

## ON THE RELATION BETWEEN $d = 4$ AND $d = 11$ SUPERGRAVITY

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We investigate the supersymmetry transformation laws in an arbitrary compactification of  $d = 11$  supergravity to four dimensions. The  $d = 4$  fields of gauged  $N = 8$  supergravity are identified in a class of  $SO(7)^-$  invariant backgrounds. The two stationary points in these background configurations correspond to the round and parallelized  $S^7$ . We explicitly demonstrate that the latter coincides with the  $SO(7)^-$  stationary point of the  $N = 8$  supergravity potential. Chiral  $SU(8)$  is found to play a crucial role in establishing these results, we speculate on its possible relevance in the full  $d = 11$  theory.

### 1. Introduction

It is by now well established that compactification of  $d = 11$  supergravity [1] leads to effective  $d = 4$  theories with and without residual supersymmetries whose properties are to a large extent determined by the ground state solutions of the  $d = 11$  theory and their symmetries (for recent reviews of the subject, see refs. [2-4]). Much progress has been made in understanding and classifying the small fluctuations in the vicinity of the ground state solution. Such considerations are for instance sufficient to elucidate the structure of the (classical) mass spectrum which has now been calculated in several cases of interest [3, 5]. On the other hand, much less is known about the nonlinear structure of the effective  $d = 4$  theory and how this nonlinear structure emerges in the compactification. Even in the conventional Kaluza-Klein theories [6] this problem did not receive much attention. The most obvious candidate to study this is  $N = 8$  supergravity. The  $d = 11$  supergravity theory admits a classical solution with background metric  $(AdS)_4 \times S^7$ , full  $N = 8$  supersymmetry and  $SO(8)$  internal symmetry [7, 8], and the massless excitations are known to constitute a standard  $N = 8$  supermultiplet with maximum spin-2 [9, 8]. One may conjecture that the resulting effective  $d = 4$  theory corresponds to gauged  $N = 8$  supergravity [10] coupled to an infinite tower of massive  $N = 8$  multiplets, but there are several subtleties which make a straightforward demonstration of this fact

rather difficult. In fact, a recent analysis of  $G_2$  invariant solutions of  $d=11$  supergravity has led the authors of [11] to cast some doubt on such a relation. A more complete understanding of the nonlinear structure of the theory is therefore important and, in fact, indispensable to clarify the correspondence between the rich variety of stationary points of gauged  $N=8$  supergravity [12] and their  $d=11$  counterparts. In this paper, we study some of the nonlinear aspects of the  $S^7$  compactification with particular emphasis on the relation between the parallelized solution of the  $d=11$  theory [14] and the  $SO(7)^-$  stationary point of [12,13]; some of the results presented and elaborated here have already been announced in [4].

As we have mentioned above it is a first necessity to analyze the spectrum of small fluctuations about a given background. Hence one assumes that the  $d=11$  space-time is compactified according to

$$\mathcal{M}^{11} \rightarrow \mathcal{M}^4 \times \mathcal{M}^7. \quad (1.1)$$

The coordinates  $z^M$  are split accordingly into  $z^M \rightarrow (x^\mu, y^m)$ . Subsequently one expands the fields of  $d=11$  supergravity, collectively denoted by  $\phi(x, y)$ , in terms of a suitable set of eigenfunctions  $Y^{(n)}(y)$  of the relevant mass operator according to

$$\phi(x, y) = \sum_n \phi^{(n)}(x) Y^{(n)}(y), \quad (1.2)$$

and determines the eigenfunctions  $Y^{(n)}(y)$  that characterize the  $y$ -dependence of the fluctuations.

However, the analysis of small fluctuations is not of much use if one wants to understand the nonlinear structure of the compactification. The first complication is that the  $y$ -dependence of the modes in (1.2) is not free of ambiguity because the ansätze are subject to  $y$ -dependent gauge transformations. Spurious modes may be eliminated by imposing a gauge condition; while such a procedure is useful in determining gauge invariant quantities such as the mass spectrum, it is of little help in studying the effective  $d=4$  theory because most gauge conditions induce compensating supersymmetry transformations which are nonlocal in terms of the extra coordinates  $y^m$ . Secondly the proper identification of the  $d=4$  fields involves nonlinear modifications as has been pointed out in [13]. Instead of specifying the  $y$ -dependence of  $\phi(x, y)$ , one may be specifying the  $y$ -dependence of  $f(\phi(x, y))$  where  $f$  is some unknown function such that  $f(\phi(x, y)) \propto \phi(x, y)$  in the linear approximation. Therefore, from the results of a linearized analysis one cannot infer anything about the full  $y$ -dependence of the fields. There is no doubt that such complications do indeed play a role because even in the simplest case of the reduction on a 7-torus [15], nonlinear field redefinitions are required for a proper identification of the effective  $d=4$  fields. Also, the supersymmetry transformation parameter has to be redefined although these modifications disappear in the supersymmetric background.

The problem of nonlinear modifications becomes acute when one considers the truncation to the massless modes of the theory, and investigates their symmetry transformations. In the case of the 7-torus, the truncation to the massless modes was effected by imposing the restriction that all fields no longer depend on the extra coordinates. This leads to a truncation of the full theory which is obviously consistent in the sense that supersymmetry transformations preserve the independence of the extra coordinates and thus do not reintroduce massive (i.e.,  $y$ -dependent) modes which have been discarded in the truncation. For the  $S^7$  compactification, this is no longer the case [13]. If one inserts the linearized ansatz of [9, 8] into the  $d = 11$  transformation rules

$$\delta\phi(x, y) = F(\phi(x, y), \hat{\epsilon}(x, y)), \quad (1.3)$$

where  $\hat{\epsilon}(x, y)$  denotes the  $d = 11$  supersymmetry transformation parameter that leaves the background invariant, one finds that the  $y$ -dependence of the left- and right-hand sides of (1.3) does not match. This means that one cannot consistently put the massive modes equal to zero because these modes will reappear through the supersymmetry transformation (1.3). Therefore the multiplet structure of the fluctuations is not manifest in the transformation rules, and this is a major obstacle in relating the  $d = 4$  field theory to  $d = 11$  supergravity. In principle the problem of defining a consistent truncation also arises in standard Kaluza-Klein theories with respect to the bosonic symmetries [16].

To examine the nonlinear modifications in more detail, we therefore adopt a strategy in which the qualitative features of the  $d = 4$  transformation rules are used as input. After redefining fields according to the "standard" procedure [15], one finds that the fermionic transformation laws contain all the generic terms of the  $d = 4$  transformation rules. At this point, it is no longer possible to make arbitrary field redefinitions, but one must restrict oneself to redefinitions that take the form of field-dependent chiral  $SU(8)$  transformations. Subsequently one exploits these  $SU(8)$  redefinitions to bring the transformation rules in a form that is consistent upon a truncation to the massless sector. These redefinitions vanish in the supersymmetric background of the round sphere, but for finite deviations from that background the proper definitions of the fields thus involve a finite chiral rotation.

The fact that we have two solutions of  $d = 11$  supergravity that may have an interpretation in pure  $N = 8$  supergravity, namely the round and parallelized  $S^7$ , gives us a unique opportunity to test the viability of this strategy. Namely, we consider the transformation rules in an  $S^7$  background where the field strength with "internal" indices is proportional to a parallelizing torsion but with an arbitrary proportionality factor. After making an appropriate chiral  $SU(8)$  rotation, we then establish the consistency of the spin-2,  $-\frac{3}{2}$ , -1 and  $-\frac{1}{2}$  transformation rules in this background. We should emphasize at this point that it is by no means guaranteed that consistency can be achieved by a mere  $SU(8)$  redefinition, so this result must be

viewed as the first nontrivial indication that our approach is correct. The linearized approximation of our results is also consistent with the work of [17] where the redefinitions of the supersymmetry transformation parameter were determined to lowest nontrivial order by requiring the consistency of the truncation to that order.

