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Testing general relativity and probing the merger history of massive black holes with LISA

Emanuele Berti[1], **Alessandra Buonanno**[2,3] **and Clifford M Will**[1]

[1] McDonnell Center for the Space Sciences, Department of Physics, Washington University, St Louis, MO 63130, USA

[2] AstroParticule et Cosmologie (APC), 11 place Marcelin Berthelot, 75231 Paris and Institut d'Astrophysique de Paris, 98bis Boulv. Arago, 75013 Paris, France

E-mail: berti@wugrav.wustl.edu

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Abstract

Observations of binary inspirals with the proposed Laser Interferometer Space Antenna (LISA) will allow us to place bounds on alternative theories of gravity and to study the merger history of massive black holes (MBH). These possibilities rely on LISA's parameter estimation accuracy. We update previous studies of parameter estimation for inspiralling compact binaries of MBHs, and for inspirals of neutron stars into intermediate-mass black holes, including non-precessional spin effects. We work both in Einstein's theory and in alternative theories of gravity of the scalar–tensor and massive-graviton types. Inclusion of non-precessional spin terms in MBH binaries has little effect on the angular resolution or on distance determination accuracy, but it degrades the estimation of intrinsic binary parameters such as chirp mass and reduced mass by between one and two orders of magnitude. The bound on the coupling parameter $\omega_{\rm BD}$ of scalar–tensor gravity is significantly reduced by the presence of spin couplings, while the reduction in the graviton-mass bound is milder. LISA will measure the luminosity distance of MBHs to better than $\sim$10% out to $z \simeq 4$ for a $(10^6 + 10^6)M_\odot$ binary, and out to $z \simeq 2$ for a $(10^7 + 10^7)M_\odot$ binary. The chirp mass of a MBH binary can always be determined with excellent accuracy. Ignoring spin effects, the reduced mass can be measured within $\sim$1% out to $z = 10$ and beyond for a $(10^6 + 10^6)M_\odot$ binary, but only out to $z \simeq 2$ for a $(10^7 + 10^7)M_\odot$ binary. Present-day MBH coalescence rate calculations indicate that most detectable events should originate at $z \sim 2$–6: at these redshifts LISA can be used to measure the two black-hole masses and their luminosity distance with sufficient accuracy to probe the merger history of MBHs. If the low-frequency LISA noise can only be trusted down

[3] UMR 7164 (CNRS, Université Paris7, CEA, Observatoire de Paris).

to 10^{-4} Hz, parameter estimation for MBHs (and LISA's ability to perform reliable cosmological observations) will be significantly degraded.

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(Some figures in this article are in colour only in the electronic version)

1. Summary of results

The Laser Interferometer Space Antenna (LISA) is being designed to detect gravitational wave (GW) signals in the frequency band between 10^{-4} Hz and 10^{-1} Hz [1]. Operating at these low frequencies, LISA can detect, among other sources, the inspiral of stellar-mass compact objects—such as neutron stars (NS) or black holes (BH)—into intermediate-mass black holes (IMBH) with masses in the range 10^2–$10^4 M_\odot$. Another strong gravitational wave source to be observed by LISA is the inspiral and merger of massive black holes (MBH) with masses in the range 10^4–$10^7 M_\odot$.

Gravitational radiation reaction drives the inspiral of stellar-mass compact objects into IMBHs. It also dominates the final stages in the evolution of coalescing MBH binaries. The amplitude and phase of the gravitational wave signal carry information about binary parameters, such as masses and spins, and about the location and distance of the binary. They may also be different in different theories of gravity. Therefore LISA can provide important astrophysical information, yield interesting tests of fundamental physics and place bounds on alternative theories of gravity. In this paper, we consider, along with standard general relativity, theories of the scalar–tensor type (the simplest exemplar being that of Brans and Dicke) and theories with an effective mass in the propagation of gravitational waves (which we call massive graviton theories, in short). In scalar–tensor theories the phasing evolution is modified predominantly by the presence of dipole gravitational radiation reaction in the orbital evolution (in general relativity the lowest radiative multipole moment is the quadrupole). In massive graviton theories the gravitational wave propagation speed depends on wavelength: this generates a distortion in the time of arrival (and in the wave phasing) with respect to general relativity, similar to the dispersion of radio waves by interstellar plasma.

In a recent paper [2] (henceforth BBW), we investigated the effect of spin–orbit and spin–spin couplings both on the estimation of astrophysical parameters within general relativity, and on bounds that can be placed on alternative theories. We restricted our analysis to non-precessing spinning binaries, i.e. binaries whose spins are parallel (or anti-parallel) to the orbital angular momentum. Our work extended results of previous papers [3–7] that derived bounds on the graviton mass, on the Brans–Dicke parameter $\omega_{\rm BD}$ and on parameters describing more general scalar–tensor theories under the assumption that the compact objects do not carry spin.

Within Einstein's general relativity, various authors have investigated the accuracy with which LISA can determine binary parameters including spin effects [8–12]. These earlier works (except for [12]) adopted analytical approximations to LISA's instrumental noise [8], augmented by an estimate of white-dwarf confusion noise [13] in the low-frequency band. In BBW (and in the present work) we model the LISA noise curve by a similar—albeit slightly updated—analytical approximation ([14]; see section IIC and figure 1 of BBW for details). This noise curve has the advantage of being given in analytical form, and reproduces very well the salient features of numerical noise curves available online from the LISA Sensitivity Curve Generator [15], a tool sponsored by the LISA International Science Team.

