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SOME CONSIDERATIONS ABOUT THE STRINGY HIGGS EFFECT

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We discuss various aspects of the Higgs mechanism that takes place in string theories, by considering the Z_3 heterotic orbifold as a specific example. Some emphasis is put on the duality invariance of the effective field theory description.

1. Introduction

The Higgs mechanism provides a consistent field theoretical description of massive Yang–Mills fields. It is realized also in string theory, as the masses of certain states depend on continuous background parameters of the internal compact space [1], that is, on the moduli [2] of the corresponding conformal field theory (CFT). At special ("critical") points in moduli space some masses can actually vanish. In particular, various vector and scalar bosons, though massive at generic points, can be massless at special points in moduli space. Thus, at these points, extra gauge symmetries occur. This phenomenon is called the stringy Higgs effect. We will discuss various aspects of this mechanism, in particular an effective lagrangian description. This description involves a gauged sigma model with spontaneous symmetry breaking. It contains just the lightest string degrees of freedom, namely, in addition to the massless moduli, those fields which can have arbitrarily small masses (like the weak scale). The other string excitations that have large masses (like the Planck scale) do not enter the effective lagrangian.

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In general, there are certain discrete transformations, acting in particular on moduli and also on other fields, that leave a CFT invariant. These transformations constitute the "target space duality" group [3–17]. Duality invariance is a stringy phenomenon and typically mixes fields with different masses. It imposes, in principle, constraints on any effective field theory [10, 16]. Specifically, the fields that are involved in the stringy Higgs mechanism mix with *infinitely* many other fields under duality transformations. We will discuss how such symmetries can be reconciled with a field theoretic description of the Higgs effect that involves only a *finite* number of fields. We will find that the situation is conceptually similar to electron bands in a solid.

The simplest and best understood example which displays the stringy Higgs effect is given by a heterotic compactification on the $\mathbb{Z}_3$-orbifold [18]. This CFT has just one (complex) modulus τ, and the gauge group that appears at any critical point τ_c is particularly simple and given by $U(1)\times U(1)$. The duality group is just given by the modular group, $PSL(2,\mathbb{Z})$. Furthermore, three copies of this CFT lead to a semi-realistic, $N=1$ supersymmetric heterotic string model that allows for an easy effective lagrangian description. We will, however, rather first focus on one copy of this model, as it already displays many interesting features. The generalization to six dimensions will then be discussed subsequently.

2. CFT and the $\mathbb{Z}_3$-orbifold

The $\mathbb{Z}_3$-orbifold model can be described as a two-dimensional torus compactification on the root lattice of SU(3), modded out by $\mathbb{Z}_3$ twists. Two moduli, the antisymmetric tensor background B and the scale of the lattice R, survive the twist and can be combined into one complex modulus, $\tau = B + i\sqrt{3}R^2$. The internal left- and right-moving momenta $p^{\pm}_{L,R} = (1/\sqrt{2})(p^1_{L,R} \pm ip^2_{L,R})$ are then

$$p^+_L = \frac{i}{\sqrt{2}\left(\sqrt{3}\,\Im\tau\right)^{1/2}}\left(m_2+\bar\rho m_1+\bar\tau(n_1-\bar\rho n_2)\right), \qquad p^-_L = (p^+_L)^\dagger,$$

$$p^+_R = \frac{i}{\sqrt{2}\left(\sqrt{3}\,\Im\tau\right)^{1/2}}\left(m_2+\bar\rho m_1+\tau(n_1-\bar\rho n_2)\right), \qquad p^-_R = (p^+_R)^\dagger. \quad (2.1)$$

Here, m_i and n_i are the momentum and winding numbers, respectively, and $\rho \equiv e^{2\pi i/3}$. It is easy to determine how duality transformations $\tau \to (a\tau+b)/(c\tau+d)$, which belong to $PSL(2,\mathbb{Z})$ ($a,b,c,d \in \mathbb{Z}$, $ad-bc=1$), act on the theory. The conformal dimensions, $\bar h = p^+_L p^-_L$ and $h = p^+_R p^-_R$, of the winding states only remain invariant if one simultaneously shifts the winding and momentum numbers. For the generators of the modular group, S: $\tau \to -1/\tau$ and T: $\tau \to \tau+1$, one finds [9, 13, 14]

$$S:\ \begin{cases} n_1 \to m_1 - m_2 \\ n_2 \to m_2 \\ m_1 \to n_1 \\ m_2 \to n_1 + n_2 \end{cases}, \qquad T:\ \begin{cases} n_1 \to n_1 \\ n_2 \to n_2 \\ m_1 \to m_1 + n_2 \\ m_2 \to m_2 - n_1 \end{cases}. \tag{2.2}$$

These transformations, in spite of leaving $\bar{h}$, h invariant, induce a background-dependent rotation [19] on the momentum vectors $p^{\pm}_{\mathrm{L,R}}$. Therefore, to get invariant vertex operators $V_{n_1,n_2,m_1,m_2} = \mathrm{e}^{ip_{\mathrm{L,R}}\cdot X_{\mathrm{L,R}}}$ (where $p\cdot X = p^+X^- + p^-X^+$), one also has to rotate the torus coordinates $X^{\pm}_{\mathrm{L,R}}$. For a general PSL(2, $\mathbb{Z}$) transformation one finds

$$X^+_{\mathrm{L}} \to \lambda\left[\frac{c\tau + d}{c\bar{\tau} + d}\right]^{1/2} X^+_{\mathrm{L}}, \qquad X^+_{\mathrm{R}} \to \lambda\left[\frac{c\bar{\tau} + d}{c\tau + d}\right]^{1/2} X^+_{\mathrm{R}}, \tag{2.3}$$

where λ is a τ-independent phase which depends on the parameters of the PSL(2, $\mathbb{Z}$) transformation: e.g. $\lambda = \rho$ for ST.

