Radiation from Quantum Weakly Dynamical Horizons in Loop Quantum Gravity

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We provide a statistical mechanical analysis of quantum horizons near equilibrium in the grand canonical ensemble. By matching the description of the nonequilibrium phase in terms of weakly dynamical horizons with a local statistical framework, we implement loop quantum gravity dynamics near the boundary. The resulting radiation process provides a quantum gravity description of the horizon evaporation. For large black holes, the spectrum we derive presents a discrete structure which could be potentially observable.

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In order to fully understand and explain the semiclassical result of Hawking's calculation [1] regarding the emission of thermal radiation from a black hole, it is strongly believed that a theory of quantum gravity is necessary. The possibility to probe the Planck scale structure of black holes with observations at much bigger wavelengths has been conjectured by [2]. Using an analogy with discrete quantum systems, the authors assume the area of the black hole to be quantized, and due to the relation between area and mass, they study the emission amplitude related to the jump from one quantized value of the mass to a lower one, analogous to the behavior of atoms. Due to the discreteness of the area eigenvalues, the full emission spectrum is given by a set of spectral lines at frequencies multiple of a fundamental one on the order of the black hole surface gravity. This would imply a drastic departure from Hawking's semiclassical result, as emphasized by [3]. However, in [4] it has been shown that replacing the ansatz of [2] for the area spectrum with the one computed from loop quantum gravity (LQG) for a macroscopical black hole, the spectral lines are very dense in frequency, recovering in this way a continuous spectrum.

This "atomic" picture of a quantum black hole was further exploited in [5] within the LQG framework soon after the first exciting ideas [6] concerning the derivation of black hole entropy from the degrees of freedom (d.o.f.) of the horizon quantum geometry started to circulate. After the introduction of a local definition of black hole through the notion of isolated horizons (IH) [7], those ideas led to models [8,9] in which the quantum horizon d.o.f. are described by a Chern-Simons theory with punctures, representing the quantum excitations of the gravitational field as described by the LQG kinematics. Counting the dimension of the Hilbert space of such a theory living on the horizon leads to an entropy in agreement with the Bekestein-Hawking result, once the large area limit is taken [10,11]. In [5], this quantum mechanical description of black hole states is used to study the radiation associated with a single puncture transition. The spectrum obtained shows a thermal envelope, even though it presents a discrete structure. However, in this analysis no dynamical process responsible for the puncture jump is taken into account. In particular, this single "atom" transition approximation leads to the awkward feature of removing the line $1/2 \rightarrow 0$, which, for macroscopic black holes, would represent the most likely transition. Moreover, the statistical framework of [5] lacks a clear connection with the usual energy canonical ensemble.

In this Letter, we investigate further the analogy between a quantum horizon with its punctures and a gas of particles by introducing the main ingredients for a grand canonical ensemble analysis and implementing the LQG dynamics locally near the horizon, in order to describe the evaporation process in the quantum gravity regime.

The basic idea is to consider the bulk and the horizon as separate in thermal equilibrium. At some point, a weakly dynamical phase takes place, and they interact with each other, allowing for the possibility of energy and particle exchange. After such a change of thermodynamic state has taken place, the two subsystems go back to equilibrium. By matching the description of this intermediate phase in terms of weakly dynamical horizons [12,13] with the local statistical description of [14], a physical time parameter can be singled out, allowing us to describe the boundary states evolution in terms of the theory Hamiltonian operator. In this way, elements coming from different frameworks, such as thermodynamics, classical general relativity, and quantum gravity, combine together to provide a local quantum dynamical derivation of the radiation spectrum.

This picture will be made more precise in the following, where we will concentrate only on the spherically symmetric case. However, let us at this point clarify the framework we are considering: no background structure is introduced at any point, and we will work in the quantum gravity regime; no matter is going to be coupled; the radiation spectrum we will derive is related entirely to emission of quanta of the gravitational field due to dynamical processes described by the LQG approach.