The central conclusions of our work are as follows. Inclusion of non-precessing spin–orbit and spin–spin terms in the gravitational wave phasing generally reduces the accuracy with which the parameters of the binary can be estimated. This is not surprising, since the parameters are highly correlated, and adding parameters effectively dilutes the available information. Such an effect has already been described within Einstein's general relativity in the context of ground-based detectors of the LIGO/VIRGO type [16, 17]. For example, for massive black-hole binaries at 3 Gpc, we find that including spin–orbit terms degrades the accuracy in measuring chirp mass by factors of order 10, and in measuring the reduced mass parameter by factors of order 20–100; including spin–spin terms further degrades these accuracies by factors of orders 3 and 5, respectively. For neutron stars inspiralling into IMBHs with masses between 10^3 and 10^4 solar masses, the corresponding reductions are factors of orders 20 and 5–30 in chirp mass and reduced mass parameter, respectively, when spin–orbit is included, and additional factors of orders 4 and 7, respectively, when spin–spin terms are included.

When we consider placing bounds on alternative theories of gravity, for technical reasons, we treat only spin–orbit terms. The source of choice to place bounds on the coupling parameter $\omega_{\rm BD}$ of scalar–tensor gravity is the inspiral of a neutron star into an IMBH. We first reproduce results of earlier work [6], apart from small differences arising from corrected normalization of the LISA noise curve. Then, we include spin–orbit effects and find that the bound on $\omega_{\rm BD}$ is significantly reduced, by factors of orders 10–20. For example, for a $1.4M_\odot$ neutron star inspiralling into a $400M_\odot$ BH, the average bound on $\omega_{\rm BD}$ from a population of binaries across the sky goes from 3×10^5 to 2×10^4 when spin–orbit terms are included. The latter (average) bound should be compared with the bound of 4×10^4 from Cassini measurements of the Shapiro time delay [18].

The effect of including spin on bounding the graviton mass is more modest. In this case, the source of choice is the inspiral of MBH binaries. For masses ranging from $10^5 M_\odot$ to $10^7 M_\odot$, the reduction in the bound induced by the inclusion of spin–orbit terms is only a factor of 4 to 5.

We also consider the effect of spin terms on the angular and distance resolution of LISA. We find that spin couplings have a mild effect on the angular resolution, on the distance and, as a consequence, on the redshift determination for MBH binaries. By contrast, for stellar mass objects inspiralling into IMBHs, neither distance nor location on the sky is very well determined. Furthermore, for NS-IMBH binaries the angular resolution is somewhat dependent on the inclusion of spin terms. Here we show that the different spin dependence of the angular resolution in the NS-IMBH and MBH-MBH cases can be traced back to different behaviour of the correlations between the chirp mass (leading the gravitational wave evolution of the binary) and the angular variables describing LISA's position in the sky.

LISA's low-frequency sensitivity affects the accuracy of estimating parameters for MBH binaries, as we showed in BBW (similar investigations can be found in [19, 20]). As default low-frequency cutoff we choose 10^{-5} Hz. However, below 10^{-4} Hz, the noise characteristics of LISA are uncertain, and if we set LISA's low-frequency cutoff at 10^{-4} Hz, the accuracy of estimating both extrinsic parameters such as distance and sky position and intrinsic parameters such as chirp mass and reduced mass, as well as the graviton mass, can be significantly lower, especially for higher-mass systems.

LISA can observe MBH binaries with large SNR out to large values of the cosmological redshift. If the corresponding mass and distance determinations are accurate enough, LISA will be a useful tool to study BH formation in the early universe. Using Monte Carlo simulations we find that LISA can provide accurate distance determinations out to redshift $z \sim 2$ for source masses $\sim 10^7 M_\odot$, and out to $z \sim 4$ for source masses $\sim 10^6 M_\odot$. Mass determinations strongly depend on an accurate treatment of spin effects. The chirp mass of a MBH binary can always be determined with excellent accuracy. Ignoring spin effects, the reduced mass can be measured within $\sim$1% out to $z = 10$ and beyond for a $(10^6 + 10^6)M_\odot$ binary, but only out to $z \simeq 2$ for a $(10^7 + 10^7)M_\odot$ binary. Present-day MBH coalescence rate calculations indicate that most detectable events should originate at $z \sim 2$–6 [21–23]: at these redshifts LISA can be used to measure the two BH masses and their luminosity distance with sufficient accuracy to probe the merger history of MBHs.

This paper is organized as follows. In section 2 we briefly summarize our procedure to study parameter estimation with LISA. All calculations presented here adopt the 'more realistic' of the two models described in BBW: we take into account the orbital motion of the detector and use Monte Carlo simulations of a population of sources across the sky. In sections 3 and 4 we extend and clarify some results presented in BBW. Section 3 shows the dependence of parameter estimation and tests of alternative theories of gravity on the binary mass (BBW only considered a few representative values of the masses). Section 4 discusses the cosmological reach of LISA and its ability to probe the merger history of MBHs, extending and complementing the discussion in section IIIC of BBW.

