# Comparing linear stability of electrostatic kinetic and gyro-kinetic ITG modes in general tokamak geometry

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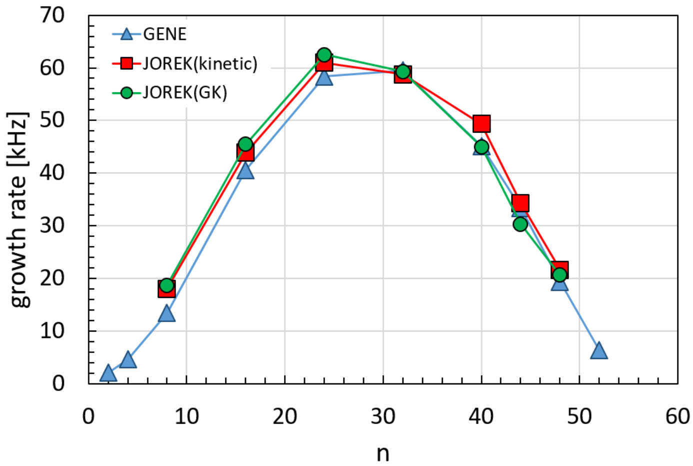
The verification and validation of nonlinear MHD fluid simulations of instabilities in tokamak plasmas requires a visco/resistive MHD model extended with kinetic effects, for example due to drifts and finite Larmor radius effects [1]. Such an extended MHD model will have to be compared (benchmarked) with a first principle electro-magnetic kinetic or gyro-kinetic model. As a first step towards this goal, an electrostatic kinetic and gyro-kinetic model have been implemented in the framework of the nonlinear MHD code JOREK [2]. In its original version, the JOREK code solves the time evolution of the fluid MHD equations in toroidal geometry. In previous years, the JOREK code has been extended with a general particle framework, coupled to the fluid MHD equations. Applications include transport studies of heavy impurities during ELMs [3], modelling of runaway electrons [4], a kinetic neutral model for improved divertor solutions [5], fast particle driven modes [6], and the use of marker particles in shattered pellet injection (SPI) in disruption mitigation simulations [7].
The new full-f (gyro) kinetic models use the existing JOREK particle implementation. The full orbit of the kinetic ions are advanced in time using the classical Boris method, whereas RK4 is used for the gyro-centre (or guiding centre) of gyro-kinetic particles. In this paper, electrons are treated as adiabatic (although guiding-centre electrons have also been implemented [8]). The electric potential is obtained from the solution of the Poisson equation for quasi-neutrality. In the gyro-kinetic model the Poisson equation includes the long wavelength form of the ion-polarization density, in the kinetic model this term does not appear. The electric potential is discretised with cubic $C^1$ Bezier finite elements in the poloidal plane and a Fourier series in the toroidal direction, guaranteeing a continuous electric field. The Poisson equation is similar in structure to the (existing) projection operations to calculate the particle moments [3]. The projection operations include filtering terms to reduce the particle noise. Two types of filters are used, hyper-diffusion in the poloidal plane and a Laplacian in the parallel direction. The time evolution uses a straightforward explicit scheme with a time-advance of the particles, followed by a solution of the Poisson equation to update the electric field. The kinetic and gyro-kinetic particles modules have been implemented on GPUs (on Marconi100) with the particles living on the GPU while the Poisson equation is solved on CPUs.

**Benchmark linear ITG in circular geometry**

Both the kinetic and gyro-kinetic models have been successfully benchmarked on the GENE/XGC case [9] for linear growth rates of ITG modes. This well-defined benchmark uses a circular plasma shape, with $R_0$=1.68 m, a=0.59 m, B=2.09 T, T=2.25 keV and q(r/a)=1.37. The normalised length scales of the temperature and density are $R_0/L_T$=6.9 and $R_0/L_n$=2.2. Figure 1 shows the linear growth rates of the ITG modes as a function of the toroidal mode number, for both the kinetic and the gyro-kinetic models, compared with the results from the GENE code [9]. For this case, $10^9$ particles have been used in a poloidal grid of 101x256 cubic finite elements. Typical time steps are $\delta t$=5x10$^{-7}$ s for the gyro-kinetic ions and $\delta t$=5x10$^{-9}$ s for the kinetic ions. One difficulty with the use of a full-f model is that the initial distribution function as prescribed by the benchmark cannot as easily be kept constant in time, compared to $\delta$f methods. A stationary distribution function is constructed with the method described in [10].



