 00 11	14
10 00 11	15
00 10 11	16
10 10 11	17
11 10 11	18
00 11 11	19
10 11 11	20
11 11 11	21

21 admissible patterns out of 64 possible patterns

Table 2: Statistical Parameters of the Testing Model of Operatory Development (Procedure by Dayton and MacReady, 1976)

Pattern		Obs N	Pred N	Chi-Square	Pattern		Obs N	Pred N	Chi-Square
000000	1	7	6.49	0.02	000001		1	0.48	0.54
100000	2	3	2.82	0.01	100001			0.18	0.18
010000			0.44	0.44	010001			0.03	0.03
110000		1	0.61	0.23	110001			0.05	0.05
001000	3	16	17.23	0.08	001001		2	1.21	0.50
101000	4	16	14.92	0.07	101001		1	1.03	0.00
011000		2	1.49	0.16	011001		1	0.12	6.30
111000	5	5	6.09	0.19	111001		2	0.62	3.07
000100			0.41	0.41	000101			0.03	0.03
100100			0.23	0.23	100101			0.03	0.03
010100			0.02	0.02	010101			0.00	0.00
110100			0.04	0.04	110101			0.05	0.05
001100		1	1.17	0.02	001101			0.10	0.10
101100			1.59	1.59	101101			0.39	0.39
011100			0.10	0.10	011101			0.05	0.05
111100			0.48	0.48	111101			0.58	0.58
000010	(6)		0.62	0.62	000011	14	1	1.02	0.00
100010	(7)		0.89	0.89	100011	(15)		0.22	0.22
010010			0.04	0.04	010011			0.08	0.08
110010			0.10	0.10	110011		1	0.25	2.14
001010	8	3	2.85	0.00	001011	16	2	2.11	0.00
101010	9	9	9,26	0.00	101011	17	2	1.95	0.00
011010			0.22	0.22	011011			0.37	0.37
111010	(10)		1.15	1.15	111011	18	2	1.96	0.31
000110		1	0.16	4.29	000111			0.09	0.09
100110			0.74	0.74	100111		1	0.34	1.23
010110			0.01	0.01	010111			0.05	0.05
110110			0.12	0.12	110111			0.56	0.56
001110	11	1	1.63	0.24	001111	(19)		0.50	0.50
101110	12	10	8.56	0.23	101111	20	4	3.99	0.00
011110			0.17	0.17	011111		2	0.57	3.58
111110	13	2	1.40	0.24	111111	21	10	6.53	1.84

Chi-Square = 36.31 with DF = 41

alpha: 0.057
beta : 0.078

Pattern : Configuration (IN1 - CI1 - CI2 - MC2 - MC3 - PD3)
Observed N : Observed frequency
Redicted N : Predicted frequency
alpha : "Positive error" (Guessing error)
beta : "Negative error" (Forgetting error)

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