We now discuss the (untwisted) spectrum and the action of the modular group on the relevant states. First, the (complex) marginal operator $\bar{\partial}X^-_{\mathrm{L}}\partial X^+_{\mathrm{R}}$ corresponds to the freedom of having the modulus τ as a free parameter. It creates a single-particle state, namely a four-dimensional scalar $|\tilde{\tau}\rangle$ whose mass vanishes for all backgrounds τ. Its vacuum expectation value $\langle\tilde{\tau}\rangle$ takes arbitrary values as a function of τ. Using (2.3) we deduce that $\tilde{\tau}$ transforms under duality transformations as

$$\tilde{\tau} \to \frac{c\bar{\tau} + d}{c\tau + d}\tilde{\tau}. \tag{2.4}$$

The winding and momentum spectrum is generated by $\mathbb{Z}_3$ twist-invariant combinations of vertex operators V_{n_1,n_2,m_1,m_2}. At any critical point τ_c there are certain states which are massless just at this point (there are infinitely many such critical points, which are images of $\tau_c = \rho$ under PSL(2, $\mathbb{Z}$)). That is, any valid choice of τ_c determines a particular set of labels $\{n_i, m_i\}$, corresponding to massless states. Taking for example $\tau_c = \rho$, there are massless, four-dimensional space-time vectors*

$$V^+_{\mu} = \frac{1}{\sqrt{3}}\left(V_{1,0,1,1} + V_{-1,1,-1,0} + V_{0,-1,0,-1}\right)\left(\partial X_{\mu\mathrm{R}} + \dots\right),$$

$$V^-_{\mu} = \frac{1}{\sqrt{3}}\left(V_{-1,0,-1,-1} + V_{1,-1,1,0} + V_{0,1,0,1}\right)\left(\partial X_{\mu\mathrm{R}} + \dots\right). \tag{2.5}$$

* We denote vertex operators and the corresponding fields by the same symbols. The dots denote other operators needed for BRS invariance.

Thus there is an extra U(1) × U(1) gauge symmetry. The left-moving momenta are the two $\mathbb{Z}_3$-invariant combinations of roots of SU(3); the right-moving momenta vanish. In addition, there are two massless (complex) scalars,

$$\tilde{\phi}^1 = \frac{1}{\sqrt{3}}\left(V_{1,0,1,1} + \bar{\rho}V_{-1,1,-1,0} + \rho V_{0,-1,0,-1}\right)\left(\partial X_{\mathrm{R}}^{+} + \dots\right),$$

$$\tilde{\phi}^2 = \frac{1}{\sqrt{3}}\left(V_{-1,0,-1,-1} + \bar{\rho}V_{1,-1,1,0} + \rho V_{0,1,0,1}\right)\left(\partial X_{\mathrm{R}}^{+} + \dots\right). \qquad (2.6)$$

For arbitrary background parameters the above fields have non-vanishing masses,

$$m^2 = p_{\mathrm{R}}^{+} p_{\mathrm{R}}^{-} = \frac{i}{\sqrt{3}} \frac{|\tau - \rho|^2}{\tau - \bar{\tau}}, \qquad (2.7)$$

and two real scalar degrees of freedom may be regarded as longitudinal modes of the vector bosons.

The fields $\bar{\tau}$, $\tilde{\phi}_1$ and $\tilde{\phi}_2$ do not have well-defined charges under the two U(1) currents. It is thus convenient to build linear combinations with definite U(1) × U(1) charges Q_α^i ($i = 1, 2$, $\alpha = 1, 2, 3$),

$$s_1 = (1/\sqrt{3})(\bar{\tau} - \tilde{\phi}_1 - \tilde{\phi}_2): \qquad Q_1 = (\sqrt{2}, 0),$$

$$s_2 = (1/\sqrt{3})(\bar{\tau} - \bar{\rho}\tilde{\phi}_1 - \rho\tilde{\phi}_2): \qquad Q_2 = \left(-1/\sqrt{2}, \sqrt{\tfrac{3}{2}}\right),$$

$$s_3 = (1/\sqrt{3})(\bar{\tau} - \rho\tilde{\phi}_1 - \bar{\rho}\tilde{\phi}_2): \qquad Q_3 = \left(-1/\sqrt{2}, -\sqrt{\tfrac{3}{2}}\right). \qquad (2.8)$$

The spectrum is anomaly free. The winding states have zero vacuum expectation value, $\langle \tilde{\phi}_i \rangle = 0$, due to internal momentum conservation. This implies that all three fields s_α have the same vacuum expectation value, $\langle s_\alpha \rangle = \langle \bar{\tau} \rangle / \sqrt{3}$.

Under general modular transformations (2.2), the fields (2.5) and (2.6) transform into different fields, with in general different masses. As the modular orbit length is infinite, it seems a priori difficult to achieve a duality-invariant effective field theory with only a finite number of massive fields. However, as any choice of τ_c defines a particular set of labels $\{n_i, m_i\}$, we can achieve a reasonable effective field theory description by considering fields which formally depend on τ_c, instead of $\{n_i, m_i\}$. That means τ_c must transform under general modular transformations like $\tau_c \to (a\tau_c + b)/(c\tau_c + d)$, as this is equivalent to transforming the labels

$\{n_i, m_i\}$. Using (2.3) the scalars $\bar{\phi}_i$ then transform as

$$\bar{\phi}_i(\tau, \tau_c) \to \lambda \left[\frac{c\bar{\tau}+d}{c\tau+d}\right]^{1/2} \bar{\phi}_i\left(\frac{a\tau+b}{c\tau+d}, \frac{a\tau_c+b}{c\tau_c+d}\right), \tag{2.9}$$

whereas the vectors $V^i_\mu(\tau, \tau_c)$ are inert. The mass formula, valid for any given choice of τ_c, is

$$m^2 = p^+_R p^-_R = \frac{|\tau-\tau_c|^2}{(\tau_c-\bar{\tau}_c)(\bar{\tau}-\tau)}. \tag{2.10}$$

It is invariant under modular transformations.