Having identified the proper  $d = 4$  fields in the chosen background configuration, we may consider the transformation rules at the two points that characterize the round and parallelized solutions of  $d = 11$  supergravity, and try to compare them with the known  $d = 4$  solutions [12, 13]. Based on the linearized approximation of the fluctuations about the round  $S^7$  it has been argued that the round  $S^7$  compactification corresponds to the supersymmetric solution of gauged  $N = 8$  supergravity, whereas the parallelized  $S^7$  compactification should correspond to an  $SO(7)^-$  invariant solution of that theory in which the pseudoscalar fields acquire an  $SO(7)^-$  invariant expectation value [18, 9]. This is indeed confirmed by our results, because we find a complete numerical agreement between the  $d = 11$  transformation rules in the parallelized  $S^7$  background and the  $d = 4$  transformations at the  $SO(7)^-$  stationary point of the  $N = 8$  supergravity potential. Since the  $d = 11$  transformations contain the inverse  $S^7$  radius  $m_7$ , whereas the  $d = 4$  results are expressed in terms of the  $SO(8)$  gauge coupling constant  $g$ , we also find the relation between  $g$  and  $m_7$  at the round and parallelized spheres

$$\begin{aligned} g &= \sqrt{2} m_7, & (\text{round } S^7), \\ g &= 4 \times 5^{-3/4} m_7, & (\text{parallelized } S^7). \end{aligned} \tag{1.4}$$

Using this relation we can express the  $d = 4$  cosmological constant in terms of  $m_7$ ; one finds  $\Lambda = -12m_7^2$  and  $\Lambda = -10m_7^2$ , respectively, in agreement with the result of the  $d = 11$  field equations which were no input in this calculation.

The reader will notice that we do not analyze the scalars and pseudoscalars and their transformation rules in any detail here. The reason is that further complications appear in this sector as will be plausible from our discussion: a chiral  $SU(8)$  transformation mixes scalars (conventionally assumed to be contained in the 7-metric) and pseudoscalars (conventionally assumed to be contained in the three-index field). It therefore appears that in order to describe these fields in a unified way, one must transcend the geometrical framework of the  $S^7$  background.

This paper is organized as follows. In sect. 2 we evaluate the  $d = 11$  transformation rules in an arbitrary background of type (1.1). In sect. 3 these transformation rules are then determined in an  $SO(7)^-$  invariant background in such a way that the truncation to the massless  $N = 8$  supermultiplet is consistent. This yields the transformation rules of the  $N = 8$  supergravity fields in a background where the pseudoscalars acquire an  $SO(7)^-$  vacuum expectation value. Agreement between  $d = 11$  and  $d = 4$  is then established. In sect. 4 we discuss the implications of our results. Particular emphasis is given to the chiral  $SU(8)$  group which plays such a

crucial role in the definition of the  $d = 4$  fields. Some useful formulae are collected in an appendix.

## 2. Transformation laws

Our starting point are the transformation laws of  $d = 11$  supergravity, which we will analyze in this section in the context of an arbitrary ground state of type (1.1). We here follow the conventions and notation of [19] and refer the reader to that paper for more details. The transformation rules relevant for our discussion are those of the elfbein  $E_M^A$  and the  $d = 11$  gravitino  $\Psi_M$  which are given by

$$\delta E_M^A = \frac{1}{2} \bar{\epsilon} \Gamma^A \Psi_M, \quad (2.1)$$

$$\delta \Psi_M = D_M(\hat{\omega}) \epsilon + \frac{1}{288} \sqrt{2} (\Gamma_M^{ABCD} - 8 \delta_M^A \Gamma^{BCD}) \epsilon \hat{F}_{ABCD}. \quad (2.2)$$

For solutions describing spontaneous compactification to four dimensions, the  $d = 11$  indices are split into curved and flat  $d = 4$  indices  $\mu, \nu, \dots$  and  $\alpha, \beta, \dots$ , respectively, and curved and flat  $d = 7$  indices  $m, n, \dots$  and  $a, b, \dots$ , respectively. For example, the  $d = 11$  gravitino decomposes according to

$$\Psi_M(x, y) \rightarrow \begin{cases} \psi_\mu(x, y) \\ \psi_m(x, y). \end{cases} \quad (2.3)$$

For the elfbein, we make use of the local  $\text{SO}(1, 10)$  invariance of the  $d = 11$  theory to fix a gauge where [15]

$$E_M^A = \begin{bmatrix} e_\mu^\alpha & B_\mu^m e_m^a \\ 0 & e_m^a \end{bmatrix}. \quad (2.4)$$

In this gauge, the local  $\text{SO}(1, 10)$  invariance is reduced to local  $\text{SO}(1, 3) \times \text{SO}(7)$ . Moreover, compensating rotations are needed in the supersymmetric transformation laws (2.1) and (2.2) to maintain the gauge choice (2.4). In order to rewrite the theory within the  $d = 4$  context and to make contact with  $N = 8$  supergravity, we then redefine the fields in the standard way [15]. Here we briefly summarize the various steps that are required in this procedure. One first re-expresses (2.2) in terms of flat indices and redefines the spin- $\frac{3}{2}$  and spin- $\frac{1}{2}$  fields such as to eliminate an off-diagonal kinetic term mixing spin- $\frac{3}{2}$  and spin- $\frac{1}{2}$  fields. One then performs a Weyl rescaling to obtain the canonical  $d = 4$  Einstein action and kinetic terms for the spin- $\frac{3}{2}$  and spin- $\frac{1}{2}$  fields; there is also a corresponding redefinition of the supersymmetry parameter. Finally, a  $\gamma^5$ -redefinition of the fermionic fields is required for the conventional parity assignments. To keep the properties of the background as manifest as possible, it proves convenient to Weyl rescale the fields *not* with respect

to the full siebenbein determinant  $\det e_m^a$ ; rather, we write

$$e_m^a(x, y) = \hat{e}_m^b(y) S_b^a(x, y), \quad (2.5)$$

where  $\hat{e}_m^a(y)$  is an orthonormal frame on  $\mathfrak{N}^7$ , and define the Weyl rescaling with respect to the factor

$$\Delta(x, y) = \det S_a^b(x, y). \quad (2.6)$$

With this choice the relation between the gravitational coupling constants in  $d = 4$  and  $d = 11$  takes the form (see for instance [20])

$$\left(\frac{1}{\kappa^2}\right)_{d=4} = \left(\frac{1}{\kappa^2}\right)_{d=11} \int_{\mathfrak{N}^7} d^7y \det \hat{e}. \quad (2.7)$$

In the final expressions, all derivatives  $\partial_m$ , which act on internal coordinates, may be replaced by derivatives  $\hat{D}_m$  which are covariant with respect to the  $\mathfrak{N}^7$  background. After going through all these redefinitions, one obtains expressions which can be still further simplified by the use of chiral notation for the spinors; the analogy with the  $d = 4$  results of [10] then becomes even more suggestive. We will use the letters  $A, B, C, \dots$  to denote spin-7 indices (although these were previously used to denote tangent-space indices in the  $d = 11$  theory there should arise no confusion because the latter will not be needed in what follows). Subsequently the indices are promoted to chiral SU(8) indices by taking chiral projections. For the redefined gravitino field  $\psi'_\mu$ , these are introduced in such a manner that

$$\gamma^5 \psi'^A_\mu = \psi'^A_\mu, \quad \gamma^5 \psi'_{\mu A} = -\psi'_{\mu A}. \quad (2.8)$$

For the redefined spin- $\frac{1}{2}$  fields  $\psi'_{m A}$ , one first eliminates the  $\mathfrak{N}^7$  vector index  $m$  by switching to the combination  $\Gamma^m_{[AB} \psi_{mC]}$ , where  $\Gamma^m$  denotes the  $d = 7$   $\Gamma$ -matrices, which are discussed in the appendix. Subsequently one introduces a chiral SU(8) notation through

$$\begin{aligned} \chi^{ABC} &= (1 + \gamma^5) \frac{3}{4} i \sqrt{2} \Gamma^m_{[AB} \psi'_{mC]}, \\ \chi_{ABC} &= (1 - \gamma^5) \frac{3}{4} i \sqrt{2} \Gamma^m_{[AB} \psi'_{mC]}. \end{aligned} \quad (2.9)$$