In order to provide the tools for a thermodynamical study of the system described above, one has to introduce a notion of local energy for the horizon. However, in the context of IH such a notion is not unique due to the fact that there can be radiation in space-time outside the horizon, and therefore, no unique time evolution vector field can be singled out. Nevertheless, in [14,15] a local first law for IH has been recently derived, whose uniqueness can be proven once a local physical input is introduced. More precisely, for a preferred local stationary observer $\mathcal{O}$ hovering outside the horizon at proper distance ℓ, this reads $dE = \bar{\kappa}/(8\pi G)dA$, where $\bar{\kappa} = 1/\ell + o(\ell)$ represents the local surface gravity measured by $\mathcal{O}$. Integrating the previous equation, one obtains a local notion of energy associated to the IH in terms of its area, namely,

$$E = \frac{\bar{\kappa}}{8\pi G} A. \qquad (1)$$

Using this result, the grand canonical partition function for the gas of punctures can be written as $\mathcal{Z}(\beta) = \sum_{N=0}^{\infty} z^N Z(\beta, N)$, where $z = \exp(\beta\mu)$, μ being the chemical potential, N the number of punctures, and $Z(\beta, N)$ the canonical partition function given by

$$Z(\beta, N) = \sum_{\{s_j\}} \prod_j \frac{N!}{s_j!} (2j+1)^{s_j} e^{-\beta s_j E_j}, \qquad (2)$$

where we assumed the punctures to have Maxwell-Boltzmann statistics; $\{s_j\}$ represents a given quantum configuration where s_j punctures carry the spin j, for all possible value of j, and such that $\sum_j s_j = N$, while E_j is given by the scaled IH area spectrum through Eq. (1), namely,

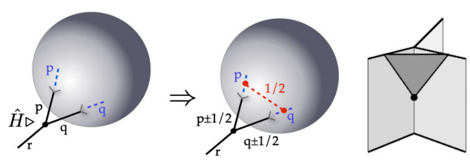
$$\hat{H}|\{s_j\}\rangle = \sum_j s_j E_j |\{s_j\}\rangle = \frac{\bar{\kappa}\gamma\ell_p^2}{G} \sum_j s_j \sqrt{j(j+1)} |\{s_j\}\rangle.$$

Notice that, so far, β can be regarded simply as the intensive parameter conjugate to the energy Eq. (1). From the grand canonical partition function $\mathcal{Z}(\beta)$, all the thermodynamical quantities can be computed. In particular, for the expectation value of s_j we get

$$\bar{s}_j = -\frac{\partial}{\partial(\beta E_j)} \log \mathcal{Z} = \frac{z(2j+1)e^{-\beta E_j}}{1 - z\sum_j (2j+1)e^{-\beta E_j}}. \qquad (3)$$

When the number of punctures is large, Eq. (3) gives $1 \approx \bar{N}/(\bar{N}+1) = z\sum_j (2j+1)e^{-\beta E_j}$, where $\bar{N} = \sum_j \bar{s}_j$ is the mean number of punctures of the gas. The previous equation provides an expression for the chemical potential matching the one found in [14]. Notice that, due to the dynamical processes taking place near the horizon (as described below), the chemical potential does not need to vanish. In fact, $\bar{N}$ is not fixed *a priori*, and a change in the number of punctures would imply a change of the horizon energy; therefore, μ can be different from zero.

Within this thermodynamical framework, we can now study the evaporation process. The topology of the null surface Δ representing the IH is assumed to be $S^2 \times \mathbb{R}$; i.e., at "a given time", the horizon surface is given by the two-sphere at the intersection between Δ and a spacelike Cauchy surface. The quantum space geometry is described by the polymerlike excitations of the gravitational field encoded in the spin networks states. Some edges of those states can now pierce through the horizon, providing local quantum d.o.f. accounting for the IH entropy. Dynamics in the bulk are implemented by the Hamiltonian constraint $\hat{H}[N] = 0$. This reflects the fact that, in a diffeos invariant theory, the canonical Hamiltonian generates evolution in the parameter of the action which is unphysical. This freedom in choosing the evolution parameter is reflected in the possibility to rescale the time vector field by the lapse N. Then, in the quantum theory, the projection operator into the kernel of $\hat{H}$ is constructed by integrating over all possible values of the lapse [16,17]. Despite the lack of a completely well-defined quantization prescription for $\hat{H}$, one clear general property is that it acts locally at the vertices of the spin networks and changes the spin of some of the edges attached to the given vertex. In order to be more quantitative, in the following we will consider Thiemann's proposal [18]. If we concentrate on the Euclidean part for simplicity, its action on a three-valent node having two edges piercing the horizon can be graphically represented as



, (4)

where the holonomies entering the regularization of $\hat{H}$ are taken in the fundamental representation. The action depicted above can be interpreted as an absorption or emission process of quanta of the gravitational field by the quantum horizon.