At any critical point, $\tau = \tau_c$, one can "re-bosonize" [20–22] the coordinates by writing $i\bar{\partial} X^{\pm}_L = (1/\sqrt{3})\Sigma_\alpha \exp(\pm i\alpha \cdot Y_L)$ ($\pm\alpha$ are the six root vectors of SU(3)). In this new basis, the vertex operators for the massless fields s_α are simply the exponentials of the bosons Y, $s_\alpha = e^{i\alpha \cdot Y_L} \partial X^+_R$. It follows then from internal momentum conservation that all fields must have zero vacuum expectation values, $\langle \bar{\tau}(\tau_c) \rangle = 0$. Moreover, since exponentials create states normalized to one, the corresponding kinetic terms in the effective action must be canonical at all critical points. Finally, adopting the above basis, it can easily be shown that all three fields s_α transform with the same $\mathbb{Z}_3$-phase under the particular modular transformation that fixes a given τ_c (e.g. ST for $\tau_c = \rho$). Thus, at the critical point, this duality transformation acts just like the finite gauge transformation $s_\alpha \to \exp\{4\pi i[(1/3\sqrt{2})Q^1_\alpha + (1/\sqrt{6})Q^2_\alpha]\} s_\alpha$ [9].

3. Effective lagrangian description

We start by considering only one copy of the $\mathbb{Z}_3$-orbifold. The usual four-dimensional $N = 1$ supergravity action for the modulus superfield t is the SU(1,1)/U(1) non-linear sigma model with Kähler potential*

$$K(t, \bar{t}) = -\log(t + \bar{t}). \tag{3.1}$$

It can be derived, for example, from string theory by computing [23] the Zamolodchikov metric $g_{t\bar{t}} = 1/(t+\bar{t})^2 = (\partial^2/\partial t\, \partial\bar{t}) K(t, \bar{t})$, or by truncating the ten-dimensional supergravity action [24, 25]. It is crucial to realize that the "standard" supergravity field t cannot be identified with the string field $\bar{t}_{\text{string}}$ that is created

* From here on we will follow supergravity conventions and use the variable $t|_{\theta=0} = -i\tau = \sqrt{3}R^2 - 2iB$. We will use t to denote both the superfield and its lowest component.

by the marginal operator $\bar{\partial} X_L^- \, \partial X_R^+$. It does not transform under duality transformations as in (2.4) but rather as

$$t \to \frac{at - ib}{ict + d}\,. \tag{3.2}$$

(This leaves (3.1) invariant up to a Kähler transformation, $\Delta K = 2\log|ict + d|$.) In addition, $\langle t \rangle = t_c \equiv -i\tau_c$ at the critical point in contrast to $\langle \tilde{t}_{\rm string} \rangle = 0$. To relate the two fields let us first perform a holomorphic field redefinition,

$$\tilde{t}(t, t_c) = \frac{t_c - t}{\bar{t}_c + t}\,, \tag{3.3}$$

which under simultaneous PSL(2, $\mathbb{Z}$) transformations of t and t_c transforms with a constant, background-independent phase,

$$\tilde{t} \to \left(\frac{-ic\bar{t}_c + d}{ict_c + d} \right) \tilde{t}\,. \tag{3.4}$$

To arrive at the string basis we have to perform the additional non-holomorphic field redefinition

$$\tilde{t}_{\rm string} = \left(\frac{\bar{\tilde{t}} + 1}{\tilde{t} + 1} \right) \tilde{t} = \frac{t_c - t}{\bar{t} + t_c}\,. \tag{3.5}$$

The field $\tilde{t}_{\rm string}$ does indeed transform as $\tilde{\tau}$ in (2.4).

Let us now include the two scalar fields $\phi_{1,2}$. As they behave like ordinary matter fields with metric $g_{\phi_i \bar{\phi}_i} = \delta_{ij}/(t + \bar{t})$ [26], they enter the Kähler potential as

$$K = -\log\left(t + \bar{t} - |\phi_1|^2 - |\phi_2|^2\right). \tag{3.6}$$

This describes a sigma model on the Kähler manifold SU(3, 1)/SU(3) × U(1). In order for K to be invariant (up to irrelevant Kähler transformations) under PSL(2, $\mathbb{Z}$) duality transformations (3.2), the fields ϕ_i have to transform with modular weight minus one,

$$\phi_i \to \frac{\lambda}{ict + d} \phi_i\,, \tag{3.7}$$

where λ is a field-independent phase. Thus, the fields ϕ_i do not have the correct modular transformation behaviour to be identified with the string fields $\tilde{\phi}_i$ discussed earlier. In addition, the above form of the effective action is not suitable for including gauge fields, since it is not manifestly gauge invariant. To achieve this, (3.3) must be accompanied by a holomorphic field redefinition of the ϕ_i*.

$$\tilde{\phi}_i(\phi_i, t, t_c) = \frac{\sqrt{t_c + \bar{t}_c}}{\bar{t}_c + t}\phi_i, \tag{3.8}$$

which transforms as

$$\tilde{\phi}_i \to \lambda \left[\frac{-ic\bar{t}_c + d}{ict_c + d} \right]^{1/2} \tilde{\phi}_i. \tag{3.9}$$

The Kähler potential then reads

$$\tilde{K} = -\log\left(1 - \left(|\tilde{t}|^2 + |\tilde{\phi}_1|^2 + |\tilde{\phi}_2|^2\right)\right), \tag{3.10}$$

up to a Kähler transformation. It is completely invariant under the duality transformation (3.4) and (3.9) which is equivalent to transforming t, ϕ_i and t_c simultaneously. Under usual PSL(2, $\mathbb{Z}$) transformations acting only on t and ϕ_i, it is invariant up to a Kähler transformation. The corresponding metric leads to a canonical kinetic term for $\tilde{t}$ near the critical point, where $\tilde{t}(t_c) = 0$. Note also that (3.10) is manifestly SU(3) × U(1) invariant. Both features are in accordance with the considerations of the last paragraph of sect. 2. Actually, linear SU(3) × U(1) invariance combined with the SU(1, 1)/U(1) coset structure implies as complete coset structure SU(1, 3)/SU(3) × U(1). This justifies (3.6) a posteriori.