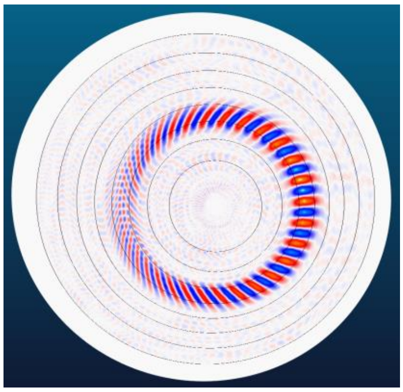



*Fig. 1 Linear kinetic and gyro-kinetic ITG growth rates from JOREK as a function of toroidal mode number compared with GENE results (left). The electric potential of the n=24 kinetic ITG mode (right).*

Good agreement is found between the kinetic and gyro-kinetic models and with the published GENE/XGC/ORB5 results. Thus, in this circular plasma case with $\rho^{\star} = 1/180, \rho_i = 5.5 \times 10^{-3} m$ and $L_\theta = 2\pi/k_\theta = 2\pi r/nq = 1/38\, m$ (for n=50) there is no significant difference between the full-f 6D kinetic and 5D gyro-kinetic models (for their linear growth rates).

**Comparison of kinetic and gyro-kinetic ITG linear growth rates in circular geometry**

To identify a regime where the kinetic and gyro-kinetic models may give different results, the benchmark case was modified with an increase of the temperature scale length to $R_0/L_T$=20 (at constant density scale length) to destabilize higher-n modes with smaller poloidal structures. The magnetic field and central temperature were scaled with the same scaling factor to vary $\rho^{\star}$, ($1/\rho^{\star}$=180, 114 and 80). Fig. 2 shows the ITG growth rates as a function of the toroidal mode number for the scaled equilibria. In this case, a locally refined grid of 151x512 cubic elements was used to resolve the small scales at high toroidal mode numbers.

Significant differences between the kinetic and gyro-kinetic linear ITG growth rates are found starting from $\rho_i > L_\theta/4$ or $k_\theta \rho_i > \pi/2$ , i.e. when the width of the Larmor orbit $2\rho_i$ is larger than half a period of the poloidal mode structure. (With $\rho_i$ and $L_\theta$ measured locally in the outer mid-plane at the maximum of mode amplitude.) The marginal stability of gyro-kinetic model has a strong (i.e. linear) dependence on $\rho^{\star}$. For this case, the marginally stable point can be expressed as $L_\theta = 2\rho_i$ or $k_\theta \rho_i \sim \pi$, i.e. when the ion orbit size equals one poloidal period of the mode structure. On the contrary, the marginal stability limit in the kinetic model varies only weakly with $\rho^{\star}$. I.e., in the kinetic model high-n modes with poloidal mode structures smaller than $\rho^{\star}$ are significantly more unstable compared to the long-wavelength gyro-kinetic model.