2. Gravitational wave detection and parameter estimation

We assume, as in [8], that two independent Michelson outputs can be constructed from the readouts of the three LISA arms if the noise in the arms is totally symmetric (but see [24] for discussion of the most general combinations of LISA outputs, and their sensitivities). Since the modulation of the measured signal due to LISA's motion occurs on timescales $\sim$1 year (much larger than the binary's orbital period), the Fourier transform of the signal can be evaluated in the stationary phase approximation, with the result

$$\tilde{h}_\alpha(f) = \frac{\sqrt{3}}{2}\left[\frac{\mathcal{M}^{5/6}}{\sqrt{30}\pi^{2/3}D_{\mathrm{L}}}\right]\left\{\frac{5}{4}\frac{\tilde{A}_\alpha(t)}{f^{7/6}}\right\}\mathrm{e}^{\mathrm{i}(\psi(f)-\varphi_{p,\alpha}(t)-\varphi_D(t))}. \tag{1}$$

Here f is the frequency of the gravitational waves; $\mathcal{M} = \eta^{3/5}M$ is the 'chirp' mass, with $M = m_1 + m_2$ and $\eta = m_1m_2/M^2$; D_{L} is the luminosity distance to the source; $\tilde{A}_\alpha(t) = \left\{[1 + (\hat{\mathbf{L}}\cdot\mathbf{n})^2]^2F_\alpha^{+2} + 4(\hat{\mathbf{L}}\cdot\mathbf{n})^2F_\alpha^{\times 2}\right\}^{1/2}$, where $\hat{\mathbf{L}}$ is the orbital angular momentum unit vector, $\mathbf{n}$ is a unit vector in the direction of the source on the sky, and the quantities $F_\alpha^{+,\times}$ are the pattern functions for the two equivalent Michelson detectors. The time variable $t = t(f)$ is given by an expansion to 2PN order, including also the Brans–Dicke parameter and the graviton-mass term (see BBW and [8] for details); $\varphi_{p,\alpha}(t)$ is the waveform polarization phase and $\varphi_D(t)$ the Doppler phase.

We have adopted the standard 'restricted post-Newtonian approximation' for the waveform, in which the amplitude is expressed to the leading order in a post-Newtonian expansion (an expansion for slow-motion, weak-field systems in powers of $v \sim (M/r)^{1/2} \sim (\pi\mathcal{M}f)^{1/3}$), while the phasing $\psi(f)$, to which laser interferometers are most sensitive, is expressed to the highest post-Newtonian (PN) order reasonable for the problem at hand. For binaries with spins aligned (or anti-aligned) and normal to the orbital plane, this is a valid approximation because the amplitude varies slowly (on a radiation reaction timescale) compared to the orbital period. But when the spins are not aligned, modulations of the amplitude on a precession timescale must be included. Preliminary studies show that precession effectively decorrelates parameters, improving parameter estimation in a significant way [11]. The study of precessional modulations is beyond the scope of this work.

The phasing function $\psi(f)$ is known for point masses up to 3.5PN order [25, 26]. But spin terms are known only up to 2PN order, so to be reasonably consistent, we include in the phasing point-mass terms only up to this same 2PN order. The needed expression for the phasing is

$$\psi(f) = 2\pi f t_c - \phi_c + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left\{1 - \frac{5(\hat{\alpha}_1 - \hat{\alpha}_2)^2}{336\omega_{\mathrm{BD}}}\eta^{2/5}(\pi\mathcal{M} f)^{-2/3}\right.$$
$$- \frac{128}{3}\frac{\pi^2 D\mathcal{M}}{\lambda_g^2(1+z)}(\pi\mathcal{M} f)^{2/3} + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)\eta^{-2/5}(\pi\mathcal{M} f)^{2/3}$$
$$- 16\pi\eta^{-3/5}(\pi\mathcal{M} f) + 4\beta\eta^{-3/5}(\pi\mathcal{M} f) - 10\sigma\eta^{-4/5}(\pi\mathcal{M} f)^{4/3}$$
$$\left. + \left(\frac{15\,293\,365}{508\,032} + \frac{27\,145}{504}\eta + \frac{3085}{72}\eta^2\right)\eta^{-4/5}(\pi\mathcal{M} f)^{4/3}\right\}, \tag{2}$$

where t_c and ϕ_c are the time and phase at coalescence. The second term inside the braces is the contribution of dipole gravitational radiation in Brans–Dicke theory: it depends on the difference of the scalar charges of the two bodies $\alpha_i^2 \simeq \hat{\alpha}_i^2 / 2\omega_{BD}$ $(i = 1, 2)$[4].

For black holes, $\alpha_i \equiv 0$, therefore we need 'mixed' binaries (e.g. NS–IMBH binaries) to place bounds on scalar–tensor theories. The third term in the braces is the effect of a massive graviton, which alters the arrival time of waves of a given frequency, depending on the size of the graviton Compton wavelength λ_g and on a distance quantity D (see BBW or [4] for the relation between D and the luminosity distance D_L). The remaining terms in the braces are the standard general relativistic, post-Newtonian terms, including spin effects.