Replacing $\tilde{t}$ and $\tilde{\phi}_i$ by the charge eigenstates s_α in eq. (2.8), the Kähler potential can easily be made gauge invariant by including gauge superfields V^i for the U(1) × U(1) subgroup of SU(3),

$$\tilde{K} = -\log\left[1 - \sum_{\alpha=1}^{3} \bar{s}_\alpha \exp\left(\sum_{i=1}^{2} Q_\alpha^i V^i\right) s_\alpha\right]. \tag{3.11}$$

In order for $\tilde{K}$ to transform properly under PSL(2, $\mathbb{Z}$), the vector fields have to be invariant under modular transformations. Note that under the modular transformation that fixes the given choice of t_c, the fields $\tilde{t}$, $\tilde{\phi}_i$ and thus s_α transform with a common $\mathbb{Z}_3$ phase that can be interpreted as a discrete gauge transformation. All this is in agreement with what we have discussed in sect. 2.

* To switch from the supergravity to the string basis, one has to perform a non-holomorphic field redefinition: $\tilde{\phi}_i^{\text{string}} = \phi_i/(t + \bar{t})^{1/2}$. $\tilde{\phi}_i^{\text{string}}$ then transforms under modular transformations as in (2.9) provided that one identifies the phases in (2.9) and (3.7).

The D-terms $D^i = \tilde{K}_\alpha Q^i_\alpha s_\alpha$, where $\tilde{K}_\alpha = \partial\tilde{K}/\partial s_\alpha = \bar{s}_\alpha/Y$ with $Y \equiv e^{-\tilde{K}}|_{V^i=0}$, are

$$D^1 = \frac{1}{\sqrt{2}Y}\left(2|s_1|^2 - |s_2|^2 - |s_3|^2\right),$$

$$D^2 = \sqrt{\frac{3}{2}}\,\frac{1}{Y}\left(|s_2|^2 - |s_3|^2\right). \tag{3.12}$$

Unbroken $N=1$ space-time supersymmetry thus implies that the vacuum expectation values of the three fields s_α are identical,

$$\langle s_1\rangle = \langle s_2\rangle = \langle s_3\rangle = s\,. \tag{3.13}$$

The undetermined parameter s describes a single flat direction of the D-term potential. This agrees with our conclusions in sect. 2 (cf. eq. (2.8)), if we make the following identification between field theory vev and string theory background fields:

$$s(R,B) = \frac{1}{\sqrt{3}}\tilde{t}\left(\langle t\rangle \equiv \sqrt{3}R^2 - 2iB, t_c\right). \tag{3.14}$$

We now study the Higgs mechanism within the effective field theory. The mass matrix for the two U(1) gauge bosons can be read off from the covariant kinetic energy terms for the scalars s_α: $m^2_{ij} = \tilde{K}_{\alpha\bar{\beta}} Q^i_\alpha Q^j_\beta s_\alpha \bar{s}_\beta$. Here, $\tilde{K}_{\alpha\bar{\beta}} = \partial^2\tilde{K}/\partial s_\alpha \partial\bar{s}_\beta$ is the Kähler metric on SU(3, 1)/SU(3) × U(1). Explicitly we obtain, using eqs. (3.13), (2.8) and (3.14),

$$m^2_{ij} = \frac{|\tilde{t}|^2}{1-|\tilde{t}|^2}\delta_{ij} = \frac{|t-t_c|^2}{(t_c+\bar{t}_c)(t+\bar{t})}\delta_{ij}\,, \tag{3.15}$$

in agreement with the modular invariant conformal field theory formula (2.10). It is interesting to observe that (2.10) anticipates, in a sense, the structure of the Kähler potential, as it involves a factor $1/Y \sim e^{\tilde{K}}$.

Note that so far we had to restrict the parameter range to some fundamental region (which depends on the particular choice of t_c in eq. (3.14)). Specifically, for $t_c = (-i)\rho$, the modulus must be restricted to $t \in (-i)\hat{\mathcal{F}}$, where*

$$\hat{\mathcal{F}} = \{\tau | \tau \in \mathbb{H}^+, -1 \leqslant \Re\tau < 0, \text{with } |\tau| > 1 \text{ for } -\tfrac{1}{2} \leqslant \Re\tau < 0,$$
$$\text{and } |\tau+1| \geqslant 1 \text{ for } -1 \leqslant \Re\tau < -\tfrac{1}{2}\}. \tag{3.16}$$

* This fundamental domain contains only one neighbourhood of a critical point, in contrast to the usual fundamental domain, which has two distinct regions that are arbitrarily close to some τ_c.