Taking into account all that has been said so far one arrives at the following results after a somewhat tedious but otherwise straightforward calculation. Up to higher order fermionic terms, the gravitino transforms as

$$\begin{aligned} \delta \psi'^A_\mu &= \left\{ \partial_\mu - B_\mu^m \hat{D}_m - \frac{1}{4} \omega_\mu^{\alpha\beta} \gamma_{\alpha\beta} - \frac{1}{4} \gamma_\mu \gamma^\nu \hat{D}_m B_\nu^m \right\} \varepsilon'^A \\ &+ \frac{1}{2} \mathfrak{B}_\mu^A{}_B \varepsilon'^B + \gamma^{\alpha\beta} \gamma_\mu \mathfrak{G}_{\alpha\beta}{}^{AB} \varepsilon'_B + i \mathfrak{O}_1^{AB} \gamma_\mu \varepsilon'_B. \end{aligned} \quad (2.10)$$

Before explaining the various symbols in (2.10), we emphasize that we have not

made any assumption about the  $y$ -dependence so far; therefore (2.10) and similar formulae below are still completely general and independent of the specific compactification one may choose to consider. So,  $\mathring{D}_m$  denotes the derivative that is covariant with respect to the given background configuration; in sect. 3 we will take it the covariant derivative in the  $S^7$  background. The primes which appear on the right-hand side of (2.10) indicate that one is dealing with Weyl-rescaled quantities; for example,  $\omega'_\mu{}^{\alpha\beta}$  is the standard spin connection expressed in terms of the rescaled vierbein field  $e'^\mu{}_\alpha = \Delta^{1/2} e_\mu{}^\alpha$  with the modified derivative  $\partial_\mu - B_\mu{}^m \mathring{D}_m$ ; similarly  $\gamma_\mu = e'^\mu{}_\alpha \gamma_\alpha$ . More importantly, the tensor  $\mathfrak{B}_\mu{}^A{}_B$  takes values in the Lie algebra of  $SU(8)$ , and has all the characteristic features of a connection field associated with a local  $SU(8)$ . We will return to the possible implications of this fact at the end of this paper. The tensor  $\mathfrak{G}_{\alpha\beta}{}^{AB}$  is a complex tensor antisymmetric in  $[AB]$  and antisymmetric and antiselfdual in  $d = 4$  Lorentz indices  $[\alpha\beta]$ . Both  $\mathfrak{B}_\mu{}^A{}_B$  and  $\mathfrak{G}_{\alpha\beta}{}^{AB}$  have been calculated explicitly, but since they are not relevant in what follows here, we refrain from giving the corresponding expressions. The remaining quantity to be defined in (2.10) is  $\mathfrak{O}_1{}^{AB}$ ; this operator is given by

$$\begin{aligned} \mathfrak{O}_1{}^{AB} = & \Delta^{-1/2} \left\{ \frac{1}{2} \Gamma_{AB}^a S^{-1}{}^b{}_a (\mathring{D}_b - \frac{1}{4} \Delta^{-1} \mathring{D}_b \Delta) + \frac{1}{8} \Gamma_{AB}^{abc} S^{-1}{}^d{}_a (S^{-1} \mathring{D}_d S)_{bc} \right. \\ & \left. - \frac{1}{4} \Gamma_{AB}^a S^{-1}{}^b{}_c (S^{-1} \mathring{D}_c S)_a{}^b \right\} + \frac{1}{48} i \sqrt{2} \Gamma_{AB}^{abc} F_{abc} + \frac{1}{8} i \sqrt{2} f \delta_{AB}. \end{aligned} \quad (2.11)$$

The quantities  $f$  and  $F_{abc}$  are derived from the field strength  $F_{MNPQ}$  by

$$\begin{aligned} f = & -\frac{1}{24} i \Delta^{-1/2} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma\delta}, \\ F_{abc} = & \frac{1}{24} \eta' \Delta^{-1/2} \epsilon_{abc}{}^{defg} F_{defg}, \end{aligned} \quad (2.12)$$

where  $F_{\alpha\beta\gamma\delta}$  and  $F_{abcd}$  denote the field-strength components with  $d = 11$  tangent-space indices taking values in  $\mathfrak{N}^4$  and  $\mathfrak{N}^7$ , respectively. The factor  $\eta'$  is related to the convention for the  $d = 7$   $\Gamma$ -matrices and is defined in the appendix.

A similar compilation leads to the final result for the spin- $\frac{1}{2}$  fields  $\chi^{ABC}$

$$\delta \chi^{ABC} = 3\sqrt{2} \mathfrak{G}_{\alpha\beta}^{[AB} \gamma^{\alpha\beta} \epsilon'^{C]} + \frac{3}{4} \sqrt{2} \gamma^\mu \mathfrak{Q}_\mu^{ABCD} \epsilon'_D + \mathfrak{O}_2{}^{ABC}{}_D \epsilon'^D, \quad (2.13)$$

where we made use of the formulae listed in the appendix. The complex tensor  $\mathfrak{Q}_\mu{}^{ABCD}$  is selfdual in  $[ABCD]$  with respect to the 8-index Levi-Civita symbol; we again refrain from giving its explicit expression here. The operator  $\mathfrak{O}_2{}^{ABC}{}_D$  is given by

$$\begin{aligned} \mathfrak{O}_2{}^{ABC}{}_D = & 3\sqrt{\frac{1}{2}} \Delta^{-1/2} i \Gamma_{[AB}^a \delta_{C]D} S^{-1}{}^b{}_a (\mathring{D}_b - \frac{1}{4} \Delta^{-1} \mathring{D}_b \Delta) + \frac{3}{8} \sqrt{2} \Delta^{-1/2} i \Gamma_{[AB}^a \Gamma_{C]D}^{bc} \\ & \times (S^{-1}{}^d{}_c (S^{-1} \mathring{D}_d S)_{ba} + S^{-1}{}^d{}_a (S^{-1} \mathring{D}_d S)_{bc} + S^{-1}{}^d{}_c (S^{-1} \mathring{D}_d S)_{ab}) \\ & - \frac{1}{4} f \Gamma_{[AB}^a \Gamma_{C]D}^a + \frac{1}{24} \Gamma_{[AB}^a (2\Gamma_a{}^{bcd} - 3\delta_a^b \Gamma^{cd})_{C]D} F_{bcd}. \end{aligned} \quad (2.14)$$

We stress once more that the results (2.10) and (2.13) are valid irrespective of the

chosen background. Also we have not imposed any restrictions so far on the  $y$ -dependence of the fields and transformation parameters. Nevertheless the structure of (2.10) and (2.13) is strongly reminiscent of the transformation rules of gauged  $N = 8$  supergravity [10], with  $\Theta_1$  and  $\Theta_2$  playing the role of the  $SU(8)$  tensors  $A_1$  and  $A_2$ .