The quantities β and σ represent spin–orbit and spin–spin contributions to the phasing, given by

$$\beta = \frac{1}{12}\sum_{i=1}^{2}\chi_i\left[113\frac{m_i^2}{M^2} + 75\eta\right]\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_i, \tag{3}$$

$$\sigma = \frac{\eta}{48}\chi_1\chi_2(-247\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2 + 721\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_1\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_2), \tag{4}$$

where $\hat{\mathbf{S}}_i$ and $\hat{\mathbf{L}}$ are the unit vectors in the direction of the spins and of the orbital angular momentum, respectively, and $\mathbf{S}_i = \chi_i m_i^2\hat{\mathbf{S}}_i$. For BHs, the dimensionless spin parameters χ_i must be smaller than unity, while for neutron stars, they are generally much smaller than unity. It follows that $|\beta| < 9.4$ and $|\sigma| < 2.5$.

We use the by now standard machinery of parameter estimation in matched filtering for gravitational wave detection that has been developed by a number of authors [16, 28–30]. Given a set of parameters θ^a characterizing the waveform, one defines the 'Fisher matrix' Γ_{ab} with components given by

$$\Gamma_{ab} \equiv \left(\frac{\partial h}{\partial\theta^a}\middle|\frac{\partial h}{\partial\theta^b}\right) \equiv 2\int_0^\infty \frac{1}{S_n(f)}\left[\frac{\partial\tilde{h}^*}{\partial\theta^a}\frac{\partial\tilde{h}}{\partial\theta^b} + \frac{\partial\tilde{h}^*}{\partial\theta^b}\frac{\partial\tilde{h}}{\partial\theta^a}\right]\mathrm{d}f. \tag{5}$$

An estimate of the rms error, $\Delta\theta^a$, in measuring the parameter θ^a can then be calculated, in the limit of large SNR, by taking the square root of the diagonal elements of the inverse of the Fisher matrix,

$$\Delta\theta^a = \sqrt{\Sigma^{aa}}, \qquad \Sigma = \Gamma^{-1}. \tag{6}$$

The correlation coefficients between two parameters θ^a and θ^b are given by

$$c_{ab} = \Sigma^{ab}/\sqrt{\Sigma^{aa}\Sigma^{bb}}. \tag{7}$$

[4] Note that the quantity $\hat{\alpha}_i = 1 - 2s_i$, where s_i is the so-called sensitivity of the body [3]. In calculations involving NSs we adopt the value $\hat{\alpha}_{\mathrm{NS}} = 0.6$, a typical value obtained from calculations of NS structure with realistic equations of state in scalar–tensor theories [27].

The assumption of large SNR is well justified for MBH binaries, which can be observed with high SNR out to very large distances.

To determine how accurately LISA can measure source locations and luminosity distances we perform Monte Carlo simulations using a population of sources across the sky. We consider in detail two classes of systems. The first are NS–IMBH binaries with a NS mass $M_{\rm NS} = 1.4 M_\odot$ and a BH mass in the range $[400\text{–}10^4] M_\odot$, a typical target system used to place bounds on the BD parameter. According to current estimates, detection rates for $10\text{–}10^2 M_\odot$ BHs spiralling into IMBHs of mass $10^2\text{–}10^3 M_\odot$ are expected to be very low [31]. In order to apply the Fisher matrix formalism in a consistent way, we assume that these NS–IMBH binaries, rare though they may be, are detected with a single-detector SNR $\rho_I = 10$. The second are MBH binaries. In this case we first consider equal-mass binaries with total *measured* mass in the range $[2 \times 10^4\text{–}2 \times 10^7] M_\odot$ at distance $D_L = 3$ Gpc—typical systems to put bounds on massive graviton theories (section 3). Then we study parameter estimation for MBH systems located at different cosmological redshifts and having mass $(10^6 + 10^6) M_\odot$ and $(10^7 + 10^7) M_\odot$ *as measured in the source rest frame* (section 4).

For each of these systems we distribute 10^4 sources over sky position and orientation, described, in the solar system barycentric frame, by angles $(\bar\theta_S, \bar\phi_S)$ and $(\bar\theta_L, \bar\phi_L)$, respectively. We randomly generate the angles $\bar\phi_S, \bar\phi_L$ in the range $[0, 2\pi]$ and $\mu_S = \cos\bar\theta_S$, $\mu_L = \cos\bar\theta_L$ in the range $[-1, 1]$ (see BBW for details). When the waveform contains a large number of highly correlated parameters, computing the inverse of the Fisher matrix can be numerically difficult. The method we used to check the robustness of our results is described in appendix B of BBW.

3. Parameter estimation and bounds on alternative theories of gravity

In BBW we carried out a set of Monte Carlo simulations for a MBH binary of mass $(10^6 + 10^6) M_\odot$ at fiducial distance $D_L = 3$ Gpc. We showed that our results are compatible with results obtained using a simpler approach [6], in which we do not take into account LISA's orbital motion and we average over pattern functions.