In order to study the radiation process, we need to perturbate the equilibrium states represented by the IH configurations by turning on some weakly dynamical effects near the horizon until another equilibrium configuration state is reached again. In this way, a physical process causes a small change of the IH state. For a large black hole, this is expected to happen quite slowly.

A description of this dynamical phase requires first to locally single out one of the partial observables to play the role of time. In this way, we can use the Hamiltonian operator in the bulk to define the evolution of the boundary states with respect to this observable. Let us now show how the contact between the dynamical horizons (DH) framework developed in [12,13] and the thermodynamical description of [14,15] would allow us to do so.

Within a framework advocated by Ashtekar, the idea is to describe the evaporation process by interpreting this intermediate evolution phase as an extension of the quantum geometry from isolated to dynamical horizons, where a proper quantum notion of gravitational energy—induced by the quantum theory of geometry in the bulk—replaces the classical one introduced in [13,19].

In [13], a dynamical process version of the first law was derived from an area balance law relating the change in the area of the DH to the flux of matter and gravitational energy. In the vacuum, its infinitesimal version provides a first law for DH, namely,

$$\frac{\bar{\kappa}_r}{8\pi G} dA = dE_r, \tag{5}$$

where $\kappa_r = (2r)^{-1}$ is an effective surface gravity, with r the radius of the two-sphere leaf of the dynamical horizon. The right-hand side of Eq. (5) is related to the bulk term in the Hamiltonian, which can be written in terms of pure geometrical quantities. In this way, the first law above provides a dynamical coupling of bulk and boundary geometries near the horizon. Let us notice that, given the spherical symmetry of the horizons studied here, the classical notion of gravitational energy defined by the (integral version of) rhs of Eq. (5) would give a vanishing flux. Of course, Hawking radiation teaches us that this does not need to hold also at the quantum level.

In [13] it is shown that, for a dynamical horizon, the first law Eq. (5) can be generalized by replacing the radius r with an arbitrary function $f(r)$, reflecting the freedom to rescale the vector field ξ with respect to which the notion of energy is associated. The function $f(r)$ encodes the dynamical nature of the previous version of the first law, but it also represents an ambiguity in the description of the dynamics.

The passage from a weakly DH to a weakly IH is a well-defined procedure described in [13]. The only ingredient missing in our construction is the matching of the surface gravity entering Eq. (5) with the one appearing in the local notion of energy in Eq. (1). This last step can now be carried out by means of the rescaling freedom described above. For our preferred family of observers located right outside the horizon, a natural choice of the field ξ_f would be given by $f = \ell$. In standard coordinates for the Schwarzschild metric, for an observer at $r = 2M + \epsilon$, the proper distance from the horizon is $\ell = 2\sqrt{2M\epsilon}$. If we now set $f(r) = \ell$, we get an effective surface gravity matching exactly the one entering the energy expression Eq. (1), i.e., $\bar{\kappa}_f = (df/dr)\bar{\kappa}_r = 1/\ell$. Therefore, the local first law derived in [14,15] is compatible with the framework of [13] if we single out, among the permissible time vector fields for the local observer $\mathcal{O}$, the one defined by $t^a = N_\ell \chi^a$, where N_ℓ is a permissible lapse associated with the radial function ℓ.