The spin-2 and spin-1 transformation rules do not require as much effort. However, whereas the compensating rotations are irrelevant for the fermions as long as one disregards higher-order fermionic terms, they must be taken into account for the bosonic transformation laws. One of them we have already mentioned: it is the ‘‘off-diagonal’’  $SO(1, 10)$  rotation required to maintain the special gauge (2.4). In addition, a compensating  $SO(1, 3)$  rotation is needed to cast the vierbein transformation law into the canonical form

$$\delta e'_\mu{}^\alpha = \frac{1}{2} \bar{\epsilon}'^A \gamma^\alpha \psi'_{\mu A} + \text{h.c.} \quad (2.15)$$

A little more work is required to determine the spin-1 transformation law in terms of chiral spinors; it reads

$$\delta B_\mu{}^m = \frac{1}{8} \sqrt{2} i \Gamma^m{}_{AB} \Delta^{-1/2} \left( 2\sqrt{2} \bar{\epsilon}'^A \psi'^B{}_\mu + \bar{\epsilon}'_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.} \quad (2.16)$$

Hence we have now given the transformation rules for the spin-2,  $-\frac{3}{2}$ , -1 and  $-\frac{1}{2}$  fields. In this we were guided by the form that these transformations take in gauged  $n = 8$  supergravity. The observation which will be crucial for what follows is that the field redefinitions that preserve the structure of these transformation rules must take the form of a (field-dependent) chiral  $SU(8)$  transformation. These transformations are naturally defined on the chiral spinors  $\psi_\mu^A$  and  $\chi^{ABC}$

$$\begin{aligned} \psi_\mu^A(x, y) &\rightarrow U^A{}_B(x, y) \psi_\mu^B(x, y), \\ \chi^{ABC}(x, y) &\rightarrow U^A{}_D(x, y) U^B{}_E(x, y) U^C{}_F(x, y) \chi^{DEF}(x, y). \end{aligned} \quad (2.17)$$

In the next section we will see that such an  $SU(8)$  redefinition is indeed important in order to extract the correct  $d = 4$  fields. Of course, in that case we will choose a specific background related to the sphere  $S^7$ . It is intriguing that the  $SU(8)$  structure of the transformation rules (2.10) and (2.13) also persists in the context of more general backgrounds. We will return to the possible implications of this fact in sect. 4.

### 3. Consistent supersymmetry in an $SO(7)^-$ invariant background

In this section we study the supersymmetry transformations (2.10), (2.13), (2.15) and (2.16) in an  $S^7$  background where

$$e_m{}^a(x, y) = \hat{e}_m{}^a(y) \quad (3.1)$$

is the (globally defined) siebenbein of  $S^7$ , and the field strength  $F_{mnp}(x, y)$  is

proportional to one of the Cartan-Schouten [21] torsions  $S_{mnp}$ , which satisfies

$$\dot{D}_m S_{npq} = \frac{1}{6} m_\gamma \eta' \epsilon_{mnpqrst} S^{rst}, \quad (3.2)$$

as well as [9, 13]

$$S^{[mnp]S^q]rs} = -\frac{1}{4} \eta' \epsilon^{mnpq[r} S^{s]tu}, \quad (3.3)$$

$$S^{mnp} S_{qrp} = 2\delta^{mn}_{qr} - \frac{1}{6} \eta' \epsilon^{mn}_{qrstu} S^{stu}, \quad (3.4)$$

where  $m_\gamma$  is inversely proportional to the  $S^7$  radius. If  $F_{mnp} = 0$ , then we are dealing with the round sphere, whereas  $F_{mnp} = \pm\sqrt{2} m_\gamma S_{mnp}$  corresponds to the parallelized sphere, which is a solution of  $d = 11$  supergravity [14]. Hence the class of background configurations that we consider contains two solutions of  $d = 11$  supergravity. Furthermore it is well known that these configurations are  $SO(7)^-$  invariant [13, 22], and the results of this section show that they are clearly related to an  $SO(7)^-$  invariant background in gauged  $N = 8$  supergravity.

Since we assume that the background is also AdS invariant we may drop the  $x$ -dependence and concentrate on the transformation rules

$$\langle \delta \psi_\mu^A(x, y) \rangle = \gamma_\mu \left( \frac{1}{2} \iota \Gamma^a \dot{D}_a - \frac{1}{48} \sqrt{2} \Gamma^{abc} F_{abc} - \frac{1}{8} \sqrt{2} f \right)^{AB} \epsilon_B(x, y), \quad (3.5)$$

$$\begin{aligned} \langle \delta \chi^{ABC}(x, y) \rangle = & \left\{ 3\sqrt{\frac{1}{2}} \iota \Gamma^a_{[AB} \delta_{C]D} \dot{D}_a - \frac{1}{4} f \Gamma^a_{[AB} \Gamma^a_{C]D} \right. \\ & \left. + \frac{1}{24} \Gamma^a_{[AB} (2\Gamma_a^{bcd} - 3\delta_a^b \Gamma^{cd})_{C]D} F_{bcd} \right\} \epsilon^D(x, y), \end{aligned} \quad (3.6)$$

$$\langle \delta e_\mu^\alpha(x, y) \rangle = \frac{1}{2} \bar{\epsilon}_A(x, y) \gamma^\alpha \psi_\mu^A + \text{h.c.}, \quad (3.7)$$

$$\langle \delta B_\mu^m(x, y) \rangle = \frac{1}{8} \sqrt{2} \iota \Gamma_{AB}^m (2\sqrt{2} \bar{\epsilon}^A \psi_\mu^B + \bar{\epsilon}_C \gamma_\mu \chi^{ABC}) + \text{h.c.}, \quad (3.8)$$

where we have dropped the primes.

For small deviations from the round  $S^7$  background ( $F_{mnp} = 0$ ) the small fluctuations can be expressed in terms of products of eight Killing spinors  $\eta^I(y)$  satisfying

$$\dot{D}_m \eta^I = -\frac{1}{2} \iota m_\gamma \Gamma_m \eta^I, \quad (3.9)$$

where  $m_\gamma$  is equal to

$$m_\gamma = \frac{1}{6} \sqrt{2} f. \quad (3.10)$$

Ultimately we want to compare our results to  $N = 8$  supergravity, so that we will try

to preserve the  $y$ -dependence of the  $d = 4$  fields irrespective of the value of  $F_{mnp}$  in the chosen background. This forces us to keep  $m_7$  constant, so that the  $S^7$  radius is fixed. On the other hand the Freund-Rubin parameter  $f$  may then change under variations of  $F_{mnp}$ , so that the relation (3.10) is only valid for the *round* sphere where  $F_{mnp} = 0$ .

Only the round  $S^7$  background is fully supersymmetric as can be verified from (3.5)–(3.8), since all the supersymmetry variations vanish in the background if  $F_{mnp} = 0$  and (3.10) holds, provided that the 8 independent supersymmetries are characterized by the  $S^7$  Killing spinors (3.9). Hence in chiral notation

$$\begin{aligned}\hat{\epsilon}^A(x, y) &= \epsilon^I(x) \eta_A^I(y), \\ \hat{\epsilon}_A(x, y) &= \epsilon_I(x) \eta_A^I(y),\end{aligned}\tag{3.11}$$

where  $\epsilon^A$  ( $\epsilon_A$ ) and  $\epsilon^I$  ( $\epsilon_I$ ) denote the positive (negative) chirality components. The massless gravitino field associated with these supersymmetries is contained in  $\psi_\mu(x, y)$  and must have the same  $y$ -dependence as the corresponding supersymmetry parameters  $\hat{\epsilon}_A(x, y)$ . Furthermore it has been shown that small fluctuations in  $F_{mnp}$  proportional to the Cartan-Schouten torsion, i.e. within the class of background configurations that we consider, are also contained in the massless  $N = 8$  supermultiplet [8, 9]. Nevertheless, if these fluctuations are inserted in the right-hand side of (3.5) with  $\epsilon_A = \hat{\epsilon}_A$ , then  $\langle \delta\psi_\mu \rangle$  does not satisfy the Killing condition (3.9). This indicates that the massive modes transform into the massless modes, so that the first ones cannot be put to zero in a consistent fashion, since they will reappear through the supersymmetry transformations. However, this result is clearly unacceptable because the small fluctuations about a background must always transform among each other under the isometries of that background, and can thus be classified according to irreducible representations of the isometry group.