Otherwise, outside this region, our effective action would not be appropriate, as there exist then other fields with lower masses, which are not taken care of. On the other hand, certainly all backgrounds $t \in (-i)\mathbb{H}^+$ are allowed. We thus seek an effective action that is equivalent to (3.11) in each of the infinitely many modular copies of $\hat{\mathcal{F}}$ in $\mathbb{H}^+$. Effective actions that display such "periodic" behaviour by means of modular functions have been discussed in refs. [10, 16]. It turns out, however, that for our situation there does not exist an easy, smooth description in terms of modular functions. Rather, denoting eq. (3.11) (together with a choice of t_c in eq. (3.14)) by $\tilde{K}_{t_c}(t)$, a lagrangian that is duality invariant and appropriate for all $t \in (-i)\mathbb{H}^+$ can be obtained by defining patchwise

$$K(t) = \tilde{K}_{t_c = (-i)\gamma \cdot \rho}(t), \qquad \text{if } t \in (-i) \hat{\mathcal{F}}_\gamma, \tag{3.17}$$

where $\hat{\mathcal{F}}_\gamma$ denotes the copy of $\hat{\mathcal{F}}$ under $\gamma \in \mathrm{PSL}(2, \mathbb{Z})$. To give this an interpretation in familiar terms, let us fix $R^2 = \Re t/\sqrt{3} = 1/2$. Then the mass spectrum depends only on $B = -\frac{1}{2}\Im t$ and can be described by infinitely many shifted, intersecting parabolas $m_i^2(B) = \frac{1}{12}(1 + 6i + 4B)^2$, $i \in \mathbb{Z}$. This is much like free electron dispersion relations in a solid. Duality invariance under $\gamma = T$ corresponds to lattice periodicity and (3.16) to the first Brillouin zone. Our prescription (3.17) amounts to including only the lowest "energy band" in the effective theory. The field theory vacuum expectation value, s, becomes a "periodic" (but non-smooth) function of the string background, and can take values only in the image of $\hat{\mathcal{F}}$ under the map (3.14) [up to a phase (3.4) that is induced by modular transformations]. Though (3.17) is defined for all background values $t \in (-i)\mathbb{H}^+$, it does not give a good description at the boundary where any two "Brillouin zones" $\hat{\mathcal{F}}_\gamma, \hat{\mathcal{F}}_{\gamma'}$ meet, and level crossing occurs. At these lines, the particle spectrum is degenerate, and our description misses certain states*.

4. Heterotic compactification on the six-dimensional $\mathbb{Z}_3$-orbifold

The discussion of the previous sections generalizes in a straightforward way to the compactification of the heterotic string on the six-dimensional $\mathbb{Z}_3$-orbifold. This model is obtained as three copies of the two-dimensional $\mathbb{Z}_3$-orbifold where the $\mathbb{Z}_3$ twist acts simultaneously on the three complex coordinates $X_i, X_{\bar{i}} = \overline{X}_i$ $(i = 1, \ldots, 3)$ as $X_i \to e^{2\pi i/3} X_i$.

* Brief inspection suggests, however, that at the one-loop level there occur direct mixings between degenerate states. This can happen as the internal momenta are not conserved in the twisted sectors. Only certain $\mathbb{Z}_3$-invariant combinations of the momentum and winding quantum numbers are. The mixings correspond to D-terms in the effective theory. One therefore expects that, similar to the situation in a solid, the degeneracies at the boundaries of the "Brillouin zones" are lifted by loop effects, and disconnected bands appear. Thus, including loop corrections, it makes conceptually perfect sense to define an effective action that describes only the lowest, periodic and presumably smooth "band".

We will first focus on the nine (complex) marginal operators of the form $\bar{\tau}_{ij} = \bar{\partial} X_{i\mathrm{L}}\, \partial X_{j\mathrm{R}}$ with the nine corresponding moduli τ_{ij} describing the metric (three radii and six angles) and antisymmetric tensor degrees of freedom. The duality group that acts on the moduli τ_{ij} has been conjectured [16] to be $\mathrm{SU}(3,3;\mathbb{Z})$. For simplicity, we will set the off-diagonal moduli to particular, fixed values such that the three SU(3) root-lattices are orthogonal to each other. The moduli matrix then reduces to $\tau_{ij} = \tau_i \delta_{ij}$, with $\tau_i = 2B_i + i\sqrt{3}R_i^2$, and the duality group breaks to $\mathrm{SL}(2,\mathbb{Z})^3$.

The momentum and winding spectrum is characterized by three pairs of momentum and winding numbers $(n_1, n_2, m_1, m_2)_i$. In analogy with the two-dimensional case one now considers six vector bosons $V_{i\mu}$, $\bar{V}_{i\mu}$ and 18 scalars of the form

$$\bar{\phi}^1_{ij} = \frac{1}{\sqrt{3}}(V_{1,0,1,1} + \rho V_{-1,1,-1,0} + \bar{\rho} V_{0,-1,0,-1})_i(\partial X_{j\mathrm{R}} + \dots),$$

$$\bar{\phi}^2_{ij} = \frac{1}{\sqrt{3}}(V_{-1,0,-1,-1} + \rho V_{1,-1,1,0} + \bar{\rho} V_{0,1,0,1})_i(\partial X_{j\mathrm{R}} + \dots). \qquad (4.1)$$

The states with labels i are massless at the critical point $\tau_{i,c} + \rho$. Thus, the maximal gauge symmetry is $\mathrm{U}(1)^6$ (this is also the case for the complete theory including all nine moduli). Away from the critical points the masses of these states are

$$m_{ij}^2 = m_i^2 = \frac{i}{\sqrt{3}} \frac{|\tau_i - \rho|^2}{\tau_i - \bar{\tau}_i}. \qquad (4.2)$$