For this paper, we performed a more extensive set of Monte Carlo simulations, spanning a wider range of binary masses. Results from these simulations are shown in figure 1. We consider nonspinning equal-mass MBHs at fiducial distance $D_L = 3$ Gpc, and we investigate the dependence of mass, distance and angular resolution determinations on the binary's total mass. From the plots on the first row we see that distance and angular resolution determinations are largely independent of the inclusion of spin terms (and even of the massive-graviton terms) in the phase. Mass determinations are more sensitive to the inclusion of spin terms and to the theory of gravity we are considering. Distance errors and angular resolution generally decrease with the binary's mass. In contrast, mass errors have a relative minimum in the given mass range. This is because the measurement of mass parameters depends on an accurate modelling of the phase of the waveform: when the MBH mass is large the binary's frequency becomes low, the binary spends a significant fraction of time out of the LISA sensitivity band, and the accuracy of mass determinations decreases.

We also explored the dependence of parameter estimation on the signal's ending frequency. Instead of using the Schwarzschild innermost stable circular orbit (ISCO) as a conventional ending frequency of the signal, we looked at the effect of truncating the signal at the minimum energy circular orbit (MECO, see section IIB of [32] for a discussion), consistently computed at 2PN and assuming dynamically small spins ($\beta = \sigma = 0$). We found that using the MECO instead of the ISCO has a completely negligible effect on distance and angular position determinations. The effect on mass determinations is completely negligible for total masses

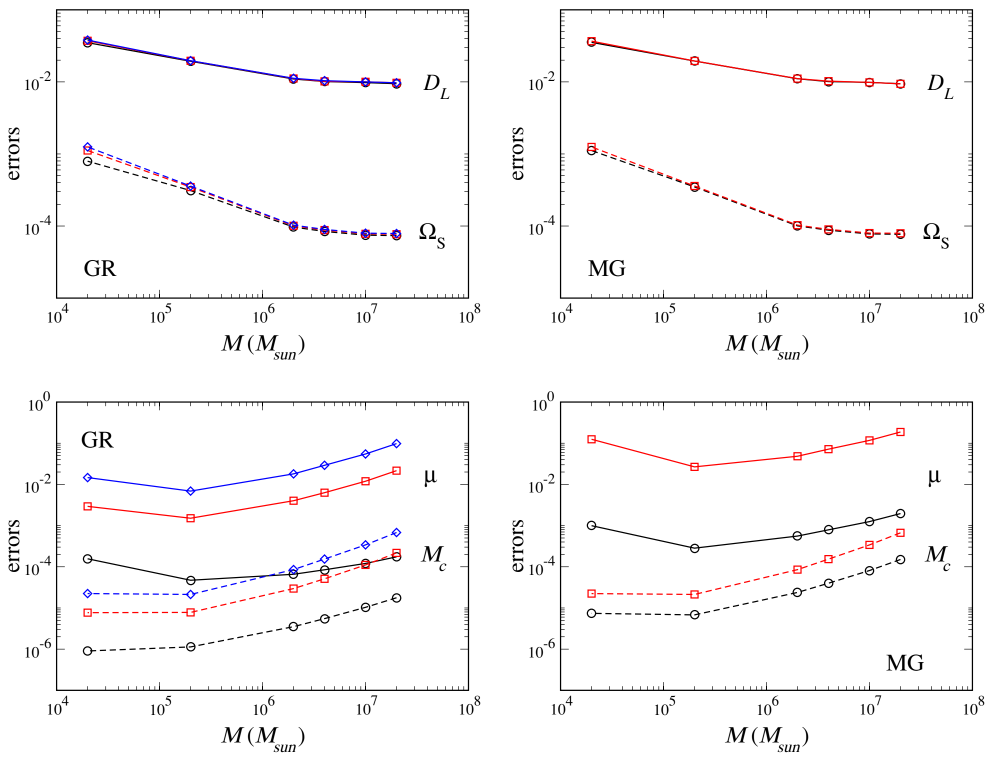

Figure 1. Top: average errors on Ω_S (dashed lines) and D_L (solid lines). Bottom: average errors on $\mathcal{M}$ (dashed lines) and μ (solid lines). Left panel refers to GR, right panel to massive graviton theories; errors are given as a function of the total mass for equal mass MBH binaries. We assume the LISA noise curve can be trusted down to $f_{\rm low} = 10^{-5}$ Hz. Black lines are computed omitting spin terms, red lines include spin–orbit terms, blue lines include both spin–orbit and spin–spin terms.

M smaller than $\sim 10^6 M_\odot$; even for values of M larger than this, using the MECO instead of the ISCO only results in slightly smaller (by factors ~ 2 or so) errors. However, the effect of using the MECO could be non-negligible if the spins are large.

In figure 2 we look at bounds to be placed on (i) the graviton Compton wavelength λ_g, using a sample of equal-mass MBHs at fiducial distance $D_L = 3$ Gpc (left panel), and (ii) the Brans–Dicke parameter ω_{BD}, using a sample of NS–IMBH binaries with single-detector signal-to-noise ratio equal to 10, as a function of the IMBH mass (right panel). These results confirm qualitatively (and improve quantitatively) results previously obtained by Will and collaborators. Those authors adopted a somewhat cruder LISA model, which does not take into account LISA's orbital motion [3–6]. Figure 2 shows that understanding the low-frequency LISA noise is important to set bounds on the graviton mass using high-mass MBH binaries. We will see in section 4 that an understanding of the low frequency noise is also important to use LISA in a cosmological context.