In the introduction we have already outlined how one may attempt to make the transformation rules consistent upon truncation to the massless sector, by introducing redefinitions of the fields and the transformation parameters. Since the transformation rules (2.10), (2.13), (2.15) and (2.16) were already in qualitative agreement with those of  $N = 8$  supergravity, one must restrict oneself to redefinitions that take the form of a field-dependent chiral  $SU(8)$  transformation. Since  $S_{mnp}$  is the only quantity in the backgrounds considered here from which such a chiral  $SU(8)$  transformation can be constructed we start from

$$U = \exp\left(\frac{1}{6}\tau S^{mnp}(y) \Gamma_{mnp} \gamma_5\right).$$

Since we use chiral components throughout we may drop the  $\gamma_5$  here and consider the  $y$ -dependent  $SU(8)$  transformation

$$U(\tau) = \exp\left(\frac{1}{6}\tau S^{mnp}(y) \Gamma_{mnp}\right).\tag{3.12}$$

This  $8 \times 8$  matrix, which is an element of the  $SU(8)/SO(8)$  coset space, can be

calculated by using (3.4), and one finds

$$U(\tau) = \frac{1}{8}(e^{-7i\tau} + 7e^{i\tau}) + \frac{1}{48}i(e^{-7i\tau} - e^{i\tau})S^{mnp}(y)\Gamma_{mnp}. \quad (3.13)$$

Using (3.2) one may also verify the useful identity

$$\dot{D}_m U(\tau) = \frac{1}{2}im_\gamma[\Gamma_m, U(\tau)]. \quad (3.14)$$

We now assume that for nonvanishing  $F_{mnp} \propto S_{mnp}$  the supersymmetry parameters of the background must be modified by the SU(8) transformation (3.12), where  $\tau$  is related to the proportionality factor between  $F_{mnp}$  and  $S_{mnp}$ . To preserve the qualitative features of the transformation rules this SU(8) transformation must act *uniformly* on all fields. Hence after redefining  $\psi_\mu \rightarrow U\psi_\mu$  and  $\varepsilon \rightarrow U\varepsilon$ , the transformation for the redefined field takes the form

$$\langle \delta\psi_\mu^A(x, y) \rangle = \gamma_\mu \left\{ U(-\tau) \left( \frac{1}{2}i\Gamma^a \dot{D}_a - \frac{1}{48}\sqrt{2}\Gamma^{abc}F_{abc} - \frac{1}{8}\sqrt{2}f \right) U(-\tau) \right\}^{AB} \hat{e}_B(x, y), \quad (3.15)$$

for the positive chirality component. The next step is to investigate whether this redefinition can now be used to remove the inconsistent term in  $\langle \delta\psi_\mu \rangle$  proportional to  $F_{mnp}$ . This turns out to be possible provided we choose

$$F_{abc} = 2\sqrt{2}m_\gamma \text{tg}(4\tau)S_{abc}, \quad (3.16)$$

$$f = \sqrt{2}m_\gamma(3 - 4\text{tg}^2 4\tau). \quad (3.17)$$

The variation (3.15) now takes the form

$$\langle \delta\psi_\mu^A(x, y) \rangle = \gamma_\mu m_\gamma \left\{ \frac{\cos^3 2\tau}{\cos^2 4\tau} (2\cos^2 4\tau + 2 - 3\cos 4\tau) - i \frac{\sin^3 2\tau}{\cos^2 4\tau} (2\cos^2 4\tau + 2 + 3\cos 4\tau) \right\} \hat{e}_A(x, y), \quad (3.18)$$

which is obviously consistent. It is gratifying that our strategy of employing only SU(8) redefinitions does indeed allow us to achieve consistency. A priori these redefinitions could also have taken the form of an Sl(8) transformation.

At this point one may wonder what would be required in case the siebenbein would also deviate from the round  $S^7$  background. This is much more complicated to analyze in general, but for small deviations it becomes rather straightforward. Upon inspection of the full transformation rule one discovers the need for an SU(8) transformation

$$U = \exp(i\xi_m \Gamma^m), \quad (3.19)$$

where at the linearized level  $\xi_m$  can be expressed in terms of the  $S^7$  derivative of the

massless fluctuations contained in the siebenbein, which is proportional to  $\bar{\eta}^{[I}\Gamma_{mn}\eta^J\bar{\eta}^K\Gamma^n\eta^{L]}$ . The redefinition of  $\dot{\varepsilon}(x, y)$  according to (3.19) now gives rise to one of the terms which were found in [17]. Combining this term with the linearized result arising from (3.12) and with the linearized effect of the Weyl rescaling of  $\dot{\varepsilon}(x, y)$  one finds all the terms of [17].

Let us now return to the previous background and continue the analysis of the transformation laws. Note that all relevant quantities have already been determined in terms of the background parameter  $\tau$ , so that the remaining calculations serve as a consistency check. It is clear that the SU(8) redefinitions do not affect the vierbein variation (3.7), so let us turn to the supersymmetry variation of the spin-1 fields. Applying the same SU(8) transformation to all the spinor quantities on the right-hand side of (3.8) yields

$$\begin{aligned} \langle \delta B_\mu{}^m(x, y) \rangle &= \frac{1}{8} i \sqrt{2} (U^T(\tau) \Gamma^m U(\tau))_{AB} (2\sqrt{2} \dot{\varepsilon}^A \psi_\mu^B + \dot{\varepsilon}_C \gamma_\mu \chi^{ABC}) \\ &\quad + \frac{1}{8} \sqrt{2} (U^T(-\tau) \Gamma^m U(-\tau))_{AB} (2\sqrt{2} \dot{\varepsilon}_A \psi_{\mu B} + \dot{\varepsilon}^C \gamma_\mu \chi_{ABC}), \end{aligned} \quad (3.20)$$

which can be written as

$$\begin{aligned} \langle \delta B_\mu{}^m(x, y) \rangle &= \frac{1}{16} \sqrt{2} e^{-2i\tau} \{ (2 \cos 4\tau + i \sin 4\tau) i \Gamma^m{}_{AB} + i \sin(4\tau) S^m{}_{np} \Gamma^p{}_{AB} \} \\ &\quad \times (2\sqrt{2} \dot{\varepsilon}^A \psi_\mu^B + \dot{\varepsilon}_C \gamma_\mu \chi^{ABC}) + \text{h.c.} \end{aligned} \quad (3.21)$$

As follows from the results presented in the appendix (3.21) is indeed consistent upon truncation to the massless sector, because if one takes  $\varepsilon$  and  $\psi_\mu$  proportional to a Killing spinor, and  $\chi$  proportional to an antisymmetric product of three Killing spinors, then the right-hand side of (3.21) is precisely proportional to the Killing vectors

$$\xi_m^{IJ}(y) = i \bar{\eta}^I(y) \Gamma_m \eta^J(y). \quad (3.22)$$

This  $y$ -dependence thus coincides with that of the massless spin-1 fluctuations contained in  $B_\mu{}^m$ .

It remains to evaluate (3.6) in this background. A straightforward but somewhat tedious calculation leads to

$$\begin{aligned} \langle \delta \chi^{ABC}(x, y) \rangle &= \left\{ 3\sqrt{\frac{1}{2}} i (U(-\tau) \Gamma^a U^T(-\tau))_{[AB} \delta_{C]D} \dot{D}_a \right. \\ &\quad - \frac{1}{4} f (U(-\tau) \Gamma^a U^T(-\tau))_{[AB} (U(-\tau) \Gamma^a U(\tau))_{C]D} \\ &\quad \left. + \frac{1}{24} (U(-\tau) \Gamma^a U^T(-\tau))_{[AB} (U(-\tau) (2\Gamma_a{}^{bcd} - 3\delta_a^b \Gamma^{cd}) U(\tau))_{C]D} F_{bcd} \right\} \dot{\varepsilon}^D \\ &= -\frac{1}{4} \sqrt{2} m_\gamma \Gamma_{[AB}^m \Gamma_{CD]}^n S_{mnp} \dot{\varepsilon}^D \\ &\quad \times e^{2i\tau} \text{tg } 4\tau (\cos 4\tau + 2 \sin 4\tau \text{tg } 4\tau + i \sin 4\tau). \end{aligned} \quad (3.23)$$