We now construct the effective low-energy action which describes the stringy Higgs effect for the six-dimensional orbifold compactification. The $N = 1$ supersymmetric action of the moduli fields $t_{ij} = -i\tau_{ij}$ is based on the $\mathrm{SU}(3,3)/\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{U}(1)$ sigma model [25,27] with Kähler potential

$$K = -\log \det(t_{ij} + \bar{t}_{ij}). \qquad (4.3)$$

We also want to include the 18 additional scalars ϕ^a_{ij} ($a = 1,2,\ i,j = 1,2,3$). As in sect. 3 one has to perform field redefinitions to switch to states with well-defined U(1) charges. There are in total 27 charge eigenstates s^α_{ij} that are built in analogy to (2.8). Their $(\mathrm{U}(1) \times \mathrm{U}(1))^3$ charges are, as in eq. (2.8), given by the roots $\boldsymbol{\alpha}$ of SU(3): $Q(s^\alpha_{ij}) \equiv Q_{\alpha,i} = \boldsymbol{\alpha}$. In terms of these fields and the six vector bosons V_m, the effective lagrangian is

$$\tilde{K} = -\log \det\left[\delta_{ij} - \sum_{\alpha,k=1}^{3} \bar{s}^\alpha_{ki} \exp(Q_{\alpha,k} \cdot V_k) s^\alpha_{kj}\right]. \qquad (4.4)$$

It represents a sigma model on the 27 complex dimensional Kähler manifold $SU(3,9)/SU(3)\times SU(9)\times U(1)$, where the $U(1)^6$ subgroup of $SU(3)^3 \subset SU(9)$ is gauged.

Studying the flat directions of the D-potential and introducing the appropriate vevs, the gauge boson masses are easily computed and found to be identical to the CFT result (4.2). Only six of the 27 bosons can get non-zero masses by the Higgs mechanism. They are given by $\phi^a_{i\bar{i}}$. The masses for the twelve remaining fields (besides the moduli), $\phi^a_{i\bar{j}}$ $(i \neq \bar{j})$, are provided by the superpotential (which is absent for the two-dimensional orbifold). In fact, the three-point couplings among vertex operators containing $\partial X_{\bar{i}\mathrm{R}}$ are only non-vanishing if all fields carry a different right-moving index $\bar{i}$. The field theory superpotential that reproduces the CFT three-point couplings is

$$W = \tfrac{1}{6}\epsilon_{\alpha\beta\gamma}\epsilon_{\bar{i}\bar{j}\bar{k}} \sum_{l=1}^{3} s^{\alpha}_{l\bar{i}} s^{\beta}_{l\bar{j}} s^{\gamma}_{l\bar{k}} \,. \tag{4.5}$$

$\tilde{K}$ and W are separately invariant under the transformations (3.4) and (3.9). The masses which are induced by this superpotential agree with eq. (4.2) if one redefines the matter fields such as to have canonical kinetic energies.

In summary, if the field $\tilde{t}_i$ gets a non-vanishing vev, four complex scalars $\tilde{\phi}^l_{i\bar{j}}$ $(l=1,2\;\; i\neq\bar{j})$ become massive due to the superpotential, whereas $\tilde{\phi}^l_{i\bar{i}}$ become massive due to the Higgs mechanism; all these masses are equal and agree with the string model. That six complex bosons become massive for each $U(1)^2$ gauge symmetry breaking reflects the fact that the untwisted sector can be obtained as a truncation of an $N=4$ supersymmetric theory.

In heterotic compactifications, there are also other marginal deformations besides the metric and antisymmetric tensor moduli*. These are the Wilson line backgrounds a. As we will show, in a similar way that the complex $t_{i\bar{j}}$ moduli are associated to vevs of the fields $s^{\alpha}_{i\bar{j}}$, the untwisted Wilson line backgrounds are associated with vevs of matter fields that transform as $(\underline{3},\underline{27})_{\bar{j}}$ under an $SU(3)\times E_6$ gauge group ($\bar{j}=1,2,3$ is a right-moving index labelling the SU(3) sublattices). In the following, we will discuss the stringy Higgs mechanism involving vevs of these fields. As we will show, the effective description is very similar to what we discussed in the previous sections. As a general conclusion, one finds that though at some multicritical point the maximal gauge symmetry of the $\mathbb{Z}_3$-orbifold is $SU(3)\times E_6\times U(1)^6$, the generic symmetry is rather just SU(3).

We consider first the remainder of the gauge boson spectrum of the heterotic $\mathbb{Z}_3$-orbifold. Before performing the $\mathbb{Z}_3$ twist, the charged gauge bosons correspond to 16-dimensional quantized momenta $P \in \Gamma_{E_8\times E_8}$ with $P^2=2$. One can easily

* We restrict ourselves to the untwisted sector; the moduli in the twisted sector are not moduli of the orbifold but rather of the underlying conformal field of which the orbifold is a critical point.

check that these gauge bosons necessarily have vanishing winding numbers n_k ($k = 1,\ldots,6$), although they may have non-vanishing quantized momenta m_k. The mass formula for these gauge bosons is given by

$$m^2 = \frac{p_R^2}{2} = \sum_{i=1}^{3} \frac{(m_1 - P_I a_1^I)_i^2 + (m_2 - P_I a_2^I)_i^2 - (m_1 - P_I a_1^I)_i (m_2 - P_I a_2^I)_i}{6R_i^2}. \tag{4.6}$$

The 96 Wilson line moduli a_k^I ($k = 1,\ldots,6$, $I = 1,\ldots,16$) correspond to marginal operators of the form $C_{Ik} = (\bar{\partial} X_I')_L (\partial X_k)_R$. Inspection of eq. (4.6) shows that among the infinitely many states characterized by the momentum numbers m_k, there are certain states that are massless if $\langle C_{Ik} \rangle = m_k - P_I a_k^I = 0$ (for given a_k^I). Thus, zero vevs of the fields C_{Ik} define infinitely many critical values for the Wilson line moduli. Generalized duality transformations [7, 8], which belong to $O(22,6;\mathbb{Z})$, act on the m_k and on the Wilson lines a_k^I such that the mass formula is invariant.