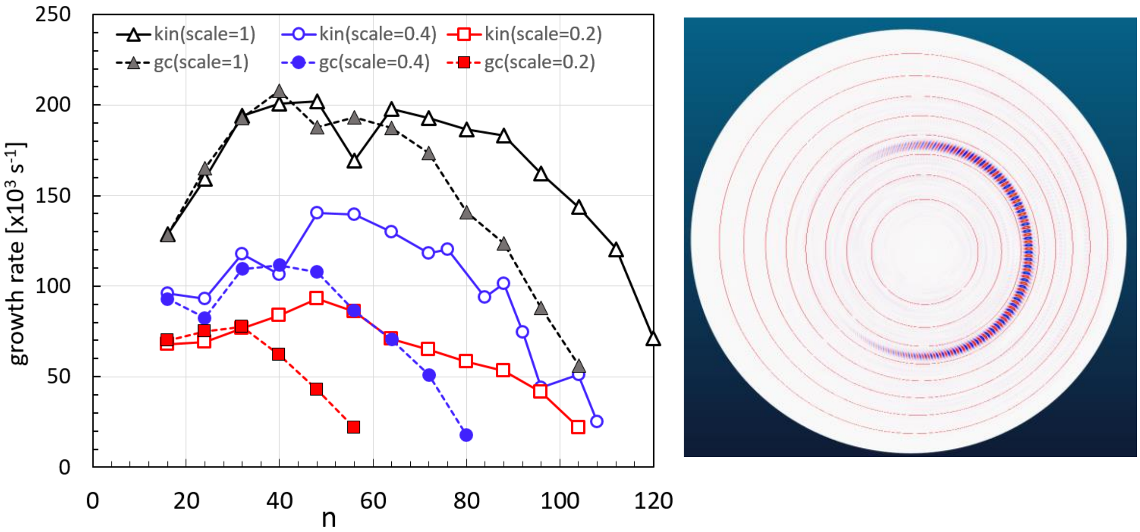



*Fig.2 Comparison of kinetic and gyro-kinetic ITG growth rates as a function of the toroidal mode number for $1/\rho^* = 180$, 114 and 80 (left). The electric potential of the n=88 kinetic ITG mode (right).*

**Padé Approximation**

One well-known method to improve on the long wavelength form of the quasi-neutrality equation is to use a Padé approximation to represent the ion polarisation density [11]. The quasi-neutrality equation becomes:

$$-\nabla\cdot\left(\frac{q_i n_0}{T_i}\rho_i^2\nabla_\perp\Phi\right)-\left(1-\nabla\cdot\rho_i^2\nabla_\perp\right)\frac{en_0}{T_e}\left(\Phi-\overline{\Phi}\right)=\left(1-\nabla\cdot\rho_i^2\nabla_\perp\right)\left(n_{i,gc}-n_{e0}(\psi)\right)$$

This arbitrary wavelength form has been implemented in the weak form in JOREK.

Fig. 3 shows the growth rates of the ITG modes using the Padé form compared to the kinetic and long-wavelength gyro-kinetic results. Up to $k_\theta\rho_i \sim 1.2$ the solutions are in good agreement. For larger values of $k_\theta\rho_i$ the Padé solution is in better agreement with the kinetic solution, giving the same marginal stability limit. Note that in [13] very good agreement was found between the Padé and kinetic solutions, in slab geometry. One disadvantage of the Padé form is an increased level of particle noise due to the second order derivatives on the particle density.

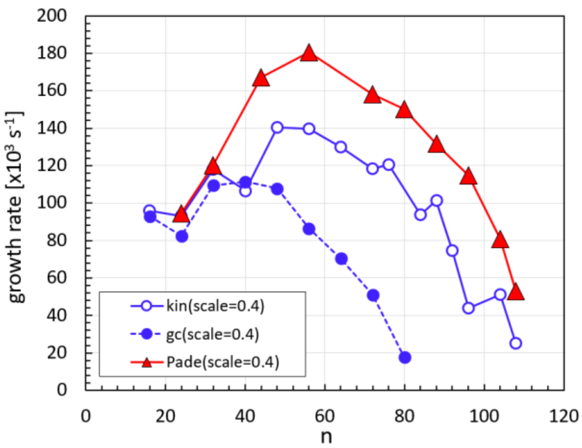
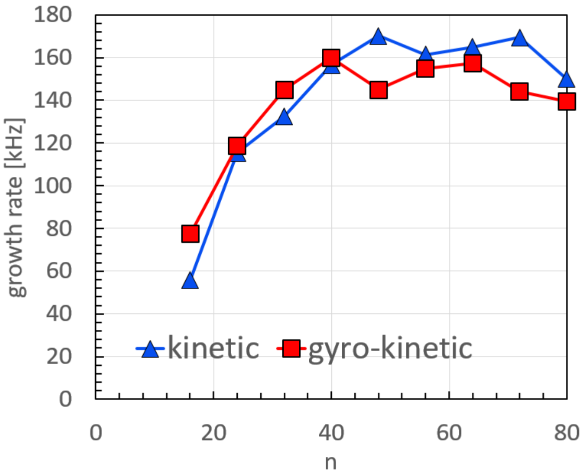