Let us now consider these transformation rules in the truncation to the massless

sector. To that order one specifies the  $y$ -dependence of the fields according to

$$\begin{aligned}\psi_\mu^A(x, y) &= \psi_\mu^I(x) \eta_A^I(y), \\ \chi^{ABC}(x, y) &= \chi^{IJK}(x) \eta_A^I(y) \eta_B^J(y) \eta_C^K(y), \\ B_\mu^m(x, y) &= B_\mu^{IJ}(x) i \bar{\eta}^I(y) \Gamma^m \eta^J(y), \\ e_\mu^\alpha(x, y) &= e_\mu^\alpha(x).\end{aligned}\tag{3.24}$$

Substituting the ansätze (3.11) and (3.23) in the right-hand side of the supersymmetry variations, one finds that the variations are again consistent with (3.23). The symmetry transformations for the  $d = 4$  fields corresponding to the massless sector can then easily be extracted and read

$$\delta e_\mu^\alpha = \frac{1}{2} \bar{\epsilon}^I \gamma^\alpha \psi_{\mu I} + \text{h.c.},\tag{3.25}$$

$$\begin{aligned}\delta \psi_\mu^I &= \gamma_\mu m_7 \left\{ \frac{\cos^3 2\tau}{\cos^2 4\tau} (2 \cos^2 4\tau + 2 - 3 \cos 4\tau) \right. \\ &\quad \left. - i \frac{\sin^3 2\tau}{\cos^2 4\tau} (2 \cos^2 4\tau + 2 + 3 \cos 4\tau) \right\} \epsilon_I,\end{aligned}\tag{3.26}$$

$$\begin{aligned}\delta B_\mu^{IJ} &= \frac{1}{16} \sqrt{2} e^{-2i\tau} \left\{ (2 \cos 4\tau + i \sin 4\tau) \delta^{IJ}_{KL} + \frac{1}{2} i \sin 4\tau C^{IJ}_{KL} \right\} \\ &\quad \times \left( 2\sqrt{2} \bar{\epsilon}^K \psi_\mu^L + \bar{\epsilon}_M \gamma_\mu \chi^{KLM} \right) + \text{h.c.},\end{aligned}\tag{3.27}$$

$$\delta \chi^{IJK} = -\frac{1}{2} \sqrt{2} m_7 C^{IJKL} \epsilon^L e^{2i\tau} \text{tg } 4\tau (\sin 4\tau - i(\cos 4\tau + 2 \sin 4\tau \text{tg } 4\tau)).\tag{3.28}$$

The antisymmetric selfdual tensor  $C^{IJKL}$ , which is introduced in the appendix, coincides precisely with the tensor that has been used to study two solutions of gauged  $N = 8$  supergravity; in these solutions the scalar or pseudoscalar fields have vacuum expectation values proportional to this tensor [13]. One of the important properties of  $C^{IJKL}$  is

$$C^{IJKP} C_{LMNP} = 6 \delta_{LMN}^{IJK} + 9 \delta \{ {}^I_L C^{JK} \}_{MN}.\tag{3.29}$$

We may now consider (3.25)–(3.28) in the background of the round and parallelized sphere, corresponding to  $F_{mnp} = 0$  and  $F_{mnp} = \pm \sqrt{2} m_7 S_{mnp}$ , respectively. This corresponds to taking  $\tau = 0$  (round sphere) and  $\text{tg } 4\tau = \pm \frac{1}{2}$  (parallelized sphere).

For the round sphere, the only nontrivial result is

$$\delta\psi_\mu{}^I = m_7 \gamma_\mu \epsilon_I. \quad (3.30)$$

For the parallelized sphere we quote (tg  $4\tau = +\frac{1}{2}$ )

$$\delta\psi_\mu{}^I = \gamma_\mu m_7 \frac{3}{8} 5^{-3/4} (1 + \sqrt{5})^{1/2} (3 + \sqrt{5} + 2i(2 - \sqrt{5})) \epsilon_I, \quad (3.31)$$

$$\begin{aligned} \delta B_\mu{}^{IJ} = & \frac{1}{64} \sqrt{2} 5^{-3/4} (1 + \sqrt{5})^{3/2} \{ ((2 + \sqrt{5}) + i(9 - 4\sqrt{5})) \delta^{IJ}{}_{KL} \\ & + (-1 + \frac{1}{2}\sqrt{5} + \frac{1}{2}i) C^{IJ}{}_{KL} \} (2\sqrt{2} \bar{\epsilon}^K \psi_\mu{}^L + \bar{\epsilon}_M \gamma_\mu \chi^{KLM}) + \text{h.c.}, \quad (3.32) \end{aligned}$$

$$\delta\chi^{IJK} = \frac{1}{8} \sqrt{2} m_7 5^{-3/4} (1 + \sqrt{5})^{1/2} (\sqrt{5} - 5 + 2i\sqrt{5}) C^{IJKL} \epsilon^L. \quad (3.33)$$

This may now be compared directly to the explicit solution of  $N = 8$  supergravity where the pseudoscalars have an  $\text{SO}(7)^-$ -invariant vacuum expectation value. For zero expectation value (3.30) follows, provided the  $\text{SO}(8)$  gauge coupling constant  $g$  is chosen according to

$$|g| = \sqrt{2} |m_7|. \quad (3.34)$$

For non-zero vacuum expectation value all results coincide with (3.31) (3.33) after adjusting for the different normalizations used in [13] provided that

$$|g| = 4 \times 5^{-3/4} |m_7|. \quad (3.35)$$

Inserting (3.34) and (3.35) into the corresponding expressions for the cosmological constant at these  $d = 4$  stationary points [12, 13] gives

$$\begin{aligned} \Lambda &= -6g^2 = -12m_7^2, \\ \Lambda &= -\frac{25}{8}\sqrt{5}g^2 = -10m_7^2, \end{aligned} \quad (3.36)$$

which coincides precisely with the values that follow from the  $d = 11$  field equations for the round and parallelized  $S^7$  solutions. This fully confirms that we have indeed succeeded in identifying the proper  $d = 4$  fields of  $N = 8$  supergravity directly from the  $d = 11$  theory, at least in the  $\text{SO}(7)^-$  invariant background. The manipulations that were required in order to obtain these results are highly nontrivial, so the conclusion that  $d = 11$  supergravity on the round  $S^7$  corresponds to gauged  $N = 8$  supergravity coupled to an infinite tower of massive supermultiplets seems hard to avoid.

#### 4. Conclusions and outlook

In this paper we have evaluated the transformation laws of  $d = 11$  supergravity in an arbitrary background that arises through the spontaneous compactification of this theory to four dimensions. These transformation rules have then been analyzed in an  $SO(7)^-$  invariant class of background configurations, and by requiring consistency in the truncation to the massless sector, we have identified the  $d = 4$  fields of gauged  $N = 8$  supergravity. Furthermore there is complete numerical agreement between the two stationary points in this background corresponding to the round and the parallelized  $S^7$  solutions and the  $SO(8)$  and  $SO(7)^-$  stationary points of the  $N = 8$  supergravity potential. As a consistency check we have determined the cosmological constants for these solutions in terms of the inverse  $S^7$  radius on the basis of the  $d = 4$  theory and found the correct values  $\Lambda = -12m_7^2$  and  $\Lambda = -10m_7^2$  that are known from the  $d = 11$  solutions. Note that this result is based on the comparison between the  $d = 11$  and  $d = 4$  transformation rules, and not as in [23] on knowledge of many of the massless ansätze and the  $d = 11$  lagrangian.