We now consider the effect of the $\mathbb{Z}_3$ twists. They act on the gauge degrees of freedom by an order-three Weyl rotation of the $E_8 \times E_8$ root lattice. (The standard embedding through shifts is only possible if the Wilson lines are quantized, i.e. at the critical points in moduli space where the theory possesses left-moving $n = 2$ world-sheet supersymmetry. Then one can perform a rebosonization procedure in analogy to the case discussed at the end of sect. 2.) This was studied in some detail in refs. [28, 29]. The gauge embedding of the twist is done by considering the first three SU(3) subgroups of E_8 and performing a simultaneous 120-degrees rotation of their weight lattices. Only linear combinations of states invariant under that rotation will survive in the spectrum. Then $\mathbb{Z}_3$-invariant gauge bosons will obtain as follows. The fourth SU(3) group inside E_8 which is left untouched by the rotation will of course remain unbroken. There will be six linear combinations of states of the form $\pm\sum_{i=0}^{2}(\sqrt{2}\rho^i, 0; 0; 0)$ and similar combinations with the entries in the second and third SU(3). $\pm\sqrt{2}\rho^i$, with $i = 0, 1, 2$, are (in complex notation) the root vectors of the SU(3) subgroups. These six linear combinations correspond to Cartan generators of the unbroken group. Finally there are linear combinations of the form $\sum_{i=0}^{2}(\rho^i d_a; \rho^i d_b; \rho^i d_c; 0)$, etc., where the $d_a = -i\sqrt{\frac{2}{3}}\rho^a$ ($a = 0, 1, 2$) are the weights of the $\underline{3}$ representation of SU(3). There are 9×8 invariant combinations of this type which, together with the $8 + 6$ generators above, correspond to the gauge bosons of $E_6 \times SU(3)$.

We turn to the $\mathbb{Z}_3$-invariant matter scalars. Only nine (complex) scalars from the original 96 oscillator states of the following form survive the $\mathbb{Z}_3$-twist,

$$C_{Ki} = (\bar{\partial} X_K')_L (\partial X_i)_R, \tag{4.7}$$

where the $\bar{\partial}X'_K$ ($K = 1,2,3$) are the (complex) Cartan subalgebra generators of the three SU(3)'s inside E_8 which are rotated by the twist. These are nine additional marginal operators of the $\mathbb{Z}_3$-orbifold CFT. The corresponding moduli are the Wilson lines a_i^K leading to four-dimensional scalars with arbitrary vev. Note that the C_{Ki} correspond to the singlet fields $\bar{\tau}_{ij}$, by replacing the torus coordinate $(\bar{\partial}X_i)_L$ by $(\bar{\partial}X'_K)_L$. In addition there are 234 matter scalars with vertex operators being linear combinations of $V = \exp(iP\cdot X'_L + ip_L\cdot X_L)\exp(ip_R\cdot X_R)\partial X_{iR}$. The linear combinations are characterized by the following combinations of lattice vectors P: $\pm\Sigma_{i=0}^{2}\rho^i(\sqrt{2}\rho^i;0;0;0)$ and similar combinations with the entries in the second and third SU(3). In addition, there are linear combinations of the form $\Sigma_{i=0}^{2}\rho^i(\rho^i d_a;\rho^i d_b;\rho^i d_c;0)$. Altogether, combining with the nine scalars C_{Ki} in eq. (4.7) there are 243 matter fields A_i that transform under the gauge group $E_6\times SU(3)$ as $(\underline{27},\underline{3})_i$.

Let us now discuss the gauge symmetry pattern in the presence of Wilson line background fields. To keep the discussion transparent, we consider first only one Wilson line along the first non-contractable loop of the torus, i.e., $C_{K1} = a_1^K P_K \neq 0$ (note that we are considering the sector with zero internal momentum, $m = 0$, and that P_K is a complexified entry of the lattice vector P, $P_K = (1/\sqrt{2})(P_{2K-1} + iP_{2K})$) and $C_{K2,3} = 0$. In addition, we restrict ourselves to Wilson lines with only non-vanishing contributions in the first SU(3) inside E_8, i.e. to symmetry breaking due to $C_{11} = P_1 a_1^1$. The gauge bosons of the fourth SU(3) subgroup will of course remain massless since the Wilson line does not touch them. For the same reason there are $9+9$ linear combinations $\pm\Sigma_{i=0}^{2}(0;\rho^i d_a;-\rho^i d_b;\rho^i d_c)$ of massless winding states. Finally, four of the six linear combinations involving SU(3) roots corresponding to the second and third SU(3) subgroups of E_8 will survive. Altogether there remain 30 generators which can be seen to correspond to an unbroken $SO(8)\times U(1)^2$ gauge group. In the same way one can check that the untwisted matter fields that stay massless transform as $(\underline{8}_v + \underline{8}_c + \underline{8}_s + \underline{1})_j$ under $SO(8)\times U(1)^2$. Thus, from $(\underline{3},\underline{27})_j$ only 25×3 states remain massless. Out of the 168 massive states, 56 states (with $K = j = 1$) correspond to the Goldstone superfields which are swallowed by the Higgs mechanism and fill up the coset $E_6\times SU(3)/SO(8)\times U(1)^2$. The remaining 112 states become massive in a way that can be described by a superpotential. This will be discussed below.