In BBW we found that LISA's angular resolution degrades when we include spin terms in the NS–IMBH case, but it is essentially unaffected in the MBH case. To explore this further, in figure 3 we show a scatter plot (out of 10^4 binaries) of the correlation $c_{\bar\phi_S \mathcal{M}}$ between the azimuthal angle $\bar\phi_S$ (describing the source position in the sky with respect to the Solar System barycenter) and the chirp mass $\mathcal{M}$ (dominating the phase evolution of the

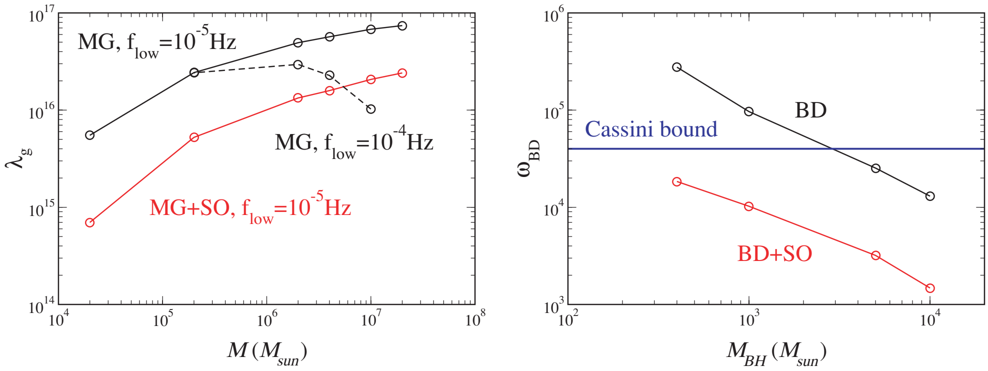

Figure 2. Left: average bounds on the graviton wavelength as a function of total mass for equal mass MBH binaries. Right: bound on the BD parameter from a NS–IMBH binary as a function of the IMBH mass. The horizontal blue line corresponds to the Cassini bound [18]. Black lines are computed omitting spin terms; red lines include spin–orbit terms as well. The dashed line shows that the bound on λ_g is reduced if the LISA noise curve can only be trusted down to $f_{\rm low} = 10^{-4}$ Hz.

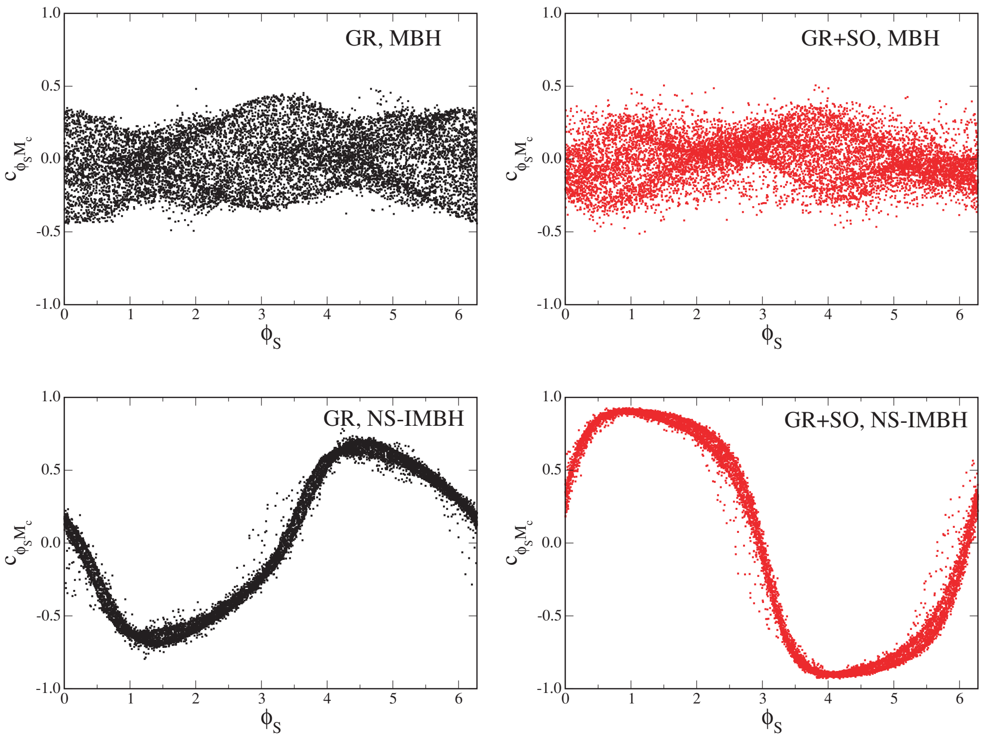

Figure 3. Scatter plot of the correlations $c_{\bar\phi_S\mathcal{M}}(\bar\phi_S)$ in GR. From left to right we observe how correlations change when we add spins. The top row corresponds to a $(10^6 + 10^6)M_\odot$ MBH binary at $D_L = 3$ Gpc; the bottom row refers to a $(1.4 + 10^3)M_\odot$ NS–IMBH system observed with single-detector SNR equal to 10.