We believe that our results constitute a proof that the parallelized solution of [14] indeed corresponds to the  $SO(7)^-$  invariant stationary point of the  $N = 8$  potential identified in [12, 13]. Previously this equivalence had been conjectured on the basis of the observation that the massless pseudoscalar fluctuations have the same  $y$ -dependence as the parallelizing torsion [18, 9]. However, this argument is not completely rigorous because it is based on the analysis of small fluctuations only. Now we understand that it is misleading, if not fallacious, for the following reason. We have demonstrated in this paper that in order to achieve consistency the proper identification of the  $d = 4$  fields of  $N = 8$  supergravity involves a field-dependent chiral  $SU(8)$  rotation. Such an  $SU(8)$  rotation inevitably mixes scalars and pseudoscalars such that the identification of the scalars with the zero-mass fluctuations of the siebenbein and of the pseudoscalars with the zero-mass fluctuations of the field strength  $F_{mnp}$  is only valid in an infinitesimal neighbourhood of the round  $S^7$  background, and fails for any *finite* value of the (pseudo) scalar fields. To see this more explicitly, we have also studied the transformation rules in a background where  $F_{mnp}$  is no longer proportional to a Cartan-Schouten torsion, and satisfies

$$\dot{D}_m F_{npq} = \frac{1}{6} m_7 \eta' \epsilon_{mnpqrst} F^{rst} \tag{4.1}$$

without further restrictions. Based on the knowledge of the small fluctuations, this background would be viewed as one in which the pseudoscalar fields of  $N = 8$  supergravity have acquired a vacuum expectation value. However, this conclusion must be false. We have attempted to achieve consistency by means of chiral  $SU(8)$  rotations in this background; eq. (A.9) in the appendix gives the relevant  $SU(8)$  transformation for this case. We have found that consistency can only be achieved if also the siebenbein deviates from the round  $S^7$  background. Therefore the back-

ground (4.1) with the standard  $S^7$  metric cannot have an interpretation within the context of *pure*  $N = 8$  supergravity, and must be interpreted as arising from the vacuum expectation values of both massless and *massive* scalar and pseudoscalar fields. It is only for the  $SO(7)^-$  invariant background where  $F_{mnp}$  is proportional to a Cartan-Schouten torsion that the scalar-pseudoscalar mixing does not play a role, because the  $SO(7)^-$  stability group of this background forbids a vacuum expectation value for the scalar fields. This observation thus explains why the attempts of this paper were successful.

Our results strongly indicate, although by no means prove, that gauged  $N = 8$  supergravity is indeed the effective  $d = 4$  theory that is obtained by compactification of  $d = 11$  supergravity on  $S^7$  and subsequent truncation to the zero-mass  $N = 8$  supermultiplet, if nonlinear modifications are properly taken into account. This seems to contradict the conclusions of [11], but we emphasize once more that in view of the scalar-pseudoscalar mixing the purely geometrical framework adopted in [11] is presumably inadequate for a unified description of the scalars and pseudoscalars. To see how this could be relevant we recall that the scalar fields in the ungauged theory arise from antisymmetric tensors by duality transformations. However, in the gauged version these scalars may acquire vacuum expectation values. In that case there is no simple local relation between these two descriptions just in the same way as there is no simple relation between electric and magnetic phases in an ordinary gauge theory. This line of argument indicates that the stationary points of the  $N = 8$  potential [12], for which so far no  $d = 11$  counterparts have been found, may in fact correspond to nonlocal solutions of the  $d = 11$  theory. Although such solutions would still describe compactification to four dimensions, they would not be of the conventional Freund-Rubin type [7], but rather resemble magnetic monopole configurations (yet dissimilar from the “black hole” solutions considered in [24]).

Another intriguing aspect of our findings is that an  $SU(8)$  structure naturally emerges for the full  $d = 11$  supergravity theory. Gauged  $N = 8$  supergravity in four dimensions possesses a local  $SU(8) \times SO(8)$  invariance, and the fields of that theory can be assigned to representations of  $SU(8) \times SO(8)$  [10]. Without making any assumption on the  $y$ -dependence of the fields, we have been able to assign the spin-2, spin- $\frac{3}{2}$ , spin-1 and spin- $\frac{1}{2}$  fields to representations of  $SU(8)$  which coincide with those of gauged  $N = 9$  supergravity in four dimensions. This suggests that local  $SU(8)$  must also be relevant for the *fields* that describe the massive multiplets. In fact, this is consistent with the structure of the massive multiplets that arise in the  $S^7$  compactification [3]; the helicity states all have the same structure as the massless supermultiplet multiplied with an extra  $SO(8)$  representation corresponding to the harmonic modes on  $S^7$ . Of course for the states one cannot make a distinction between  $SU(8)$  and  $SO(8)$ , but an  $SU(8) \times SO(8)$  assignment of the associated fields is clearly possible. The locally  $SU(8)$  symmetric form of  $N = 8$  supergravity also points in this direction, because in that formulation supersymmetry implies  $SU(8)$  invariance through the supersymmetry commutation relations.

In view of the  $SU(8)$  structure of the transformation rules (2.10) and (2.13), we may pursue the analogy with  $N = 8$  supergravity even further, and conjecture that the  $SU(8)$  group of the full  $d = 11$  theory may in fact be realized as a local symmetry of the equations of motion for arbitrary compactifications to four dimensions (for an early but somewhat different speculation regarding the  $SU(8)$  group, see [25]). The  $SU(8)$  structure would then be a universal characteristic of *any* compactification, whereas the isometry group depends on the specific properties of the manifold  $\mathcal{M}^7$  on which  $d = 11$  supergravity happens to be compactified. Cremmer and Julia [15] have suggested that the local  $SU(8)$  of ungauged  $N = 8$  supergravity becomes dynamical at the quantum level and relevant for the physical spectrum. In [26, 10], the additional assumption was introduced that the local  $SO(8)$  group of gauged  $N = 8$  supergravity provides the forces that bind the preons and lead to eternal confinement. Extending this “preconfinement hypothesis” to the full  $d = 11$  supergravity theory leads to the conjecture that all physical states must now be singlets of the isometry group. The philosophy underlying such a scenario is completely opposite to the conventional Kaluza-Klein philosophy [6]. There one generally assumes that it is the isometry group which is relevant for the physical spectrum, whereas possible “hidden” symmetries are commonly ignored. Here we consider the possibility that it is the hidden symmetry group which becomes physically relevant whereas, through dynamical effects, the erstwhile physical symmetry group becomes a hidden symmetry which is no longer manifest. The intriguing question is then what the nature is of the physical ground state in connection with the large variety of possible spontaneous compactifications of  $d = 11$  supergravity to four dimensions.

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### Appendix

For the reader’s convenience, we here summarize our conventions for the  $d = 7$  Clifford algebra and list some important formulae needed in the main body of this paper. We use hermitean  $8 \times 8$   $\Gamma$ -matrices which satisfy

$$\{ \Gamma^m, \Gamma^n \} = 2\delta^{mn}, \quad (m, n = 1, \dots, 7), \quad (\text{A.1})$$

$$\Gamma^{mnpqrst} = -i\eta' \epsilon^{mnpqrst}, \quad (\text{A.2})$$

where  $\eta' = \pm 1$  is an arbitrary duality phase. Antisymmetrized products of  $\Gamma$ -matrices

are defined with strength one, i.e.

$$\Gamma^{m_1 \dots m_k} = \Gamma^{m_1} \Gamma^{m_2} \dots \Gamma^{m_k}, \quad (\text{A.3})$$

In the context of the parallelized solution [14] and the associated stationary point of the  $N = 8$  potential [12, 13], the following combinations of  $\Gamma$ -matrices play an important role

$$\Gamma_{AB}^{abc} = -\frac{1}{24} \eta' \epsilon^{abcdefg} \Gamma_{AB}^{defg}, \quad (\text{A.4})$$

$$\Gamma_{[AB] [CD]}^{[ab] [cd]} = \frac{1}{24} \eta' \epsilon^{abcdefg} \Gamma_{[AB] [CD]}^{defg}, \quad (\text{A.5})$$

where we note the opposite duality phase in the indices  $a, b, \dots$  in (A.4) and (A.5). The expression (A.5) is furthermore selfdual in the indices  $[ABCD]$  with arbitrary duality phase [15]; the corresponding expression  $\Gamma_{[AB] [CD]}^a$ , which is of opposite duality in the indices  $[ABCD]$ , is not needed in this paper. The importance of (A.4) and (A.5) resides in the fact that they provide the basic input in solving the selfduality equation

$$D_m F_{npq} = \frac{1}{6} \eta' m \epsilon_{mnpqrst} F^{rst} \quad (\text{A.6})$$

in terms of covariantly constant spinors

$$(D_m \pm \frac{1}{2} i m \Gamma_m) \eta_{\pm}^I = 0. \quad (\text{A.7})$$

Namely, (A.6) is solved by [8, 9]

$$\begin{aligned} F_{abc}(B) &= i \bar{\eta}_-^I \Gamma_{abc} \eta_-^J B^{IJ} \\ &= i \bar{\eta}_+^{[I} \Gamma_{[ab} \eta_+^J \bar{\eta}_+^K \Gamma_c] \eta_+^L] B^{IJKL}. \end{aligned} \quad (\text{A.8})$$

