LINEAR STABILITY OF EXPONENTIAL DENSITY PROFILES

by

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Abstract: The exact dispersion relation for a plasma with density profiles $n(x)\exp(-ax)$ in a homogeneous magnetic field $B = i_z B_o$ is derived. As example, we consider the low-frequency drift instability.

The problem fo the linear stability of a Vlasov plasma whose equilibrium distribution function depends on space coordinates leads to an integro- differential equation for the Laplace transform of the electric potential φ. In general, considering only wavelengths much larger than the mean Larmor radius R. Then, in the further limit where the wavelengths are smaller than the inhomogeneity length, one applies the WKB method. The most severe of these two limitations is, of course, the first one.

In this paper we shall consider exponential density profiles $n(x) \sim \exp(-ax)$. The results have a physical meaning for profiles with $n'/n \sim -a$ over a length L if the considered wavelengths λ_x in the x-direction satisfy $\lambda_x < L$.

Since the density is a monotonic function of x with n'/n =Cte, the electrostatic waves cannot go back and forth in a finite region. Then it can be expected that in the dispersion relation the part of the density profile with $n \to \infty$ will play an important role. In other words, the dispersion relation can be expected to be very similar to that obtained in the approximation of quasi-neutrality. We will show that the exact dispersion relation is in fact the same as in the quasi-neutrality case. It follows that the proposed profile can always be used to get a dispersion relation valid also for $|k_x R| \gg 1$, if the approximation of quasi-neutrality is reasonable.

Let us consider a plasma inhomogeneous in the x-direction, in a magnetic field $\vec{B} = \hat{i}_z B_o$ with B_o= const. We treat the following case of equilibrium distribution functions:

$$f_{oj} = g_j(v_\perp^2, v_\parallel^2)\exp\left[-a(x + v_y/\omega_{cj})\right]; \ (j = e, i; \ \omega_{cj} = e_j B_o/m_j)$$

where $\lim\limits_{v_\perp \to \infty} g(v_\perp^2, v_\parallel^2)\exp(\beta v_\perp) = 0$ for any positive constant β.

We shall consider electrostatic perturbations, although the treatment could be extended to electromagnetic ones. If the initial perturbation decreases faster than $e^{-a|x|}$ for $|x| \to \infty$, the same will be true for the solution φ (electric potential) at all times t. Then

$$\int dx\, e^{i(k_x + ia)x}\varphi(x) = \varphi(k_x + ia)$$

and the integral equation for φ becomes

$$(k_x^2 + k_y^2 + k_z^2)\varphi(k_x) = G(\omega, k_x) + C(\omega, k_x + ia)\varphi(k_x + ia)$$

where G contains the initial conditions and

$$C(\omega, k_x) \equiv i\sum_{e,i}\frac{4\pi e_j^2}{m_j}\int d^3v \int_0^{-\infty} du\, e^{-i\omega u} e^{i\vec{k}\cdot(\vec{r} - \vec{R})} e^{-a v_y/\omega_{cj}}\left(\vec{k}\cdot\vec{\nabla}_v g_j - \frac{k_y a}{\omega_{cj}} g_j\right)$$

It can be shown that $\varphi(k_x)$ has poles in the complex k_x-plane at $k_x = k_o(\omega) + ina$ $(n = 0, 1, 2, ..)$ where k_o is a solution of

$$C(\omega, k_x; k_y, k_z) = 0$$

with $\mathrm{Im}(k_x) > 0$ when ω lies in its convergence half-plane, which is the same as in the "quasi-neutral" approximation. The above condition is necessary and sufficient for the singularities.

As an example we give here the stability condition for the low-frequency drift instability for an isotropic Maxwell distribution g(and $T_i = T_e$). In the range $v_{ith} < \left|\frac{\omega}{k_\parallel}\right| \ll v_{eth}$ wich is the most unstable, and in the limit $1 < k_\perp R_i < (m_i/m_e)^{1/2}$ where the local approximation is expected not to be valid, one gets the following results:

$$\lambda_D/\lambda - 1 \simeq -2k_\perp R_i\sqrt{\pi} \quad ; \lambda = \frac{\omega}{k_\parallel v_{ith}}, \ \lambda_D = \frac{k_y a R_i^2 \omega_{ci}}{k_\parallel v_{ith}}$$

$$-2a k_x/k_\perp^2 - \left(\pi\frac{m_e}{m_i}\right)^{1/2}(\lambda_D + \lambda) > 0 \quad \text{for stability}$$

or:

$$|k_x/k_\perp| > \frac{1}{4}\left(\pi\frac{m_e}{m_i}\right)^{1/2} k_\perp R_i \ |k_y/k_\parallel|$$

with the restriction $1 < (4\pi)^{-1}\left|\frac{a}{k_\parallel}\ \frac{k_y}{k_\perp}\right| \ll \left(\frac{m_i}{m_e}\right)^{1/2}$

One sees clearly the stabilizing effect of k_x.

In the limit $|k_\perp R_i| < 1$, one obtains the known results (exept for corrections of the order of $|a/k_x|$) of the local approximation.

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