gravitational waveform), as a function of $\bar\phi_S$, in the two cases. The correlation has a 'clean' periodic behaviour in the NS–IMBH case; in the MBH–MBH case $c_{\bar\phi_S\mathcal{M}}$ is *not* a clean periodic function of the binary's azimuthal angle $\bar{\phi}_S$, showing some beating phenomena. An important feature is that, when we include spin terms, the maximum correlation between mass and angular position clearly grows in the NS–IMBH case, but not in the MBH–MBH case. This is consistent with the fact that the angular resolution depends on spin couplings in the NS–IMBH case, but it does not in the MBH–MBH case.

4. Probing the merger history of supermassive black holes with LISA

Astrophysical estimates of MBH coalescence rates depend on many complex physical phenomena. The study of MBH binary evolution in galaxy mergers is a very active field of research [33]. A class of models predicts that the LISA mission should be able to *detect* at least $\sim$10 MBH coalescence events out to large values of the redshift, and that most of these events would originate at $z \sim 2$–6 [21–23]. Research in this field is very active and the estimates are still quite uncertain: a partial list of references includes [34–39].

To use LISA as a cosmological probe it would be desirable to have an accurate measurement of the binary parameters, and not just a simple detection. In this context the redshift z can be large, and the distinction between observed masses and masses as measured in the source rest frame—given by the simple rescaling $M_{\rm observed} = (1 + z)M_{\rm source}$, where z is the cosmological redshift—is important[5]. LISA can only measure redshifted combinations of the intrinsic source parameters (masses and spins), so it cannot measure the redshift z to the source. But if the MBH masses, luminosity distances and angular resolution are determined with sufficient accuracy, LISA can still provide information on the growth of structures at high redshift. If cosmological parameters are known, the relation between luminosity distance and redshift $D_L(z, H_0, \Omega_M, \Omega_\Lambda)$ (where $H_0 = 72$ km s^{-1} Mpc^{-1} is Hubble's constant and we assume a zero-spatial-curvature universe with $\Omega_\kappa = 0$, $\Omega_\Lambda + \Omega_M = 1$, according to the present observational estimates) can be inverted to yield $z(D_L, H_0, \Omega_M, \Omega_\Lambda)$. Then LISA measurements of the luminosity distance can be used to obtain intrinsic BH masses as a function of redshift, thus constraining hierarchical merger scenarios [9]. Alternatively, if we can obtain the binary's redshift by some other means, e.g. from an electromagnetic counterpart, then LISA measurements of D_L can be used to improve our knowledge of the cosmological parameters [40, 41, 19].

These exciting applications depend, of course, on LISA's measurement accuracy at large redshifts. Here, as in BBW, we look at the redshift dependence of measurement errors for two representative MBH binaries having masses $(10^6 + 10^6)M_\odot$ and $(10^7 + 10^7)M_\odot$ *as measured in the source rest frame*. We also assume that the LISA noise can be extrapolated down to $f_{\rm low} = 10^{-5}$ Hz; more conservative assumptions on $f_{\rm low}$ could significantly affect our conclusions (see [2, 19, 20]). We compute the errors performing Monte Carlo simulations of 10^4 binaries for different values of the redshift and then averaging over all binaries.

As discussed in section 3, distance determination and angular resolution for MBHs are essentially independent of the inclusion of spin terms. The (average) relative error on D_L for the $(10^6+10^6)M_\odot$ system is $\sim$2% at $z = 1$, $\sim$5% at $z = 2$ and $\sim$11% at $z = 4$. This reduction in accuracy is due to the fact that the signal spends less and less time in band as the redshift is increased. We only consider values of the redshift such that the binary spends at least one month in band before coalescing: for the case $(10^7 + 10^7)M_\odot$, this corresponds to $z \sim 4$. The distance determination error for this high-mass binary grows quite rapidly, being $\sim$2% at $z = 1$, $\sim$6% at $z = 2$ and $\sim$21% at $z = 4$. LISA's angular resolution is rather poor even at small redshifts, and it rapidly degrades for sources located farther away, the degradation being

[5] Up to this point we denoted by $\mathcal{M}$ and M the *observed* chirp and total masses, respectively.