The tensors  $B^{IJ}$  and  $B^{IJKL}$  in (A.8) belong to inequivalent 35-dimensional representations of  $\text{SO}(8)$ :  $B^{IJ}$  is symmetric and traceless in the indices  $I, J$  whereas  $B^{IJKL}$  is antisymmetric and (anti)selfdual in  $[IJKL]$ . To calculate the chiral  $\text{SU}(8)$  rotation needed in sect. 3, one may in principle use either of these representations. For practical purposes, it is, however, more convenient to work with the first representation. For arbitrary  $B^{IJ}$ , the relevant exponential is given by

$$\exp\left(\frac{1}{6} F_{abc}(B) \Gamma^{abc}\right) = \frac{1}{8} \text{tr}(e^{-8iB}) + \frac{1}{48} \Gamma^{abc} F_{abc} (e^{-8iB} - \frac{1}{8} \text{tr}(e^{-8iB})). \quad (\text{A.9})$$

An  $SO(7)^-$  invariant solution is obtained by specializing to

$$B^{IJ} = \frac{1}{8}\tau \begin{bmatrix} 7 & & & \\ & -1 & & \\ & & \ddots & \\ & & & -1 \end{bmatrix}, \quad (\text{A.10})$$

which leads to the result (3.13) used in the text with  $F_{mnp}$  proportional to a Cartan-Schouten torsion  $S_{mnp}$ . For this case the following identities are also of interest

$$(S \cdot \Gamma)^2 = -252 - 36iS \cdot \Gamma, \quad (\text{A.11})$$

$$S \cdot \Gamma \Gamma_m S \cdot \Gamma = 36\Gamma_m + 6i\Gamma_m S \cdot \Gamma + 6iS \cdot \Gamma \Gamma_m, \quad (\text{A.12})$$

$$S \cdot \Gamma \Gamma_{mn} S \cdot \Gamma = 36\Gamma_{mn} - 6\eta' \epsilon_{mnpqrs} S^{abc} \Gamma^{de} - 72iS_{mna} \Gamma^a, \quad (\text{A.13})$$

$$\Gamma^m S \cdot \Gamma \Gamma_m = -S \cdot \Gamma, \quad (\text{A.14})$$

$$U(\tau) \Gamma_m U(\tau) = \frac{1}{2} e^{-2i\tau} (2 \cos 4\tau + i \sin 4\tau) \Gamma_m + \frac{1}{4} e^{-2i\tau} \sin 4\tau S_{mnp} \Gamma^{np}, \quad (\text{A.15})$$

$$\begin{aligned} U(-\tau) \Gamma_m U(\tau) &= (1 - \frac{1}{2} \sin^2 4\tau) \Gamma_m + \frac{1}{24} \sin 4\tau \cos 4\tau (\Gamma_m S \cdot \Gamma - S \cdot \Gamma \Gamma_m) \\ &\quad - \frac{1}{24} i \sin^2 4\tau (\Gamma_m S \cdot \Gamma + S \cdot \Gamma \Gamma_m), \end{aligned} \quad (\text{A.16})$$

where  $U(\tau)$  is defined in (3.13).

We next list some of the formulae needed in sect. 3 to convert a  $d = 7$  vector spinor  $\psi_{aA}$  into a three-index spinor  $\lambda_{ABC}$  by means of the definition [15]

$$\lambda_{ABC} = i \Gamma_{[AB}^a \psi_{aC]}. \quad (\text{A.17})$$

The inverse relation is

$$\psi_{aC} = \frac{3}{4} i \left\{ \Gamma_{AB}^a \lambda_{ABC} - \frac{1}{9} (\Gamma^a \Gamma^b)_{CA} \Gamma_{BD}^b \lambda_{ABD} \right\}. \quad (\text{A.18})$$

As in [15] formula (A.18) as well as other relations below are most easily checked backwards. Inserting  $\lambda_{ABC} = \Gamma_{[AB}^X \Gamma_{CD]}^Y \epsilon_D$ , where  $X = a$  or  $[ab]$  and  $Y = c$  or  $[cd]$  one may calculate the corresponding expression for  $\psi_{aA}$  through (A.18). This expression can then be converted again by using (A.17). In that way one finds a useful identity

$$\begin{aligned} \Gamma_{[AB}^X \Gamma_{CD]}^Y &= \Gamma_{[AB}^a \left\{ -\frac{1}{4} \Gamma^X \Gamma_a \Gamma^Y - \frac{1}{4} \Gamma^Y \Gamma_a \Gamma^X + \frac{1}{36} \Gamma_a \Gamma^b \Gamma^X \Gamma_b \Gamma^Y + \frac{1}{36} \Gamma_a \Gamma^b \Gamma^Y \Gamma_b \Gamma^X \right. \\ &\quad \left. + \frac{1}{8} \text{tr}(\Gamma^X \Gamma^b) (\delta_{ab} - \frac{1}{9} \Gamma_a \Gamma_b) \Gamma^Y + \frac{1}{8} \text{tr}(\Gamma^Y \Gamma^b) (\delta_{ab} - \frac{1}{9} \Gamma_a \Gamma_b) \Gamma^X \right\}_{CD}. \end{aligned} \quad (\text{A.19})$$

For instance, choosing  $X = a$ ,  $Y = [bc]$  and antisymmetrizing in  $[abc]$  we find

$$\Gamma_{[AB}{}^{[a}\Gamma_{CD]}^{bc]} = -\frac{1}{6}\Gamma_{[AB}^d(\Gamma_d^{abc} - 6\delta_d^{[a}\Gamma^{bc]})_{C]D}. \quad (\text{A.20})$$

Another useful identity can be found by similar techniques

$$\Gamma_{[AB}^{ab}\delta_{C]D} = -\frac{1}{3}\Gamma_{[AB}^c(\Gamma_c^{ab} - 4\delta_c^{[a}\Gamma^{b]})_{C]D}. \quad (\text{A.21})$$

In sect. 3 we also need

$$\begin{aligned} \frac{1}{6}\Gamma_{[AB}^a(\Gamma_a S \cdot \Gamma - S \cdot \Gamma \Gamma_a)_{C]D} &= -\frac{1}{12}iS_{abc}\Gamma_{[AB}^{bc}(\Gamma^a S \cdot \Gamma - S \cdot \Gamma \Gamma^a)_{C]D} \\ &= (\Gamma_{[AB}^a\Gamma_{C]D}^{bc} - \Gamma_{[AB}^{ab}\Gamma_{C]D}^c)S_{abc}, \end{aligned} \quad (\text{A.22})$$

which can be derived by utilizing the properties of the Cartan-Schouten torsion (3.3), (3.4).

Finally we recall from [13] the following identity for Killing vectors

$$S_{mnp}\bar{\eta}^l\Gamma^{np}\eta^j = iC^{IJKL}\bar{\eta}^K\Gamma_m\eta^L, \quad (\text{A.23})$$

where  $C^{IJKL}$  is a *constant* antisymmetric selfdual tensor expressed by

$$C^{IJKL} = \frac{1}{2}iS^{mnp}(y)\bar{\eta}^{[I}(y)\Gamma_{mn}\eta^J(y)\bar{\eta}^K(y)\Gamma_p\eta^{L]}(y), \quad (\text{A.24})$$

where the torsion  $S_{mnp}$  and Killing spinors  $\eta^I$  are defined by (3.2)–(3.4) and (3.9), respectively.

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