If additional Wilson lines are turned on, further symmetry breaking occurs. Consider, for instance, the more generic case $C_{Ki} \neq 0$, which means that the Wilson line in the Kth SU(3) inside E_8 along the ith non-contractable loop of the torus is non-vanishing. One can easily check that the unbroken gauge group will be just SU(3), since the fourth subgroup of E_8 is untouched by the Weyl rotation as well as the Wilson lines. We thus see how the gauge symmetry of the standard $\mathbb{Z}_3$-orbifold is generically just SU(3), and only at a multicritical point is it enlarged to $SU(3)\times E_6$ ($\times U(1)^6$ if there is further enhancement from the internal sector). The mass of the gauge bosons, respectively matter fields, due to non-vanishing

Wilson line a_i^K is given by (see eq. (4.6))

$$m_{Ki}^2 = |C_{Ki}|^2/6R_i^2 . \tag{4.8}$$

We like to give a field theoretical description of the stringy Higgs effect in the chiral matter sector. The Kähler potential involving the singlet moduli t_{ij} as well as the chiral matter fields $A_j \sim (\underline{27}, \underline{3})_j$ is given by a gauged sigma model based on the coset $\mathrm{SU}(3,3+3\times 27)/\mathrm{SU}(3)\times\mathrm{SU}(84)\times\mathrm{U}(1)$ [30, 31],

$$K = -\log\det\left[t_{ij} + \bar{t}_{ij} - \bar{A}_i \exp(V^{\mathrm{SU}(3)} + V^{\mathrm{E}_6})\, A_j\right] . \tag{4.9}$$

Note that in order to relate the supergravity matter fields A_i to the corresponding string variables, one has to perform a non-holomorphic field redefinition, $A_i^{\text{string}} = A_i/(t_i + \bar{t}_i)^{1/2}$. Note also that the E_6 gauge couplings are not compatible with the SU(84) symmetry. It is straightforward to determine the flat directions of the D-term potential. These are just given by the nine fields C_{Ki}. For only one non-vanishing vev the surviving gauge symmetry is $\mathrm{SO}(8)\times\mathrm{U}(1)^2$, just as we found using string arguments in the previous paragraph. Giving vevs to additional matter fields, the gauge symmetry breaks further to SU(3). The gauge boson masses that follow from (4.9) are given by

$$m_{Ki}^2 = \frac{|C_{Ki}|^2}{t_i + \bar{t}_i - |C_{Ki}|^2} . \tag{4.10}$$

This is not the same as the string theory formula (4.8). In fact, the effective field theory action reproduces the correct string theory mass formula only if we perform a further non-holomorphic field redefinition*,

$$t_i^{\text{string}} = t_i^{\text{SG}} - \tfrac{1}{2}\sum |A_i|^2 . \tag{4.11}$$

This shift is irrelevant for the matter fields that have vanishing vev. Note that the supergravity field t^{SG} now contains both types of moduli, $\{R_i, B_i\}$ and the Wilson lines, so the non-compact directions of the underlying Kähler space are shared by both types of marginal deformations. The necessity of this field redefinition can also be traced back to the behaviour of the backgrounds with respect to duality transformations. That is, the bilinear $a_n^K a_m^K$ transforms along with the $G+B$ background [7, 8] as follows:

$$S:\quad (G+B)_{nm} + \tfrac{1}{4}a_n^K a_m^K \to \left((G+B) + \tfrac{1}{4}a^K a^K\right)_{nm}^{-1} . \tag{4.12}$$

* This shift was already discussed in the early days of string compactification [24, 32].

The effective action of the untwisted matter sector also involves a superpotential. whose cubic part is

$$W = \tfrac{1}{6}\epsilon_{lmn}\epsilon_{\bar{i}\bar{j}\bar{k}} A_{l\bar{i}} A_{m\bar{j}} A_{n\bar{k}} \,. \tag{4.13}$$

It gives a mass to e.g. $C_{K\bar{i}}$, $\bar{i} = 2,3$ when $\langle C_{K\bar{1}} \rangle \neq 0$. That three (super)fields get the same mass due to the Higgs effect is again a consequence of the truncated $N = 4$ supersymmetry in the untwisted sector.

To summarize, we want to display the complete effective action of the $\mathbb{Z}_3$-orbifold that contains all untwisted fields of the theory. The Kähler potential describes a SU(3, 90)/SU(3) × SU(90) × U(1) sigma model, with additional gauge fields,

$$\tilde{K}\left(s_i, \tilde{A}_i, \bar{s}_i, \bar{\tilde{A}}_i\right) = -\log\det\left[\delta_{ij} - \bar{s}_i \exp\left(Q \cdot V^{\mathrm{U}(1)^6}\right) s_j - \bar{\tilde{A}}_i \exp\left(V^{\mathrm{SU}(3)} + V^{\mathrm{E}_6}\right) \tilde{A}_j\right]. \tag{4.14}$$

(To obtain this form, one has to redefine the matter fields A_j above. Also, we have suppressed all gauge indices.) The superpotential is given by the sum of (4.5) and (4.13). For generic vacuum expectation values, eq. (4.14) describes gauge symmetry breaking from $E_6 \times SU(3) \times U(1)^6$ down to SU(3).

5. Conclusions

The stringy Higgs mechanism that takes place in the $\mathbb{Z}_3$-orbifold compactification of the heterotic string can perfectly be described by a conventional, gauged supersymmetric sigma model. This form of the effective action reproduces various features of the underlying string theory. The appropriate Kähler manifold is larger than the usual manifold that describes the geometry of moduli space.

The symmetry breaking order parameters in the effective field theory correspond to marginal deformations of the underlying conformal field theory. The fields are related by certain non-linear transformations whose precise form is dictated by duality symmetries. This allows for an effective field theory description that contains only a finite number of fields, despite the fact that duality transformations mix infinitely many gauge and Higgs bosons in the string model. The price one has to pay in order to achieve this is that in the effective field theory the vacuum expectation values are not arbitrary, but are restricted to live in fundamental domains. In particular, s in eq. (3.13) can take only values in the image of $(-i)\hat{\mathscr{F}}$ under the map (3.14) (this applies to the other vacuum expectation values as well, but the precise forms of the corresponding fundamental domains are not explicitly known). Outside these domains, the effective action does not describe the string spectrum appropriately. This feature of a non-trivial global structure of parameter space is the main difference between the effective actions described here and conventional supersymmetric unified theories.

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