*Fig.3 ITG growth rates versus toroidal mode number, ($1/\rho^* = 114$), of the kinetic, Padé and long wavelength gyro-kinetic solutions.*

**X-point geometry**

The kinetic and gyro-kinetic models in JOREK can be used in any of the available finite element grids, including x-points and open field lines. The boundary condition sets the electric potential to zero, here also at the end of the open field lines. As a first example, the linear stability of kinetic and gyro-kinetic ITG modes in the H-mode pedestal are compared in a COMPASS-like H-mode equilibrium [12] with: $R_0 = 0.54m, a = 0.16m, B_0 = 1.2T, q_{95} = 2.55, n_0 = 8\times 10^{19}m^{-3}$, $P_{ped} = 2.5\ kPa$. (All equilibrium flows are set to zero.) The pedestal width is 1.0-1.5cm, about $4\rho_i$ . With these pedestal parameters ITG modes are stable or close to marginal stability, stabilised by the large density gradient. For this study the density pedestal in the simulations were widened to obtain unstable ITG modes in the pedestal. Fig 4 shows the mode structure of the potential of an n=64 kinetic ITG mode. The radial width of the mode fills the pedestal, with a half-width at half maximum of 7 mm. In the midplane, the poloidal width of one period in the midplane is $L_\theta \sim 2cm$. However near the x-point, where the poloidal field goes to zero, the poloidal mode period becomes much smaller with $L_\theta \sim 1.5\rho_i$ $(k_\theta \rho_i \sim 4)$ i.e. in the regime where, in principle, a significant difference can be expected between kinetic and long wavelength gyrokinetic solutions. A comparison of the linear growth rates (see Fig 4) shows however no significant difference in the growth rates. For higher toroidal mode numbers, the modes become more localised poloidally in the outer mid-plane reducing the effect of the x-point region. At the poloidal end of the mode, the scale length $L_\theta \sim 2$ to $2.5\rho_i$ independent of n.



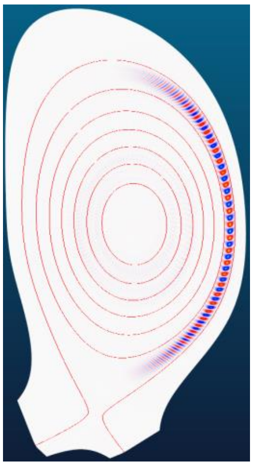



*Fig.4 Comparison kinetic (blue) and gyro-kinetic (red) ITG growth rates in H-mode pedestal as a function of the toroidal mode number (left), mode structure of an n=64 kinetic ITG mode in Compass-like geometry (right).*

## Conclusion

The particle module in the JOREK code has been extended to include a full-f kinetic and gyro-kinetic model. Significant differences have been found between kinetic and long-wavelength gyro-kinetic solutions at high toroidal mode numbers in circular geometry with $k_\theta \rho_i > \pi/2$. The use of the Padé approximation significantly improves on the long-wavelength gyro-kinetic model. In Compass-like x-point geometry in the H-mode pedestal no significant difference in the linear growth rates of the kinetic and gyro-kinetic solutions has been found.

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