Moreover, if we interpret ϵ as a parameter controlling the slow evolution of the horizon area, i.e., $\mathcal{L}_\chi r_H \sim \epsilon$, the previous expression for the surface gravity $\bar{\kappa}_f$ matches the requirements of [19] for the definition of slowly evolving horizons. In this case, the local surface gravity entering the relation (1) would be approximately constant. Henceforth, one can regard the horizon as making continuous transitions from one equilibrium state to another, and the geometrical surface gravity $\bar{\kappa}$ can be interpreted as a (slowly varying) parameter controlling this process. In this sense, it seems very natural to interpret $\bar{\kappa}$ as the temperature of the radiation emitted during this transition phase. This strengthens the connection between thermodynamics and our statistical analysis and puts on more solid ground the entropy derivation of [14].

At the quantum level, the local notion of energy in Eq. (1) acquires a definition in terms of the LQG area operator, whose eigenstates are given by the spin networks coming from the bulk. The Hamiltonian operator action modifies these states, inducing in this way a jump to a different area eigenvalue. Therefore, the local deparametrization of the system described above can be used to interpret the bulk quantum dynamics as a generator of boundary states evolution, providing a quantum notion of weakly DH. More precisely, the action of $\hat{H}$ on nodes close to the horizon can be used, in the formalism of [17], to define a physical scalar product to interpret as transition amplitudes between one boundary state to another in the physical time parameter defined by N_ℓ. Given a state $|\{s_j\}\rangle$ on a given two-sphere cross section, the action of $\hat{H}$ on the vertices near the horizon will evolve it in a state $|\{s_{j'}\}\rangle$, through the change of spin of some of the punctures. The transition probability when the two states differ only at a given vertex v, in an infinitesimal interval of time, is given by $P_{jj'} = |H_{jj'}|^2$, where

$$\begin{aligned} H_{jj'} &\equiv \langle\{s_{j'}\}|\{s_j\}\rangle_{\text{phys}} = \langle\{s_{j'}\}| \int dN O_\ell e^{-i\hat{H}[N]} |\{s_j\}\rangle \\ &= -iN_\ell \langle\{s_{j'}\}|\hat{H}_v|\{s_j\}\rangle + o(N_\ell^2). \end{aligned} \tag{6}$$

Let us explain the last passage in the previous equation. The diff-invariant nature of the physical scalar product is usually obtained by removing the dependence on the regulator in the action of $\hat{H}$ through the diff-invariant nature of the DN integral. In the case of an internal boundary though, the situation is more subtle. In particular, IH boundary condition are such that only diffeos tangent to the horizon are to be regarded as gauge transformations [7]; these diffeos are compatible with the "gauge fixing" performed in the evaluation of Eq. (6), which encodes the local deparametrization of the system. Such a selection of a physical time variable can be performed by inserting a local observable $O_\ell = \delta(N, N_\ell(v))$ in the path integral, which singles out the constant value N_ℓ for the scalar function $N(x)$ at the vertices near the horizon.

Let us now observe that the terms generated by the action of $\hat{H}$ on a generic three-vertex near the horizon are of two different kinds: two of them create new punctures piercing the horizon and one [shown in (4)] does not.

Since the new puncture created by $\hat{H}$ has to be coplanar to the two links it is attached to, the inclusion of the first kind of terms would lead to a breaking of the boundary diffeomorphism symmetry and, potentially, to an infinite entropy. Therefore, we only allow the action of $\hat{H}$ that does not create new punctures. Notice that this term corresponds to the absorption inside the horizon of a quantum of the gravitational field associated with an emission process, due to the jump to a lower area eigenvalue. This dynamic is reminiscent of the heuristic picture of Hawking radiation where one antiparticle is absorbed while one particle escapes to infinity.

Given these considerations, the $H_{jj'}$ in Eq. (6) corresponds to the transition amplitude where two of the punctures piercing the horizon have jumped to a different energy level. For the case of a macroscopic black hole, only the first $SU(2)$ irreps are relevant for the punctures; therefore, we will consider only the cases $q = j$ and $j \leq p \leq j + r$, with $r = 1/2, 1, 3/2$. Higher values of r would not be relevant for the spectrum. We now choose the holonomies in $\hat{H}$ to be in the fundamental representation, implying that, in the emission process, q can only jump to $q - 1/2$ while p to $p \pm 1/2$. Among the two possible orderings of $\hat{H}$, as defined by Thiemann, we will choose the one in which the volume operator acts before the creation of the new link, in order to have a nonvanishing probability for the jump $1/2 \to 0$ of one of the punctures. Notice that the action of $\hat{H}$ always preserves the IH gauge symmetry, and the unpleasant feature of removing the $1/2 \to 0$ transition from the spectrum is not present anymore.