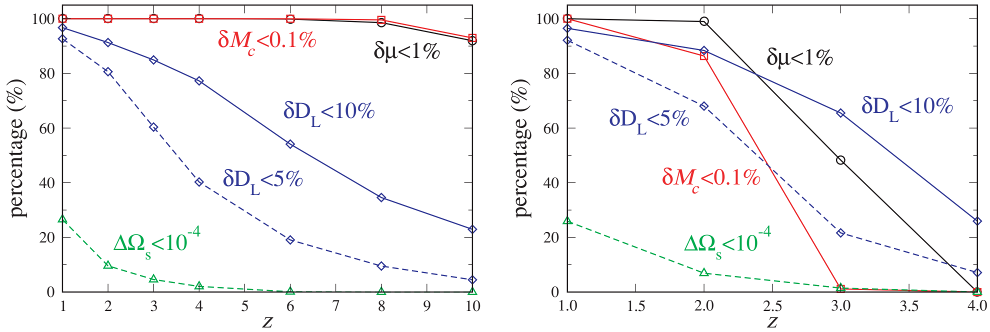

Figure 4. Percentage of binaries for which: $\delta\mu = \Delta\mu/\mu < 1\%$ (black circles), $\delta\mathcal{M} = \Delta\mathcal{M}/\mathcal{M} < 0.1\%$ (red squares; $\delta\mathcal{M} = \Delta\mathcal{M}/\mathcal{M} < 1\%$ in all cases considered), $\delta D_L = \Delta D_L/D_L < 10\%$ (blue diamonds), $\delta D_L = \Delta D_L/D_L < 5\%$ (blue diamonds, dashed lines), $\Delta\Omega_S < 10^{-4}$ (green triangles). On the left we consider a BH–BH binary of mass $(10^6 + 10^6)M_\odot$, on the right a BH–BH binary of mass $(10^7 + 10^7)M_\odot$. We assume $f_{\rm low} = 10^{-5}$ Hz.

more pronounced for higher-mass binaries. Better distance determinations can be obtained if we are lucky enough to locate the source in the sky by some other means: for example, associating the gravitational wave event with an electromagnetic counterpart. In this case angles and distance would be decorrelated, allowing order of magnitude improvements in the determination of D_L [19]. For $(10^6 + 10^6)M_\odot$ general relativistic nonspinning binaries, a least-squares fit of mass and distance errors in the interval $z \in [1, 10]$ yields:

$$\Delta\mathcal{M}/\mathcal{M} = (-0.1148 + 0.7236z + 0.5738z^2) \times 10^{-5},$$
$$\Delta\mu/\mu = (-0.614\,31 + 1.9018z + 0.437\,21z^2) \times 10^{-4},$$
$$\Delta D_L/D_L = (-0.656\,51 + 2.6935z + 0.061\,595z^2) \times 10^{-2}.$$

It is important to stress that in our discussion above we refer to *average* errors from a population of 10^4 sources randomly distributed across the sky. In a single 'lucky' detection, parameter estimation errors could be much smaller than the average. The most useful quantity to measure LISA's cosmological capabilities is perhaps the *percentage of binaries whose parameters can be measured within some given accuracy*, as a function of z. This percentage is shown in figure 4.

Upper limits on measurement errors of the various parameters have been chosen in order to answer the following questions. Out to which value of z can we determine the chirp mass to better than 0.1%, and the reduced mass to better than 1%? Out to which z does LISA beat the present uncertainty (∼10%) on cosmological parameters, allowing us to determine the luminosity distance to the source? Which fraction of binaries can be observed with an angular resolution error $\Delta\Omega_S < 10^{-4}$, roughly corresponding to the angular diameter of the Moon (and the Sun) as seen from the Earth?

The answer to these questions depends, of course, on the binary's mass and on the low-frequency sensitivity of LISA. Following BBW, in our calculations we assume that the noise curve can be extrapolated down to $f_{\rm low} = 10^{-5}$ Hz and we consider two representative binaries with masses $(10^6 + 10^6)M_\odot$ and $(10^7 + 10^7)M_\odot$, respectively.

The main conclusions to be drawn from figure 4 are the following. (i) The fractional error on the chirp mass can be better than 0.1% out to $z = 10$ and beyond for a $(10^6 + 10^6)M_\odot$ binary, but only out to $z \simeq 2$ for a $(10^7 + 10^7)M_\odot$ binary. (ii) In the absence of spins, the fractional error on the reduced mass can be better than ∼1% out to $z = 10$ and beyond for a $(10^6 + 10^6)M_\odot$ binary, but only out to $z \simeq 2$ for a $(10^7 + 10^7)M_\odot$ binary. (iii) For ∼80% of observed binaries, LISA's distance determination will be better than $\sim$10% (beating present errors on cosmological parameters) out to $z \simeq 4$ for a $(10^6 + 10^6)M_\odot$ binary, and out to $z \simeq 2$ for a $(10^7 + 10^7)M_\odot$ binary. (iv) LISA's angular resolution is comparatively quite poor: only a small fraction of binaries can be observed with $\Delta\Omega_S < 10^{-4}$ at any redshift.

Unimportant as they are for distance determination and angular resolution, spin effects have a dramatic impact on mass measurement accuracy. In BBW we showed that when we include spin–orbit and spin–spin terms, the error on the chirp mass is still pretty small: it only becomes larger than a few per cent for massive ($M \sim 2 \times 10^7 M_\odot$) binaries located at $z > 2$. However, non-precessional spin effects seriously limit our ability to measure the reduced mass. When we include both spin–orbit and spin–spin terms, the reduced mass error becomes larger than 5% at all redshifts even for 'low mass', $(10^6 + 10^6)M_\odot$ binaries. Our conclusions on mass determinations for spinning binaries could change when we consistently include *precessional* spin effects. These effects can induce modulations in the waveform, possibly improving the mass measurements in a significant way [11]. Therefore the study of precession is very important to assess LISA's ability to measure MBH masses in galactic mergers; we plan to return to this study in the future.

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