STELLARATOR EQUILIBRIUM BY LOW-BETA-EXPANSION

J. Kisslinger, H. Wobig

Max-Planck-Institut fuer Plasmaphysik
EURATOM ASSOCIATION, D-8046 Garching

Abstract

A numerical code is developed which calculates a Stellarator equilibrium following the low beta expansion as proposed by L. Spitzer /1/. In every iteration step the plasma currents are calculated and the resulting magnetic field is obtained from Biot-Savart's law. Since no boundary conditions are imposed, the method describes the free boundary equilibrium. Results are given for Wendelstein VII-A .

Low-beta expansion

In the ideal MHD-model of toroidal plasma equilibrium the scalar pressure p and the magnetic field $\vec{B}$ are calculated from

$$0 = -\nabla p + \vec{j} \times \vec{B} \quad ; \quad \nabla \times \vec{B} = \vec{j} \tag{1}$$

Several codes have been developed to solve these equations in three dimensions/2//3//4/. In all these codes boundary conditions are imposed on the last magnetic surface. In a steady state plasma, however , a conducting wall is only effective on a time scale short compared with the resistive diffusion time. Therefore a conducting wall is only effective for MHD-stability . The AJW-code /5/ calculates the Stellarator equilibrium in axisymmetric approximation, this method includes the free boundary equilibrium.

The iterative process proposed by L. Spitzer tries to solve the system (1) in the following way:

$$\left\{ \begin{array}{c} 0 = -\nabla p + \vec{j}_{n+1} \times \vec{B}_n \\ \nabla \cdot \vec{j}_{n+1} = 0 \qquad \nabla \times \vec{B}_{n+1} = \vec{j}_{n+1} \qquad \nabla \cdot \vec{B}_{n+1} = 0 \end{array} \right\} \tag{2}$$
$$n = 1, 2, \ldots$$

there is neither a prove that this iteration process converges nor is the topology of magnetic surfaces preserved. As pointed out by A. Boozer /6/ magnetic islands and field line ergodisation may occur. Nonetheless an attempt is made to calculate $\vec{B}$ from the system (2). The plasma current $\vec{j}_{n+1} = p'(\psi) \quad \nabla\nu \times \nabla\psi$ is calculated on the magnetic surface ψ = const. of the nth-iteration step. We only calculate $\vec{j}_{n+1}$ on magnetic surfaces without islands and ergodisation thus representing a pressure profile with $p' = 0$ in the island region. The stream function ν is calculated from

$$\vec{B}_n \cdot \nabla\nu = 1 \tag{3}$$

The constant of integration is chosen such that the current lines ν = const. are poloidally closed curves, thus representing a net current free Stellarator. In figure 1 this system of equilibrium currents on a magnetic surface of W VII-A is shown. Figure 1 exhibits one period of a magnetic surface, horizontal axis being the toroidal coordinate and the vertical axis being the poloidal coordinate.

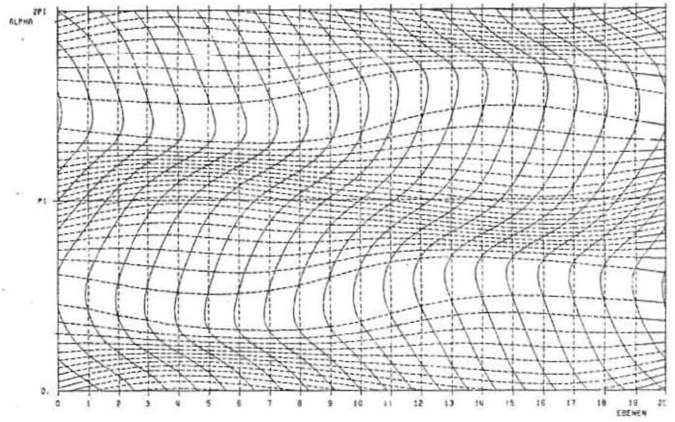

Fig. 1: Plasma currents on a magnetic surface of W VII-A (solid lines).

The typical S-shape of the plasma currents (solid curves) is caused by the Pfirsch-Schlueter currents $j_{\|}$ giving rise to the Shafranov shift. For calculating the magnetic field the continuum of these plasma currents is replaced by a finite set of current filaments $\nu =$ const with all filaments on a magnetic surface carrying the same plasma current. Since the current is proportional to the pressure gradient the radial current distribution is fixed by the pressure profile. Every current filament is discretized into a maximum of 64 straight elements and the field $\vec{B}^*_{n+1}$ at a point $\vec{x}$ added up over all current filaments following Biot - Savart's law. The element crossing $\vec{x}$ is omitted thus avoiding the divergence of $\vec{B}$ at $\vec{x}$. With the help of a relaxation parameter α the next iteration step $\vec{B}_{n+1} = \alpha\,\vec{B}^*_{n+1} + (1-\alpha)\,\vec{B}_n$ is defined. By field line integration the surfaces of $\vec{B}_{n+1}$ are found and the procedure is continued. The points $\vec{x}$ are chosen on the magnetic surfaces of $\vec{B}_n$. If the procedure converges - i.e. if the surfaces of $\vec{B}_n$ and $\vec{B}_{n+1}$ approach each other - the result is independent of the the parameter α.

As a practical measure of convergence the profile of the rotational transform $\iota\!\!-$ is used. In applying the code to W VII-A it was found that after 10 iterations the Shafranov - shift and the shape of the magnetic surfaces saturate. About 20 - 30 iterations are necessary to make the error in $\iota\!\!-$ less than 10^{-3}. The plasma currents lead to a modification of the profile of the rotational transform. In W VII-A the transform increases at the magnetic axis and decreases at the edge thus giving rise to an appreciable amount of shear.