We now have all the ingredients to derive the radiation spectrum for our local observer $\mathcal{O}$. The energy of the quantum of radiation emitted after the action of $\hat{H}$ on two punctures with spins p, q is $\Delta^{\pm}_{pq} = E_p - E_{p\pm 1/2} + E_q - E_{q(1/-2)}$ and the intensity of the lines is given by

$$I_{\rm jr} = \bar{s}_p \bar{s}_q |N_\ell A(2p, \pm 1; 2q, -1, r)|^2 \Delta^{\pm}_{pq}, \qquad (7)$$

where the matrix elements A are explicitly computed in [20], for Thiemann's version of the Euclidean constraint. In FIG. 1 we plot the energy distribution $I_{\rm jr}$ as a function of $x \equiv \beta \Delta^{\pm}_{pq}$. The plot shows a discrete structure formed by four different sets of spectral lines. The first two groups of lines correspond to the case $p \to p + 1/2$, while the last two to the case $p \to p - 1/2$.

Let us now conclude and make some comments.

We have used the statistical mechanical framework introduced in [14,15] to study the properties of IH near equilibrium in the grand canonical ensemble. This setting seems to be the most appropriate, given the nature of the emission process. Compatibility of this framework with the weakly DH description of the intermediate transition phase allowed us to single out a physical time variable in terms of which describe the boundary states evolution. By means of

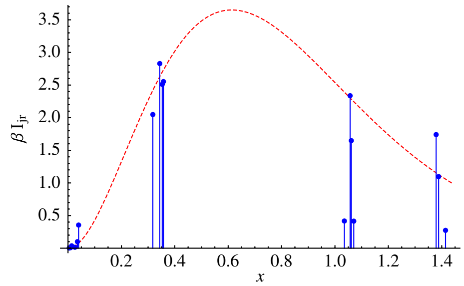


FIG. 1 (color online). Emission lines intensity and the thermal envelope. For the x variable, we used $\beta \bar{\kappa} \gamma \ell_p^2 / G \sim o(1)$. The normalization factor $N_\ell \bar{N}$ of the intensity is left unspecified. The gap between the different sets of lines is on the order $\Delta\omega \sim \bar{\kappa}$.

this deparametrization, we implemented the LQG dynamics near the horizon, providing a quantum gravity description of the evaporation process. For large black holes, the spectrum presents a neat discrete structure.

Analogous to [4], one could ask if the inclusion of other lines would make the spectrum effectively continuous. The ambiguity in the holonomies irrep present in the definition of $\hat{H}$ can be easily checked to not alter the discrete pattern. Another possibility would be to consider the simultaneous action of $\hat{H}$ on more than one vertex in the expansion Eq. (6). However, those lines could be significantly damped by the transition amplitude [for instance, this would be the case if $N_\ell \bar{N} \sim o(1)$]. Therefore, the LQG dynamics might provide the possibility to observe a departure from the semiclassical scenario. However, further investigation in this direction is necessary, and our analysis should be regarded as a first step towards a fully quantum dynamical description of the emission process.

While our attention here has been limited to the canonical analysis, the near-horizon dynamics could be implemented using spin foams. More precisely, given the increasing evidence for the spin foam amplitude to implement the Hamiltonian constraint [21], one could use the vertex amplitude to compute transition probabilities between boundary states containing the right part of (4) as a subgraph. This could turn out to be a useful application of the duality between the two formalisms.

Finally, the process described here differs considerably from the Hawking effect. The latter simply requires a curved background and a scalar field propagating on it: the gravitational d.o.f. are frozen. The emission process is intrinsically related to the definition of particles possible due to this switching off of gravity. On the contrary, in our analysis, it is the evolution of the quantum gravitational d.o.f. on the horizon which is responsible for the energy emission. Moreover, due to this dynamical framework, the whole description of the process is local.

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