In figure 2a the modification of the transform with increasing β is shown. For comparison figure 2b shows the result of the AJW-code (see also ref. /7/). The maximum shear at $\beta(0) = 1.5\%$ is $\delta\iota/\iota = 20\%$.

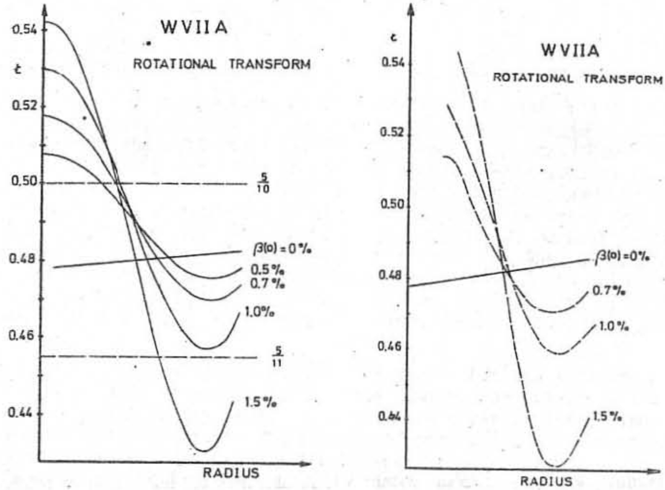

Fig. 2b (right):ι-profile of W VII-A as a function of plasma pressure. 2a(left): ι-profile W VII-A from AJW-code.

In order to improve the convergence the relaxation parameter α can be changed during the iteration. Convergence depends on the β -value chosen. At low β fewer iteration steps suffice, whereas at higher values ($\beta(0) > 1.5\%$ in W VII-A) the iteration process collapses due to island formation. It is not yet clear wether this is an indication of a real β -limit or wether numerical errors determine this limit.

Magnetic surfaces. The magnetic surfaces of W VII-A are shown in figure 3. The peak value of β is 1.5% in this case. As can be seen the Shafranov shift varies with toroidal angle, the maximum shift arises in the plane of the horizontal ellipse. At $\beta = 1.5\%$ ergodisation of magnetic surfaces begins to appear, calculation of plasma currents, however, averages over this fine structure and a further increase does not occur during the iteration process. At $\beta = 2\%$ island formation and ergodisation limit the iterative process and an equilibrium cannot be found. Island formation predominantly arises around $\iota = 5/10, 5/11, 5/12...$ with $10, 11, 12, ...$ islands. In a system with stronger shear and more field periods around the torus a reduction of this effect is expected. As an excample Heliotron E has been investigated. Heliotron E has a strong shear and 19 field periods. Here equilibria with $\beta(0) = 5\%$ can be calculated without serious island formation.

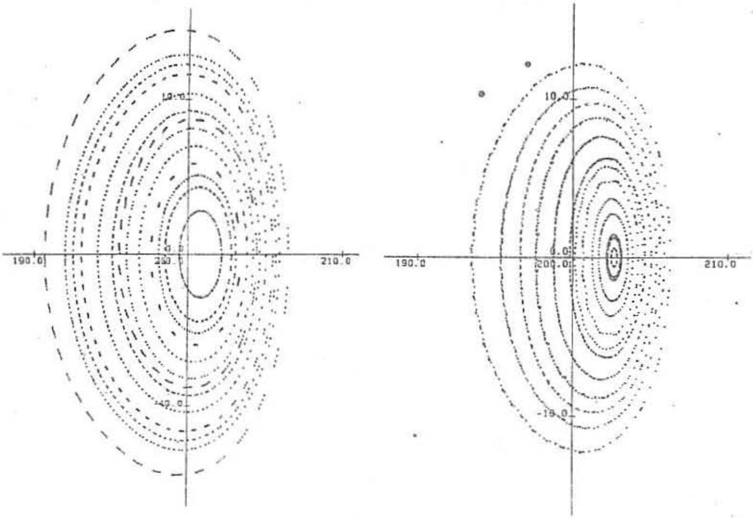

Fig. 3: Magnetic surfaces of Wendelstein VII A at finite β. left: $\beta(0) = 0.5\%$ right: $\beta(0) = 1.5\%$

Conclusions. The low-β iterative process permits to calculate 3-dimensional Stellarator equilibria with a free boundary. At sufficiently low β convergence can be achieved. Depending on the specific configuration (number of field periods, shear) ergodisation and island formation on magnetic surfaces determine the maximum β. So far it cannot be decided whether this break-up of surfaces is due to numerical errors or due to a physical effect. Other effects like rotational transform, magnetic well and Shafranov shift can be calculated with an accuracy sufficient for practical purposes.

References

/1/ L. Spitzer, Phys. Fluids 1958, Vol. 1 p.253 - 264

/2/ R.Chodura, A. Schlueter, J. Comput. Phys. 41 (1981) 68

/3/ F. Bauer ,O.Betancourt, P.Garabedian, A Computational Method in Plasma Physics, Springer Verlag, New York

/4/ T.C.Hender, B.A.Çarreras, L.Garcia, J.A.Rome, V.E.Lynch, ORNL-report ORNL/TM 9239

/5/ G.Anania, J.L.Johnson, K.E.Weimer, Phys. Fluids 26,2210 (1983)

/6/ A. Boozer, Phys. Fluids 27 (8) Aug. 1984

/7/ F.Herrnegger, Proc. of the IAEA Techn. Com. Meeting on Plasma Confinement and Heating in Stellarators. Sept. 1984 EUR 9618 Vol.I p.401